Modeling and analysis of opportunistic spectrum access schemes in cognitive cellular networks under imperfect sensing

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Abstract. Cognitive radio (CR) is an exciting technology to solve the spectrum scarcity problem in the future cellular networks. As the grade-of-service (GoS) performances directly reflect the user's experiences in the cognitive cellular networks (CCNs), the GoS performance analysis of the OSA scheme in the CCNs has attracted much attention. In this article, we derive GoS performance metrics with a three-dimensional continuous-time Markov chain (CTMC) to model the opportunistic spectrum access (OSA) schemes with non-hopping and with hopping in the CCNs under imperfect sensing. Under imperfect sensing, for a given primary user traffic, the range of achievable user equipment (UE) traffic is derived with guaranteeing the GoS of the UE. Simulations are conducted to verify the validity of the proposed CTMC model and corresponding analysis.

Keywords: Cognitive cellular network, Opportunistic spectrum access, Markov chain, Spectrum handoff, Imperfect sensing.

1 Introduction

Cognitive radio (CR) has emerged as one of the exciting technologies that could solve the spectrum scarcity problem in the future cellular networks[1]. Since the grade-of-service (GoS) performances, which are quantified by the service blocked probability and dropped probability, directly reflect the SU's communication quality in the CRNs, the GoS performance analysis for the OSA scheme in the CRNs has drawn much attention. Some recent related work has been done in [2]-[5]. In [2], the GoS performance metrics of the OSA scheme were derived. Then, for a given primary radio traffic, the achievable secondary radio traffic was also analyzed. In [3], the authors employed the Markov approach to derive the GoS performance metrics of the OSA scheme in the absence or presence of buffering mechanism. In [4], the authors modeled the spectrum handoff in the CR networks, and deeply analyzed the data delivery time of the secondary connections. In [5], we obtain the GoS performance metrics in the cognitive cellular networks under perfect sensing. However, the work that model and analyze the OSA scheme to derive the GoS

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The performance metrics in the cognitive cellular networks under imperfect sensing has not been carried out.

The main contributions of this article are to employ a three-dimension continuous-time Markov chain (CTMC) to model the OSA schemes in the cognitive cellular networks under imperfect sensing. Both the OSA schemes with non-hopping and with hopping are investigated and compared. Moreover, we investigate the OSA schemes with imperfect sensing in the cognitive cellular networks. The GoS performance metrics which are quantified by the service blocked probability and service dropped probability are derived. Simulations are also conducted to verify the validity of the proposed CTMC model and corresponding analysis.

2 OSA-based cognitive cellular networks

As shown in the Fig.1, compared with the conventional CRNs, which entirely depend on the licensed spectrum of the PU, as a commercial network, the cognitive cellular network has its own licensed spectrum. However, the limited licensed spectrum resource of the cognitive cellular networks is unable to support numerous emerging wireless services. The UE opportunistically accesses the licensed spectrum owned by PU when the PU is absent and the licensed cellular channels are full. The main difference between conventional and CR cellular networks lies in the priority between UE and PU. Different from the conventional cellular networks, in which only UEs use the spectrum, in the OSA-based cognitive cellular networks, the primary connection always have higher priority over UE connection on the licensed PU channels. To avoid causing interference to the PU, when the PU comes back, the UE must stop transmission and return the licensed PU channel to the UE. Moreover, in the OSA scheme with hopping, compared with the conventional cellular networks, in addition to classical inter-cell handoff due to the mobility of the UE, the cognitive cellular networks have to provide a new types of handoff: spectrum handoff due to the coming back of the PU\textsuperscript{[6]}.

According to the strategy of selecting the target channel for spectrum handoff, the operation mode of the cognitive radio networks can be divided into two categories: non-hopping and hopping\textsuperscript{[7]}. The non-hopping mode is the basic mode of the IEEE 802.22, in which, the SU always keeps idle and stays on its current channel when it is interrupted. In other words, the target switching channel of the non-hopping mode is always its current channel. Since the delay of the non-hopping based OSA scheme lies on the PU activity, the non-hopping mode is only suitable for the short transmission CRN. The hopping mode is also used in IEEE 802.22, for instance: the phase-shifting hopping method. In the hopping mode, the interrupted SU can stay on its current channel or change to another idle channel. Compared with the non-hopping mode, the hopping mode is suitable for more general CRN...
due to the smaller delay performances. However, the hopping mode is not always preferred due to the high energy consumption of handoff[8].

