

# UAV trajectory tracking based on ADRC control algorithm

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**Abstract.** UAV flying in the air is affected by external airflow factors, and the system has the characteristics of underdrive, nonlinear and uncertainty, which makes the flight state of the UAV become complicated. The trajectory tracking of UAV is studied by using Active disturbance rejection control (ADRC) algorithm. ADRC control algorithms include tracking differentiator, extended state observer and error feedback control law. The active disturbance rejection and decoupling control of UAV track tracking is studied, The active disturbance rejection decoupling control introduces static decoupling matrix B to decouple the virtual control law U. The dynamically coupled disturbances are regarded as total disturbances, which are observed in real time by ESO of each main channel and compensated by control laws. Finally, the 2d trajectory tracking of UAV is realized.

**Keywords:** UAV control, ADRC, Total disturbance, Decoupling control.

## 1 Introduction

In the past 50 years, the vigorous development of control theory has provided a strong theoretical support for the research of UAV flight control. The main input control equipment of general UAV is rudder and engine. The rudder uses the steering gear to provide rudder force to maintain or change the direction of UAV movement, and the engine controls the forward speed of UAV through the main engine and transmission equipment. In order to make the UAV fly along the pre-set route as accurately as possible, the purpose of UAV trajectory tracking is to compensate the horizontal and vertical position errors of the UAV to zero, so as to achieve the ideal navigation state. The research on UAV trajectory tracking has very important practical and theoretical significance. Accurate trajectory tracking is particularly important for UAV formation flying in close range and flight guidance.

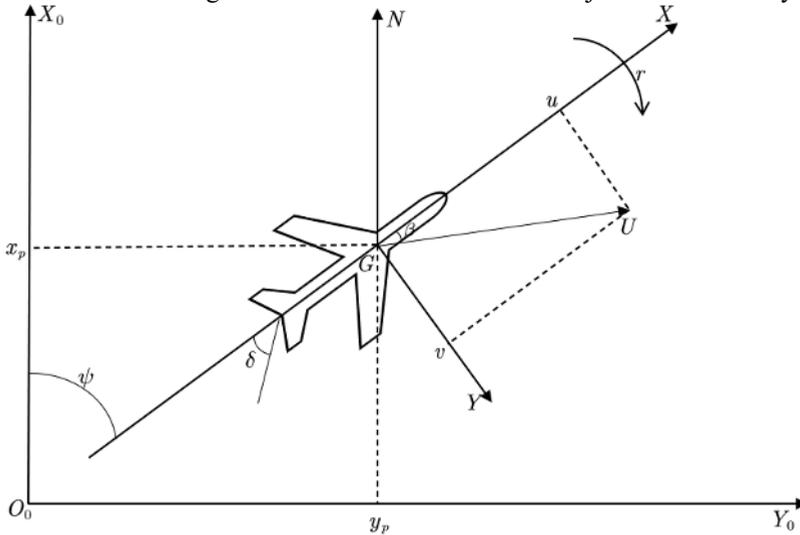
## 2 ADRC control algorithm

In practical engineering control problems, the internal model/mechanism of the controlled

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object is not easy to obtain, and at the same time, due to the complexity of external interference, the established controlled object model is often obtained through simplification. ADRC has good control effect for uncertain object model or fuzzy system.

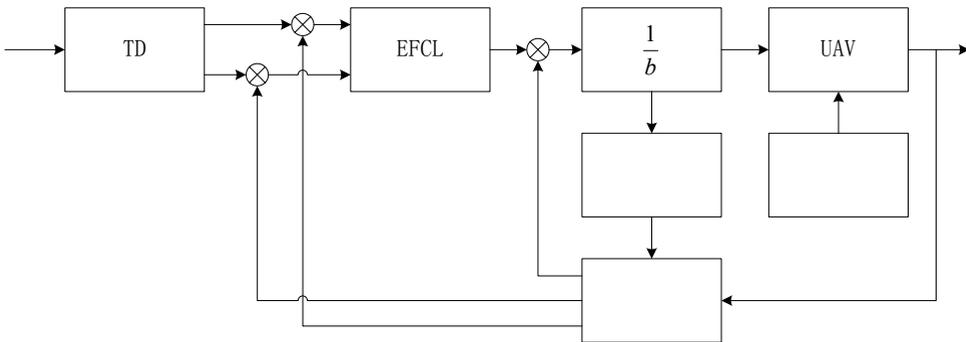


**Fig. 1.** Diagram of UAV track tracking.

Second order system differential equation:

$$\ddot{y} = f + bu \tag{1}$$

The ADRC control block diagram is shown below:



