Dual-station direction finding location based on disturbance detection

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Abstract. The application of direction finding location technology in electronic warfare is becoming more and more extensive. Aiming at the problem that the traditional location method will greatly affect the location accuracy when the measurement information of each observation station is disturbed, a dual-station direction finding location algorithm based on disturbance detection is proposed. The algorithm uses the least squares method to obtain the target's position information at each moment, then designs an improved Kalman filter algorithm with disturbance detection to improve the location accuracy. The simulation results show that the algorithm can still accurately track the target when the measurement information is disturbed, which has practical application value.

Keywords: Direction finding location, The least squares method, Kalman filter, Disturbance detection.

1 Introduction

The status of electronic warfare in modern warfare is constantly improving, and precise target location is of great significance for grasping the war situation[1]. Under the trend of more and more emphasis on concealment in target location, the passive location technology has been continuously improved. It has the advantages of strong concealment and is not easy to reveal itself compared with active location[2,3]. Direction finding location is a passive location technology widely used in military systems. It uses more than two observation stations to obtain the orientation information of the target. Each observation station obtains a location line, and the intersection of each location line is the target location[4]. This paper mainly studies the dual-station direction finding location.

The earliest direction finding location method uses angle information to obtain the position, but its location error variance is large and can only measure the target position[5]. Taking the measurement of the target angle as the input of the unscented Kalman filter can achieve target tracking, but the algorithm design is complicated to solve the nonlinear problem[6]. After modeling and simulating passive positioning, various factors affecting location accuracy can be analyzed[7,8]. Using the time difference of arrival-angle of arrival
joint positioning in the three-dimensional coordinate system improves the location accuracy\cite{9}. Consider the optimal station problem in a two-dimensional coordinate system to improve direction finding accuracy\cite{10}. However, there is a lack of research on localization methods when the measurement information is disturbed.

This paper proposes a dual-station direction finding location method based on disturbance detection based on the above problems. The improved Kalman filter algorithm with disturbance detection reduces the positioning error. The algorithm is easy to implement and increases the robustness. The target can still be accurately tracked in the case of disturbance.

## 2 Dual-station direction finding location based on disturbance detection

### 2.1 Least squares estimation

When two observation stations realize the co-location of the target, the sensor on each observation station can measure angle information. A positioning line can be determined according to the position and angle of the observation station. The intersection of the two location lines is the position of the target. There is an error in the angle measurement information of the target, so the two location lines do not necessarily intersect. Least squares estimation finds the point where the sum of the squares of the distances between the two location lines is the smallest, and takes it as the estimated position of the target\cite{11}.

The location distribution of the set targets and observation stations is shown in Fig. 1. The coordinates of the observation station $A$ and the observation $B$ station are $X_A = [x_1 \ y_1 \ z_1]^T$, $X_B = [x_2 \ y_2 \ z_2]^T$. The coordinates of the target $T$ are $X_T = [x \ y \ z]^T$. $l_1$ and $l_2$ are two location lines. The direction cosines of the target $T$ measured by the observation station $A$ and the observation $B$ are $[\alpha_1 \ \beta_1 \ \gamma_1]^T$, $[\alpha_2 \ \beta_2 \ \gamma_2]^T$.

![Fig. 1. Schematic diagram of the principle of direction finding location.](image)

Then the equations of $l_1$ and $l_2$ are expressed as:

$$
\begin{align*}
   l_1: \ & \frac{x-x_1}{\alpha_1} = \frac{y-y_1}{\beta_1} = \frac{z-z_1}{\gamma_1} \\
   l_2: \ & \frac{x-x_2}{\alpha_2} = \frac{y-y_2}{\beta_2} = \frac{z-z_2}{\gamma_2}.
\end{align*}
$$

(1)
The sum of squares of the distances from the target to $l_1$ and $l_2$ is:

$$f(x, y, z) = d_1^2(x, y, z) + d_2^2(x, y, z),$$ (2)

where $d_1$ and $d_2$ are the distances from a point in space to $l_1$ and $l_2$ respectively.