3 Performance analysis under imperfect sensing

In the OSA-based CR cellular networks, the UE is allowed to operate in the frequency bands originally allocated to the PU only when the PU is absent. However, when the PU comes back into operation, the UE should vacate the spectrum instantly to avoid interference with the PU. As a result, reliable spectrum sensing whose function is to detect the presences of PUs is a key element in the OSA-based CR cellular networks. However, the perfect spectrum sensing is impractical in practice due to the limited sensing capability and short sensing time. The imperfect sensing event are divided into two categories: missed detection and false alarm. The missed detection event will lead the interference to the PU, and the false alarm event will decrease the chances of the UE to utilize the idle licensed channel.

The spectrum sensing strategy in the OSA-based CR cellular networks are divided into two categories: on-demand sensing and periodic sensing. The on-demand sensing is performed before the SUs first access a PU channel or switch to the other channel. At this time, the wide band spectrum sensing is employed to detect the presences of PUs on all the channels except the channels that have already occupied by the SUs. When the SUs are transmitting on the PU channels, the periodic spectrum sensing is carried out to detect whether the PU has been coming back or not. Compared with the [2], the sensing errors only occur when the SU is searching an idle channel to access, we consider the two cases that not only the initiating secondary users will make sensing errors but also the SUs when they are transmitting may make sensing errors.

3.1 OSA with non-hopping under imperfect sensing

Suppose that the licensed cellular spectrum is partitioned into $N$ channels, and the cognitive spectrum is partitioned into $M$ channels. The mean rates of PU and UE service requests are denoted as $\lambda_p$ and $\lambda_s$, respectively. The PU and UE service time have mean service rates $\mu_p$ and $\mu_s$, respectively. Since the cognitive cellular network has limited licensed spectrum and the UE can access the two different spectrum bands, the two-dimension Markov models are not suitable in this case. To solve this problem, we use a three dimension CTMC to model the process of spectrum occupation for the OSA-based cognitive cellular networks. The states in the Markov model are denoted by $(i,j,k)$, where $i$ and $j$ represent the number of the existing PUs and UEs in the cognitive spectrum, respectively, and $k$ represents the number of the existing UEs in the licensed cellular spectrum. Obviously, $0 \leq i \leq M$, $0 \leq j \leq M$, $i + j \leq M$ and $k \leq N$.

We assume that the sensing result on all $M$ licensed channels are statistically independent. Let $P_m$ and $P_f$ denote the miss detection probability and the false alarm probability of each channel, respectively. The sensing interval for the period sensing is denoted as $1/\tau$. When the miss detection event happens, both the PU and UE drop the channel due to the collision event[14]. Denote $r$ and $c$ as the number of the channels which are incorrectly sensed as idle and busy, respectively. For any state $(i,j,k)$, the probability of the SU accesses the idle licensed channel and busy licensed channel can be derived as follows.
\begin{align}
P_{idle}(i, j, k) &= \sum_{r=0}^{i} \sum_{c=0}^{M-i-j-1} P'_m P'_j \frac{M-i-j-c}{r+M-i-j-c} \\
P_{busy}(i, j, k) &= \sum_{r=1}^{i} \sum_{c=0}^{M-i-j-1} P'_m P'_j \frac{r}{r+M-i-j-c}
\end{align}