**Fig. 2.** ADRC control block diagram.

### 2.1 Trace differentiator

The main function of the tracking differentiator (TD) is to extract the continuous signal from the non-continuous signal or the signal with noise and differentiate the signal, which can realize the fast tracking of the reference input without overshoot. The robustness of the controller can be improved by placing a tracking differentiator in the feedforward channel of the control system. The equation of a nonlinear tracking differentiator with discrete structure given by Han Jingqing<sup>[1]</sup> is as follows:

$$\begin{cases} x_1(k+1) = x_1(k) + h_0 x_2(k) \\ x_2(k+1) = x_2(k) + h_0 \text{fst}(x_1(k) - v(k), x_2(k), r_0, h) \end{cases} \quad (2)$$

Where,  $h_0$  is the sampling step;  $h$  is the filtering factor;  $v(k)$  is the input signal at time  $K$ ;  $r_0$  affects the speed of tracking, which is called "speed factor".  $\text{fst}(\cdot)$  is the fastest synthesis function in Han Jingqing<sup>[1]</sup>, and the expression is:

$$\begin{cases} d = r_0 h \\ d_0 = h d \\ y = x_1 + h x_2 \\ a_0 = \sqrt{d^2 + 8 r_0 |y|} \\ a = \begin{cases} x_2 + \frac{(a_0 - d)}{2} \text{sign}(y), (|y| > d_0) \\ x_2 + \frac{y}{h}, (|y| \leq d_0) \end{cases} \\ \text{fst} = - \begin{cases} r_0 \text{sign}(a), (|a| > d) \\ r_0 \frac{a}{d}, (|a| \leq d) \end{cases} \end{cases} \quad (3)$$

The TD expression of Equation (2) can be realized  $r_1 \rightarrow r$ ,  $r_2 \rightarrow \dot{r}$ . If the input signal is noisy, TD can also achieve the function of filtering.

## 2.2 Extended state observer

Extended state observer (ESO) plays an important role in ADRC control. ESO regards the internal uncertainty and external disturbance of the system as the total disturbance, and then references the input and output of the controlled object as the input quantity of ESO, and then uses ESO to observe the internal state of the controlled object and the total disturbance it receives.

In order to facilitate computer implementation, Liu Jinkun<sup>[2]</sup> gave the function form of nonlinear discrete ESO:

$$\begin{cases} e_1(k) = z_1(k) - y(k) \\ z_1(k+1) = z_1(k) + h_0(z_2(k) - \beta_1 e_1(k)) \\ z_2(k+1) = z_2(k) + h_0(z_3(k) - \beta_2 \text{fal}(e, 0.5, \delta)) \\ z_3(k+1) = z_3(k) - h_0 \beta_3 \text{fal}(e, 0.25, \delta) \end{cases} \quad (4)$$

Where,  $\beta_i (i=1,2,3)$  is the observer gain parameter, and  $\text{fal}$  is the nonlinear saturation function, which is defined as follows:

$$fal(e, \alpha, \delta) = \begin{cases} \frac{e}{\delta^{1-\alpha}}, & |e| \leq \delta \\ |e|^\alpha \operatorname{sgn}(e), & |e| > \delta \end{cases} \quad (5)$$

Where  $\delta$  is the constant affecting the filtering effect. After the observation of ESO in Equation (4) can be realized  $z_1 \rightarrow y, z_2 \rightarrow \dot{y}, z_3 \rightarrow f$ .

### 2.3 Error feedback control law

The input and output of the control quantity of the controlled object are estimated by ESO observation. The expanded state  $x_3$  of the controlled object(The total disturbance  $f$ ) is tracked by the  $z_3$  of the expanded state observer. The effect of  $z_3$  is offset by the control law in Equation (6), that is, the original controlled object model is transformed into a series form of double integrators. Due to the setting of parameters such as the initial value of the state quantity, ESO also has some estimation errors when estimating the state quantity.

$$\begin{cases} u_0 = K_p fal(e_1, \alpha_1, \delta_1) + K_d fal(e_2, \alpha_2, \delta_2) \\ K_p = \omega_c^2, \quad K_d = -2\omega_c \\ u = (u_0 - z_3) / b \end{cases} \quad (6)$$

In Formula (6),  $\omega_c$  is the controller bandwidth constant, and there are many ways to get  $K_p$  and  $K_d$ ,  $fal$  is defined in the same formula (5).