According to the distance formula from point to line, we have:

$$\tilde{d}_1 = \begin{bmatrix} 0 & -\gamma_1 & \beta_1 \\ \gamma_1 & 0 & -\alpha_1 \\ -\beta_1 & \alpha_1 & 0 \end{bmatrix} \begin{bmatrix} X_A - X \end{bmatrix} = V \begin{bmatrix} X_A - X \end{bmatrix}$$

$$\tilde{d}_2 = \begin{bmatrix} 0 & -\gamma_2 & \beta_2 \\ \gamma_2 & 0 & -\alpha_2 \\ -\beta_2 & \alpha_2 & 0 \end{bmatrix} \begin{bmatrix} X_B - X \end{bmatrix} = W \begin{bmatrix} X_B - X \end{bmatrix}$$ (3)

where $V = \begin{bmatrix} 0 & -\gamma_1 & \beta_1 \\ \gamma_1 & 0 & -\alpha_1 \\ -\beta_1 & \alpha_1 & 0 \end{bmatrix}$, $W = \begin{bmatrix} 0 & -\gamma_2 & \beta_2 \\ \gamma_2 & 0 & -\alpha_2 \\ -\beta_2 & \alpha_2 & 0 \end{bmatrix}$.

According to Eq. (3), there are:

$$f(X) = d_1^2(x, y, z) + d_2^2(x, y, z)$$

$$= X_A^T V^T V X_A + X_B^T W^T W X_B - X_A^T V^T V X - X_B^T W^T W X + X_A^T V^T V X_A + X_B^T W^T W X_B.$$ (4)

The least squares method is to find the point that minimizes $f(X)$, solve equation

$$\frac{\partial f(X)}{\partial X}\bigg|_{(x, y, z, z)} = 0;$$

$$X = \left(V^T V + W^T W\right)^{-1} V^T V X_A + \left(V^T V + W^T W\right)^{-1} W^T W X_B.$$ (5)

Eq. (5) is the coordinate of the target point determined by the least squares method. During the tracking process of the target by the observation station, the observation value is the azimuth angle and the pitch angle. After the target's position is obtained according to the least squares estimation, this can be used as a pseudo-observation, and then filtered to obtain a more accurate target position.

### 2.2 Target system model

The following equation of state describes the motion state of the target:

$$x_k = A_k x_{k-1} + w_{k-1},$$ (6)
where $w_k$ is the state noise at time $k$, which is assumed to be Gaussian noise; $x_k$ is the flight state at time $k$, in the three-dimensional Cartesian coordinate system, when the target moves in a straight line at a constant speed in the direction of the $ox$ axis, $x_k = (x^x_k, x^y_k)$, $x^x_k$ is the displacement in the positive direction of the $ox$ axis at time $k$, $v^x_k$ is the velocity in the positive direction of the $ox$ axis at time $k$; $A_k$ is the state transition matrix, $A_k = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$.

The following measurement equation describes the observation model of the observatory:

$$y_k = C_k x_k + V_k,$$

where $V_k$ is the state noise at the time $k$, assumed to be Gaussian noise; $C_k$ is the measurement matrix, $C_k = \begin{bmatrix} 1 & 0 \end{bmatrix}$; $y_k$ is the measured value at the time $k$.

### 2.3 Filtering algorithm with disturbance detection

In the complex and changing actual environment, the measurement information obtained by the observatory is subject to disturbance.

![Fig. 2. Location schematic when the measurement is disturbed.](image)

At a certain sampling moment, disturbance from enemy forces, transient sensor failure, and other disturbing factors can cause the error of the angular measurement information to exceed the goniometric accuracy, resulting in the solved value of the target position deviating wildly from the actual state, as shown in Fig. 2.

Due to the disturbance received by the angle measurement information of station $B$, the location line determined by station $B$ is $l_3$, and the deviation angle is $\theta$. At this point $l_1$ and $l_3$ determine the target position $T_d$, and $T_d$ deviates greatly from the true state $T$.

For the observation series $y_{k-1}, y_{k-2}, \cdots, y_{k-n}$, its single moving average is:

$$M_{k-1}^1 = \frac{(y_{k-1} + y_{k-2} + \cdots + y_{k-n})}{n} = M_{k-2}^1 + \frac{(y_{k-1} + y_{k-2} + \cdots + y_{k-n-1})}{n},$$

where $M_{k-1}^1$ is the single moving average at the moment $k$ and $n$ is the number of terms.
The new observation sequence \( M_{k-1}^1, M_{k-2}^1, \cdots, M_{k-n}^1 \) can be obtained according to Eq. 8, then we have Eq. 9:

\[
M_{k-1}^2 = (y_{k-1} + y_{k-2} + \cdots + y_{k-n}) / n = M_{k-2}^2 + (y_{k-1} + y_{k-2} + \cdots + y_{k-n-1}) / n, \quad (9)
\]

Then the one-step linear trend forecasting model is:

\[
y_{p,k} = a_{k-1} + b_{k-1}, \quad (10)
\]

where \( a_{k-1} = 2M_{k-1}^1 - M_{k-2}^2 \), \( b_{k-1} = (2M_{k-1}^1 - M_{k-2}^2) / (n - 1) \), \( y_{p,k} \) is the observed value of the prediction at the moment \( k \).