Fig. 2 shows the transit rate diagram of the Markov chain model for the OSA scheme with non-hopping under imperfect sensing. Note that, the UEs on the $N$ channels belong to the licensed cellular spectrum will be not affected by the sensing errors due to only UEs use the $N$ channels. There are three things result in the transition form state $(i,j,k)$ to $(i,j-1,k)$. Firstly, the transition rate resulting from the completion of the SU transmission is given as $j \mu_s(1-P_f)$. Secondly, when the PU accesses the licensed channel where the UE is transmitting and miss detection event happens, the number of the PUs in the system will be not increased and the UE drops the channel due to the collision between the PU and UE. The corresponding transition rate is given as $\lambda_p P_m/(M-i)$ . Thirdly, the event of false alarm will also result in the UE drops the channel. Since the sensing interval for the period sensing is $1/\tau$, the corresponding transition rate is given as $j \tau P_f$. Compared with the perfect sensing case, when $k=N$, beside the completion of the PU transmission, the UE accesses to the busy licensed PU channel due to the miss detection event will also results in the transition from state $(i,j,k)$ to $(i-1,j,k)$. Thus, the transition probability from state $(i,j,k)$ to $(i-1,j,k)$ is given as $i \mu_p + \lambda_p P_{busy}(i,j,k)$.

![Rate diagram of state $(i,j,k)$ in the OSA scheme with non-hopping under imperfect sensing.](image-url)
Let \( P(i,j,k) \) denote the steady-state probability of the state \((i,j,k)\). \( U(x-x_0)=1 \) if \( x>=x_0; U(x-x_0)=0 \) elsewhere. \( \delta(x)=1 \) if \( x=0\); \( \delta(x)=0 \) elsewhere. Let \( \pi \) is the steady-state probability vector. \( \mathcal{S} = \{P(i,j,k)\} \) is the set of all the states of the Markov model, and \( q_{m,n} \) is the transition rate from state \( m \) to \( n \), where \( m \in \mathcal{S} \) and \( n \in \mathcal{S} \). From Fig.1, we can derive the transition rate matrix \( Q \), where the \( q_{m,n} \) is the element of \( Q \). Then, we can obtain \( P(i,j,k) \) from the following equations: \( \pi Q = 0 \) and (3).

\[
\sum_i \sum_j \sum_k P(i, j, k) = 1
\]  

(3)

The UE services are dropped in the following three cases. Firstly, the PU returns to the licensed PU channel and the UE sensed correctly. Secondly, the PU returns to the licensed PU channel and the miss detection event happens, the UE dropped the channel due to the collision with the PU. Thirdly, the UE service is dropped due to the false alarm event. Let \( P_{\text{drop, } u} \) denote the UE dropped probability, which can be expressed as a fraction of the number of the dropped UE services over the number of UE arrivals in period \( T^{[6]} \). Thus, the UE service dropped probability could be derived as follows

\[
P_{\text{drop, } u} = \sum_{i,j,k, j \neq 0} j \lambda_p (1-P_m) P(i, j, k) + \sum_{i,j,k} j \lambda_p P_m P(i, j, k) + \sum_{i,j,k} j \tau P(i, j, k)
\]

(4)

The UE services are blocked in the following cases. Firstly, the UE services are blocked at the states where \( i+j+k=M+N \) when all the active PU channels are sensed correctly. Secondly, even if there are some idle licensed PU channels, if all the idle channels are incorrectly sensed as busy, the newly arrived UE will be blocked. Thirdly, the newly arrived UE accesses the PU channel where the PU is active due to the miss detection event. However, the UE can not obtain the service due to the collision. Thus, the UE blocked probability can be derived as follows

\[
P_{\text{block, } u} = \sum_{i,j,k, i+j+k=M+N} (1-P_m)^i P(i, j, k) + \sum_{i,j,k, i+j+k=M+N, k=N} (1-P_m)^i P(i, j, k) + \sum_{i,j,k, i+j+k=M+N, k=N} P(i, j, k) \]

(5)

The PU services are interrupted by the UE due to the miss detection with the probability

\[
P_{\text{int, } p} = \sum_{i,j,k} \lambda_p P_{\text{busy}} P(i, j, k) + \sum_{i,j,k, j \neq 0} \frac{j \tau P(i, j, k)}{M-i}
\]

(6)