### 2.4 ADRC decoupling control for multivariable systems

Decoupling of multivariable systems is always a difficult problem in the control field. With the development of modern control theory, many control algorithms are based on system model to achieve decoupling, and need a lot of calculation. ADRC decoupling control is an effective method to deal with coupled systems. The basic principle of ADRC is to regard the coupling parts of each subsystem as the external disturbance of the system and compensate them as a part of the total disturbance.

The schematic diagram of ADRC decoupling control is shown below:

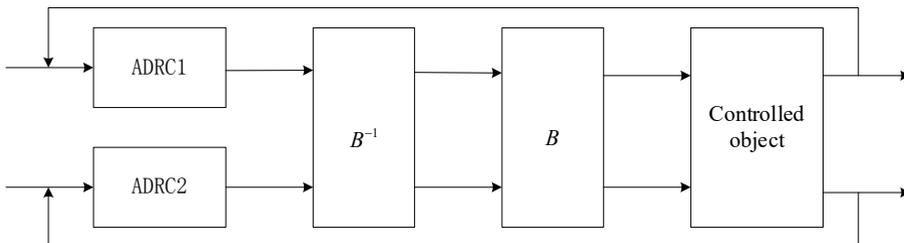


Fig. 3. Diagram of ADRC decoupling control.

Assume a dual input-dual output system:

$$\begin{cases} \ddot{x}_1 = f_1(x_1, \dot{x}_1, x_2, \dot{x}_2) + b_{11}u_1 + b_{12}u_2 \\ \ddot{x}_2 = f_2(x_1, \dot{x}_1, x_2, \dot{x}_2) + b_{21}u_1 + b_{22}u_2 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases} \quad (7)$$

Where,  $b_{ij}$  is the magnification coefficient of the control quantity, and is a function of time and state variables. ADRC has its unique decoupling characteristics and is not dependent on the specific controlled object model, and the computation amount is greatly reduced compared with conventional control algorithms. Active disturbance rejection control divides the coupling into "static coupling" and "dynamic coupling".  $b_{11}u_1 + b_{12}u_2$  and  $b_{21}u_1 + b_{22}u_2$  are regarded as the static coupling part of the system, and  $f(x_1, x_2) = [f_1 \ f_2]^T$  is regarded as the dynamic coupling part of the system. Set:

$$f = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (8)$$

The "virtual control quantity"  $U = B \cdot u$  is introduced, thus the equation of state of Equation (8) becomes:

$$\begin{cases} \ddot{x} = f_i + U_i \\ y_i = x_i \end{cases} \quad (9)$$

Han Jingqing gives the 2 order ADRC algorithm for the first channel of dual-input-dual-output system as:

$$\begin{cases} v_{11} = v_{11} + hv_{12} \\ v_{12} = v_{12} + h_0fst(v_{11} - v_1, v_{12}, r_0, h) \\ e = z_{11} - y_1 \\ z_{11} = z_{11} + h_0(z_{12} - \beta_1 e) \\ z_{12} = z_{12} + h_0(z_{13} - \beta_2 fal(e, 0.5, h) + U_1) \\ z_{13} = z_{13} + h(-\beta_3 fal(e, 0.25, h)) \\ e_1 = v_{11} - z_{11}, e_2 = v_{12} - z_{12} \\ U_1 = -fst(e_1, e_2, r, h_1) - z_{13} \end{cases} \quad (10)$$

Han Jingqing gives the 2 order ADRC algorithm for the second channel of dual-input-dual-output system as:

$$\begin{cases} v_{21} = v_{21} + hv_{22} \\ v_{22} = v_{22} + h_0fst(v_{21} - v_2, v_{22}, r_0, h) \\ e = z_{21} - y_2 \\ z_{21} = z_{21} + h_0(z_{22} - \beta_1 e) \\ z_{22} = z_{22} + h_0(z_{23} - \beta_2 fal(e, 0.5, h) + U_2) \\ z_{23} = z_{23} + h(-\beta_3 fal(e, 0.25, h)) \\ e_1 = v_{21} - z_{21}, e_2 = v_{22} - z_{22} \\ U_2 = -fst(e_1, e_2, r, h_1) - z_{23} \end{cases} \quad (11)$$