The idea of improving the filtering algorithm is to determine whether the measurement information obtained from the observation station at the time \( k \) is disturbed before the filtering calculation. If it is not disturbed, the next calculation is performed; if it is disturbed, the trend moving average method is used to construct the predicted observation at the time \( k \) instead of the disturbing observation value, and then the next calculation is performed.

Taking the displacement in the positive direction of the \( ox \) axis as an example, the predicted observation at the time \( k \) is \( y_{p,k} \), the measurement value of the observation station is \( y_k^x \), and the standard deviation of the sensor measurement error noise is \( \sigma \).

According to the 3 \( \sigma \) principle:

\[
P(|y_{p,k} - y_k^x| > 3\sigma) \leq 0.003. \quad (11)
\]

According to the 3 \( \sigma \) principle, the probability of the sensor measurement error exceeding three times the standard deviation is 0.003, which is a small probability event. It is considered that the measurement value is disturbed at this time.

The five formulas of Kalman recurrence are as follows\(^{[12]}\):

\[
\begin{align*}
\hat{x}_k^\prime &= A_k \hat{x}_{k-1}^\prime \\
\hat{x}_k &= \hat{x}_k^\prime + H_k (y_k - c_k \hat{x}_k^\prime) \\
H_k &= P_k^C^T (C_k P_k C_k^T + R_k)^{-1}, \\
P_k^\prime &= A_k P_{k-1}^\prime A_k^T + Q_k, \\
P_k &= (I - H_k C_k) P_k^\prime
\end{align*}
\]

where \( \hat{x}_k^\prime \) is the prior estimate, \( \hat{x}_k \) is the posterior estimate, \( P_k^\prime \) is the prior variance, \( P_k \) is the posterior variance, \( Q_{k-1} \) is the input noise variance, \( R_k \) is the observation noise variance.

### 2.4 Algorithm flow chart

The flow chart of dual-station direction finding location based on disturbance detection is shown in Fig. 3.
3 Simulation analysis

3.1 Simulation scenarios

In the geostationary coordinate system, suppose the starting coordinates of observatory 1 are (0, 10 km, 1 km), the starting coordinates of observatory 2 are (10 km, 0, 1 km), and the starting coordinates of the target are (30 km, 30 km, 1.1 km). Observatory 1 and observatory 2 both move at 300 m/s along the positive direction of the ox axis, and the target moves at -204 m/s along the positive direction of the oy axis. The goniometric accuracy of both station 1 and station 2 is 0.05°. The sampling time is 0.01 s and the number of samples is 100.

The position error $e_k = \sqrt{(\hat{x}_k - x_k)^2 + (\hat{y}_k - y_k)^2 + (\hat{z}_k - z_k)^2}$ is utilized as a performance metric, where $(\hat{x}_k, \hat{y}_k, \hat{z}_k)$ denotes the estimated position at sampling moment $k$ and $(x_k, y_k, z_k)$ denotes the actual position at sampling moment $k$.

3.2 Analysis of simulation results

If the measurement information of the observatory is not disturbed, the tracking trajectory is presented in Fig. 4, and the filtering results of the target trajectory are shown in Fig. 5. Suppose the measurement information is disturbed at sampling moment 8. In that case, the target position error is given in Fig. 6, and the results of the target position error from 8 to 16 sampling moments are shown in Table 1.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Sampling moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td>Traditional</td>
<td>817.905</td>
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</table>
Analysis of the simulation results shows that:

(1) when the measurement information is not disturbed, the tracking accuracy of traditional and improved algorithms is the same. The tracking error can eventually converge to within 15 m;
(2) when the measurement information is disturbed, the maximum tracking error is 43.262 m for the improved algorithm and 817.905 m for the traditional algorithm from the time the disturbance occurs. The result indicates that the improved algorithm has a better tracking effect on the target than the traditional one and has stronger robustness and practical value.

4 Conclusion

Aiming at the problem that the disturbance of the measurement information affects the location accuracy in the traditional location algorithm, a dual-station direction finding location algorithm based on disturbance detection is proposed. The algorithm enhances the robustness through the improved Kalman filtering algorithm with disturbance detection in the filtering process. The simulation experiment verifies that the algorithm can accurately track the target when the measurement information is interfered. Compared with the traditional location algorithm, it has stronger robustness and has practical application value. Further research is needed on how to solve the problem of anti-jamming when multiple observation stations track multiple targets.

References