### 3.2 OSA with hopping under imperfect sensing

Fig.3 shows the transit rate diagram of the Markov chain model for the OSA scheme with hopping under imperfect sensing. Note that, since the UE do not know the returning of the PU due to the miss detection, the dropped UE resulting form the collision event will not carry out switching. However, the dropped UE due to the false alarm event will carry out the switching for the reason that the UE incorrectly sense the channel as busy. Moreover, since the UEs need to carry out spectrum sensing to decide which channel to switch, the imperfect sensing results will also influence the accessing of the UE during the switching duration. For instance, when \( k = 0 \), there are four things result in the transition form state \((i,j,k)\) to \((i,j-1,k)\). Firstly, with no false alarm, any of the UE transmission will be completed, and the corresponding transition rate from \((i,j,k)\) to \((i,j-1,k)\) is \( j \mu_s (1-P_f) \). Secondly, the probability that the PU accesses the licensed channel where the UE is active and the UE
incorrectly sense the channel as idle is $\lambda_p P_{mj}/(M-i)$. Thirdly, when the false alarm event happens and the switching is unsuccessful, the UE service will be dropped with probability $j\mu/(1-P_m)P_{jt,j}(i-j)/(M-i)$. Lastly, when the PU accesses the licensed channel where the UE is transmitting and the UE switches to the busy PU channel due to the miss detection, the UE service will be dropped with probability $\lambda_p(1-P_m)P_{busy}(i+1,j-1,k)/(M-i)$.

Figure 3. Rate diagram of state $(i,j,k)$ in the OSA scheme with hopping under imperfect sensing.

The UE services are dropped in the following cases. Firstly, the PU returns to the licensed PU channel and the UE sensed correctly, the UE service will be dropped due to all the channels are sensed as busy or the switching to the busy PU channel resulting from the miss detection event. Secondly, the PU returns to the licensed PU channel and the miss detection event happens, the UE dropped the channel due to the collision with the PU. Thirdly, the UE service is dropped due to the false alarm event and the unsuccessful
switching that no channels are sensed as idle or the switching to the busy PU channel resulting from the miss detection event. Thus, the UE service dropped probability can be derived as follows

\[ P_{\text{drop, } u} = \sum_{i,j,k, j \neq 0, k=N} j \hat{\lambda}_p (1 - P_u) P_f^{M-i-j} P(i, j, k) + \sum_{i,j,k, j \neq 0, k=N} j \tau P_f (1 - P_m) P_f^{M-i-j+1} P(i, j, k) \]
\[ + \sum_{i,j,k, j \neq 0, k=0} j \hat{\lambda}_p (i+1, j-1,k) P(i, j, k) + \sum_{i,j,k, j \neq 0, k=0} j \hat{\lambda}_p P_m P(i, j, k) (M-i) \hat{\lambda}_s \]
\[ + \sum_{i,j,k, j \neq 0, k=0} j \hat{\lambda}_p (1 - P_m) P_f^{i+j} P(i, j, k) (M-i) \hat{\lambda}_s \]  

(7)

The PU services are interrupted due to the miss detection events which occurs at the wide band or periodic spectrum sensing. We can derived the PU interrupted probability as follows

\[ P_{\text{int, } p} = \sum_{i,j,k, j \neq 0, k=N} \lambda_p P_{\text{busy}} (i+1, j-1,k) P(i, j, k) + \sum_{i,j,k, j \neq 0, k=N} j(l - P_m) P_{\text{busy}} (i+1, j-1,k) P(i, j, k) \]
\[ + \sum_{i,j,k, j \neq 0, k=0} j \tau P_f P_{\text{busy}} (i, j, k) P(i, j, k) + \sum_{i,j,k, j \neq 0, k=0} j P_m P(i, j, k) \]
\[ + \sum_{i,j,k, j \neq 0, k=0} j \hat{\lambda}_p P_f^{i+j} P(i, j, k) \]  

(8)

4 Simulation results and analysis

In the simulations, we set \( M=3, N=3 \), and focus on the voice traffic. Both the PU and UE service requests are set to follow the Poisson arrival processes. The mean rate of the PU service requests is set to be \( \lambda_p=3 \text{min}^{-1} \). The PU and UE service requests are set to have negative exponential service time distributions with mean duration \( \mu_p=1/3 \text{ min} \) and \( \mu_s=2 \text{ min} \), respectively. Note that, our proposed three-dimension CTMC model and corresponding numerical analysis are independent of the distributions that the PU and UE service requests and service time follow, and we only need to know the mean rates of the service request and time.