Using Equations (10) and (11), combined with figure 3, the simulation diagram is set up as follows:

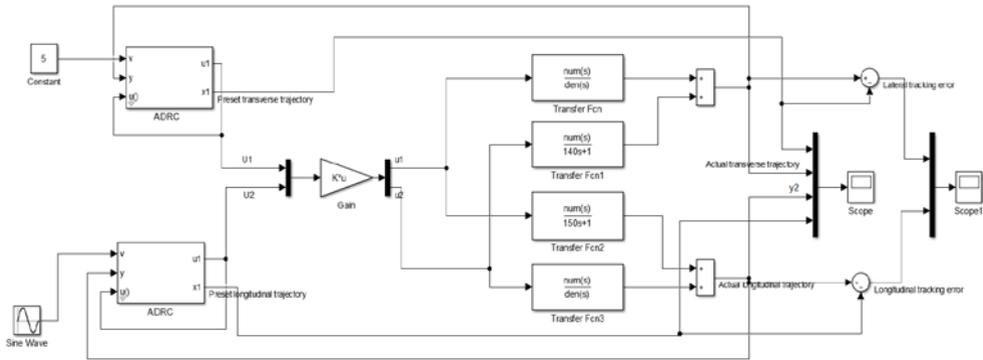


Fig. 4. ADRC decoupled control simulink module.

### 3 UAV trajectory tracking control

Considering the cross influence (coupling) between UAV lateral control and longitudinal control, the UAV trajectory tracking control system can be transformed into a dual-input-dual-output coupling (2X2) control system.

$$\begin{cases} \ddot{x} = f_1 + b_{11}\delta_r + b_{12}n \\ \ddot{y} = f_2 + b_{21}\delta_r + b_{22}n \end{cases} \quad (12)$$

For dual-input-dual-output UAV system, its transfer function is:

$$G(s) = \frac{Y(s)}{U(s)} = \begin{bmatrix} \frac{K_{11}}{D_{11}(s)} & \frac{K_{12}}{D_{12}(s)} \\ \frac{K_{21}}{D_{21}(s)} & \frac{K_{22}}{D_{22}(s)} \end{bmatrix} \quad (13)$$

The  $K$  value in Equation (13) is the open loop gain, so the static coupling matrix can be:

$$B = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \quad (14)$$

ADRC decoupling control for multivariable systems

### 3.1 One order UAV system coupling simulation

It is assumed that the two main channels of the UAV trajectory tracking control system are first-order forms, as follows:

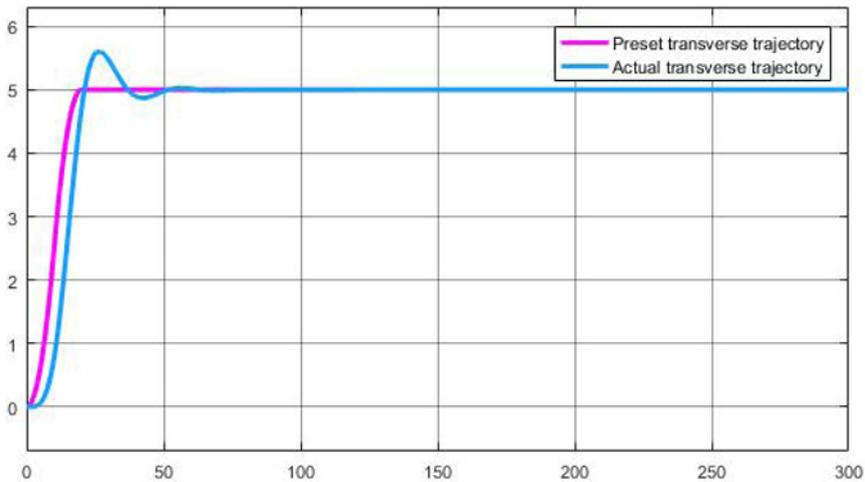
$$G(s) = \begin{bmatrix} \frac{0.7}{140s+1} & \frac{0.2}{(90s+1)(50s+1)} \\ \frac{0.3}{(80s+1)(50s+1)} & \frac{0.8}{(135s+1)} \end{bmatrix} \quad (15)$$

According to the open loop gain in Equation (15), the static coupling matrix and its inverse matrix of the coupled system are:

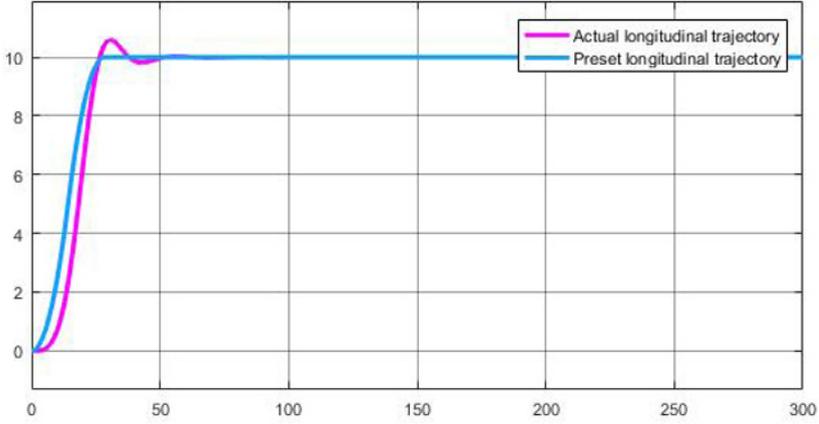
$$B = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1.6 & -0.4 \\ -0.6 & 1.4 \end{bmatrix} \quad (16)$$

Assume that the initial position of UAV  $x=y=0$ , the expected trajectory of UAV  $x_d=5, y_d=10$ , and design ADRC decoupling controller according to Equation (10,11). The controller parameters are (ADRC1 and ADRC2 have the same parameters):

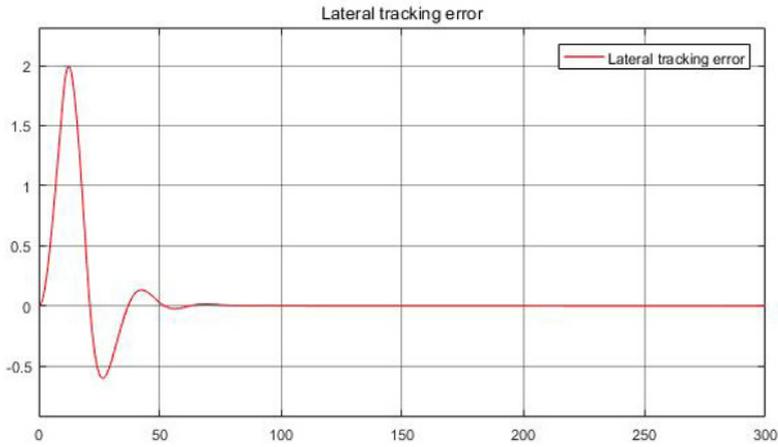
$$r_0=0.05 \quad h=0.1 \quad \beta_1=10 \quad \beta_2=400 \quad r=0.3$$



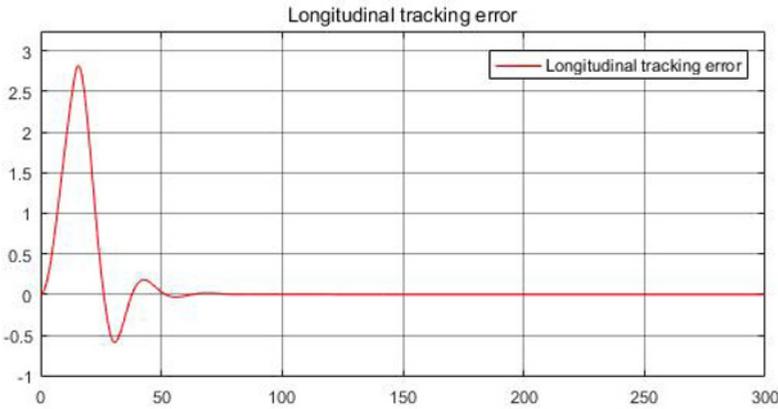
a) Lateral trajectory tracking.



b) Longitudinal trajectory tracking.