In Fig.4, we investigate the service blocked and dropped probabilities of the UE service for the OSA scheme with non-hopping under imperfect sensing in terms of the UE service request rate \( \lambda_s \), respectively. Obviously, With the increase of the \( \lambda_s \), the service blocked and dropped probabilities of the UE increase due to the busy UE service traffic. Moreover, we compare the performance metrics of OSA schemes with non-hopping under perfect sensing ,i.e., \( P_f=0 \) and \( P_m=0 \),and imperfect sensing. Firstly, for the OSA scheme with non-hopping under imperfect sensing, the service blocked probabilities of the UE is higher than that of the case under perfect sensing due to the false alarm event. However, the miss detection event causes the service blocked probabilities of the UE to increase. Compared with the case under perfect sensing, both the miss detection and false alarm event will causes the service blocked probabilities of the UE to increase.
Fig. 5 shows the service blocked and dropped probabilities of the UE service for the OSA scheme with hopping under imperfect sensing in terms of the UE service request rate. Compared with the Fig. 3, when the UE traffic is busy, for instance $\lambda_s = 12 \text{ min}^{-1}$, for the OSA scheme with hopping, the service blocked probabilities of the UE is higher than that of the OSA scheme with non-hopping due to the UE can successfully handoff whenever there are other idle channels. This causes the probability that all channels are occupied to increase and results in higher $P_{\text{block},u}$ compared with the OSA scheme with non-hopping. However, as shown in Fig. 4 (b), for the OSA scheme with hopping, the service dropped probabilities of the UE service $P_{\text{drop},u}$ is much lower than that of the OSA scheme with non-hopping due to the hopping strategy.

![Fig. 4](image)

(a) Service blocked probabilities for the OSA scheme with non-hopping under imperfect sensing; (b) Service dropped probabilities for the OSA scheme with non-hopping under imperfect sensing.

Fig. 6 shows the service interrupted probabilities of the PU service for the OSA scheme with non-hopping and hopping under imperfect sensing in terms of the UE service request rate $\lambda_s$. As shown in Fig. 5 the PU service is interrupted only when the miss detection event occurs. Compared with Fig. 5 (a) and (b), for the OSA scheme with hopping, the service interrupted probabilities of the PU service is higher than that of the OSA scheme with non-hopping since the miss detection event also happens in the hopping strategy.

![Fig. 5](image)

(a) Service blocked probabilities for the OSA scheme with hopping under imperfect sensing; (b) Service dropped probabilities for the OSA scheme with hopping under imperfect sensing.

Finally, we can use the results from Fig. 4 - Fig. 6 to derive the achievable UE traffic with GoS guarantee under imperfect sensing. For instance, assume the miss detection and false alarm probabilities are $P_m = 0.1$ and $P_f = 0.1$, for a target GoS which is represented by
(\text{P}_{\text{int},p}=0.1, \text{P}_{\text{block},u}=0.2, \text{P}_{\text{drop},u}=0.1), \text{ the OSA scheme with hopping can support } \lambda_u \text{ up to 9 min}^{-1} \text{ for the reason that it can satisfy the } \text{P}_{\text{int},p}, \text{P}_{\text{block},u} \text{ and } \text{P}_{\text{drop},u} \text{ concurrently when } \lambda_u \leq 9. \text{ For the OSA scheme with non-hopping, it can only support } \lambda_u \text{ up to 8 min}^{-1} \text{ due to the stricter bound of the } \text{P}_{\text{drop},u}. \text{ Note that, since the dropped UE connection will bring the worse user experience than the blocked event, the target UE service dropped probability is set to be smaller than the target UE service blocked probability.}

![Graph](image)

**Fig. 6.** (a) Service interrupted probabilities of PU for the OSA scheme with non-hopping under imperfect sensing; (b) Service interrupted probabilities of PU for the OSA scheme with hopping under imperfect sensing.

### 5 Conclusion

In this paper, the GoS performance metrics of the OSA schemes with non-hopping and with hopping under imperfect sensing was derived by employing a three dimension CTMC. Simulations were conducted to verify the validity of the proposed CTMC model and corresponding analysis. Under imperfect sensing, for a given primary user traffic, the proposed CTMC model is helpful in deriving the range of achievable UE traffic with guarantying the GoS.

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### References