c) Lateral tracking error

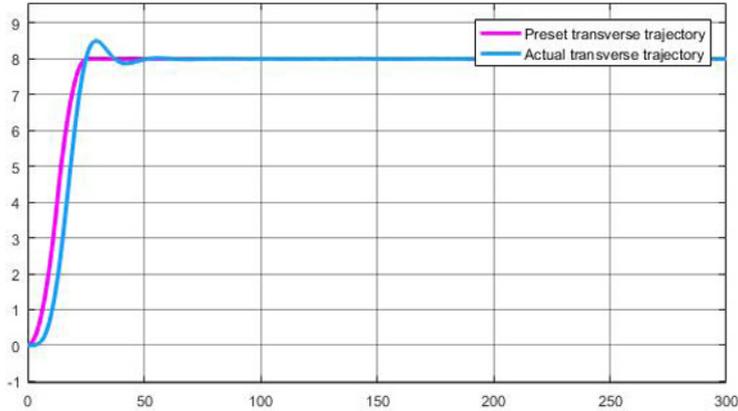


d) Longitudinal trajectory tracking

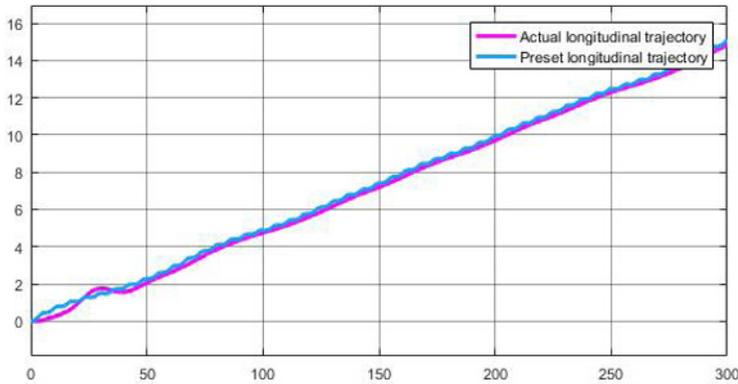
Fig. 5. First-order system for UAV trajectory tracking(one).

Assume that the initial position of UAV  $x=y=0$ , the expected trajectory of UAV  $x_d=8, y_d=0.05t+4\sin(t)$ , and design ADRC decoupling controller according to Equation (10,11). The controller parameters are (ADRC1 and ADRC2 have the same parameters):

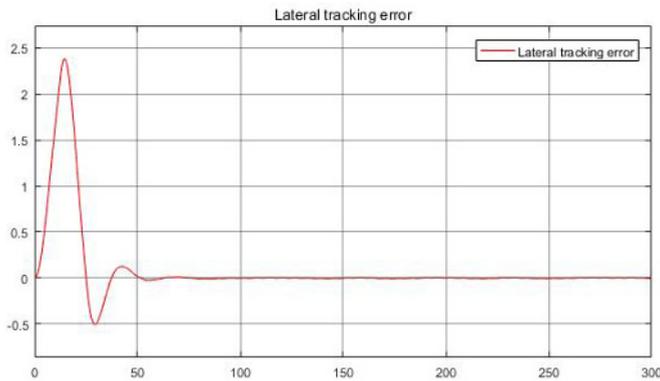
$$r_0=0.05 \quad h=0.1 \quad \beta_1=10 \quad \beta_2=400 \quad r=0.3$$



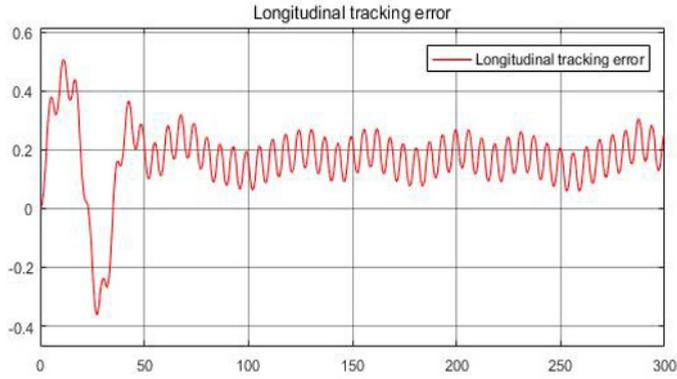
a) Lateral trajectory tracking



b) Longitudinal trajectory tracking



c) Lateral tracking error



d) Longitudinal trajectory tracking

Fig. 6. First-order system for UAV trajectory tracking(two).

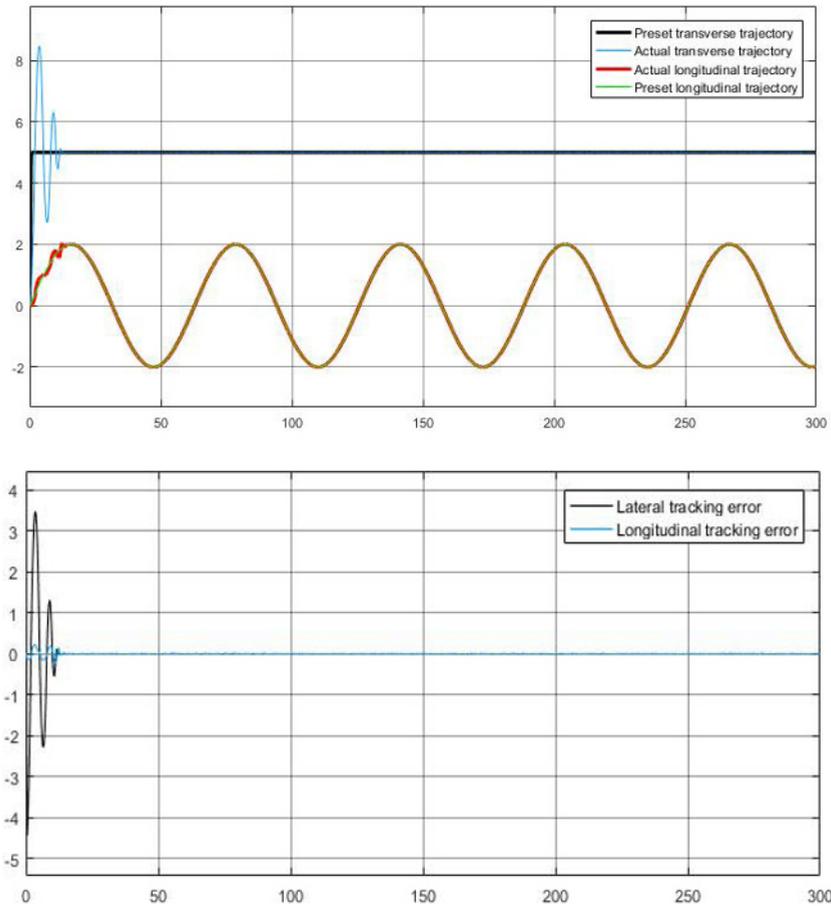


Fig. 7. Second-order system for UAV trajectory tracking.

The first-order ADRC decoupling control has remarkable effect on dual input-dual output UAV trajectory tracking control.

### 3.2 Two order UAV system coupling simulation

For example, the transfer function of UAV flight can be obtained through system identification:

$$G(s) = \begin{bmatrix} \frac{0.478}{s(216s+1)} & \frac{0.6}{140s+1} \\ \frac{0.9}{150s+1} & \frac{0.2}{(80s+1)(60s+1)} \end{bmatrix} \quad (17)$$

According to the open loop gain in Equation (17), the static coupling matrix and its inverse matrix of the coupled system are:

$$B = \begin{bmatrix} 0.478 & 0.6 \\ 0.9 & 0.2 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} -0.45 & 1.35 \\ 2.03 & -1.08 \end{bmatrix} \quad (18)$$

Assume that the initial position of UAV  $x=y=0$ , the expected trajectory of UAV  $x_d=5, y_d=2\sin(0.1t)$ , and design ADRC decoupling controller according to Equation (10,11). The controller parameters are (ADRC1 and ADRC2 have the same parameters):

$$h_0=0.01 \quad r_0=50 \quad h=0.01 \quad \beta_1=20 \quad \beta_2=300 \quad \beta_3=1000 \quad \delta=0.02 \quad b=1.5$$

The simulation results show that it basically meets the actual requirements.

## 4 Summary

The first-order active disturbance rejection and decoupling control has remarkable effect on dual input-dual output UAV trajectory tracking control, and the calculation amount is reasonable. Moreover, the tracking effect of step trajectory signal is good, and the errors of nonlinear trajectory tracking are all within a reasonable range.

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