Integrated optimization of batch production process scheduling based on USEBCTM and explicit model predictive control

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Abstract. An integration model of batch process production scheduling and control is established by using synchronous method, which generates mixed integer dynamic (MIDO) problem. In order to solve the online computing burden of MIDO problem, explicit model predictive control (E-MPC) is applied to solve the online implementation of scheduling and control integration problem. Using the method based on USEBCTM to establish the scheduling level constraints, and used the MPT toolbox in MATLAB to solve the E-MPC dynamic problem. The explicit control solution was converted into the control level constraint, then with scheduling level constrain optimization objective together.

Keywords: Batch process, Integration of scheduling and control, Specific unit time point, Explicit model predictive control, Mixed integer nonlinear programming.

1 Introduction

The batch process due to its inherent flexibility and ability to respond quickly to dynamic changes in market demand, it occupies an increasingly important position in the process industry[1].At present, the modeling method for the scheduling problem of batch production process[2] is mainly divided into discrete time and continuous time modeling method[3]. Unit-specific Event-based Continuous Time Models (USEBCTM).The USEBCTM method is still static scheduling model[4]. Integrating scheduling model with control model can reflect dynamic behavior for the batch production process is an effective way to solve the above problems. There are mainly synchronous methods and decomposition methods for the integration. Through the synchronization method, the MIDO problem is reformulated as a MINLP problem. The decomposition method is to iterate between the original problem of the integrated the main problem [5]. In this paper, integrated with the control model by using the synchronization method. The e-MPC method is introduced[6-8]. The main idea is to put the online calculation of the optimization solution offline through the idea of parameter programming[9-10], reduce the difficulty of model solving and meet the needs of real-time model.

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2 Scheduling and control integration framework

The integration framework is shown in Figure 1.

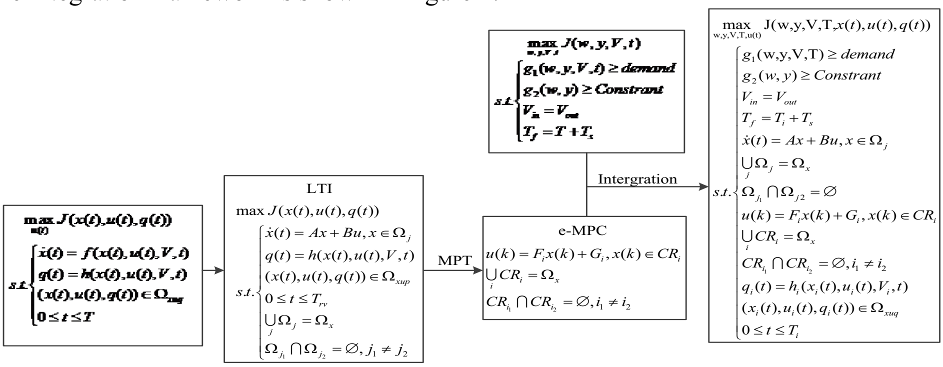

Fig. 1. Integration of scheduling and control framework.

Figure 2 shows the variables V_i and T_i of the integration model. These variables integrate scheduling and control hierarchies while processing shared information.

3 Scheduling and control integration framework

3.1 Constraints at scheduling level.

Equipment allocation constraint

$$\sum_{i\in Ij} w(i,n) = y(j,n) \quad \forall j \in J,\ n \in N \tag{7}$$

Capacity constraint

$$V_{ij}^{\min} w(i,n) \le V(i,j,n) \le V_{ij}^{\max} w(i,n) \quad \forall i \in I, j \in J_i,\ n \in N \tag{8}$$

Storage constraint

$$ST(s,n) \le ST(s)^{\max} \ \ \forall s \in S, n \in N \tag{9}$$

Material balance constraint

$$ST(s,n) = ST(s,n-1) - d(s,n) + \sum_{i\in Is} \rho_{si}^{P} \sum_{j\in Ji} V(i,j,n-1) - \sum_{j\in Ji} \rho_{si}^{C} \sum_{j\in Ji} V(i,j,n) \quad \forall s \in S, n \in N \tag{10}$$

Reaction time constraint

$$T^{f}(i,j,n) = T^{s}(i,j,n) + T_{rv}(i,j,n) \ \ \forall i \in I\ , n \in N, \mathrm{j} \in J_i \tag{11}$$

Sequence constraint
Same task in the same unit.

$$T^{s}(i,j,n+1) \ge T^{f}(i,j,n) - H(2 - w(i,n) - y(j,n)) \ \ \forall i \in I_j, j \in J_i,\ n \in N, n \ne N \tag{12}$$

Different tasks in the same unit.

$$T^{s}(i,j,n+1) \ge T^{f}(i',j,n) - H(2 - w(i',n) - y(j,n)) \;\; \forall i \in I_j, i' \in I_j, i \ne i', j \in J_i,\; n \in N, n \ne N \quad (13)$$

Different tasks in different units.

$$T^{s}(i,j,n+1) \ge T^{f}(i',j',n) - H(2 - w(i',n) - y(j',n)) \;\; \forall i \in I_j, i' \in I_j, i \ne i', j' \in J_i,\; n \in N, n \ne N \quad (14)$$

Completion of previous tasks.

$$T^{s}(i,j,n+1) \ge \sum_{n' \in N, n' \prec N} \sum_{i' \in I_j} (T^{f}(i',j,n') - T^{s}(i',j,n')) \;\; \forall i \in I_j, j \in J_i,\; n \in N, n \ne N \quad (15)$$

Time horizon constraints.

$$\begin{aligned} &T^{f}(i,j,n) \le H \;\; \forall i \in I_j, \mathrm{j} \in J_i,\; n \in N \\ &T^{s}(i,j,n) \le H \;\; \forall i \in I_j, \mathrm{j} \in J_i,\; n \in N \end{aligned} \quad (16)$$

3.1 Constraints at control level.

The state transition equations of linear time-invariant models were expressed as:

$$x(k+1) = A_j x(k) + B_j u(k) + C_j \quad (17)$$

$$x(k) \in \Omega_j \;\; \Omega_j = \{x : V_j x \le W_j\}, j \in J = \{1,...,N_j\} \quad (18)$$

$$\bigcup_j \Omega_j = \Omega_x \;\; \Omega_{j1} \cap \Omega_{j2} = \varnothing, j1, j2 \in J and\; j1 \ne j2 \quad (19)$$

Using MPT toolbox for obtaining the explicit control solutions by solving e-MPC.

$$u(k) = F_i x(k) + G_i \;\; x(k) \in CR_i \;\; CR_i = \{x : H_i x \le K_i\}, i \in I = \{1,2,...,N_I\} \quad (20)$$

$$\bigcup_i CR_i = \Omega_x \;\; CR_{i1} \cap CR_{i2} = \varnothing, i_1, i_2 \in I and i_1 \ne i_2 \quad (21)$$

Introducing binary variables $y1_i$ $y2_j$, then convert 17-21 into an explicit linear constraint:

$$-M(1 - y1_i) + H_i x(k) \le K_i \quad (22)$$

$$F_i x(k) + G_i - M(1 - y1_i) \le u(k) \le F_i x(k) + G_i + M(1 - y1_i) \quad (23)$$

$$-M(1 - y2_j) + V_j x(k) \le W_j \quad (24)$$

$$A_j x(k) + B_j u(k) + C_j - M(1 - y2_j) \le x(k+1) \le A_j x(k) + B_j u(k) + C_j + M(1 - y2_j) \quad (25)$$

The processing time can be calculated from equation 28, h is step size, N_k is step number.

$$T_{rv} = N_k h \tag{26}$$

4 The objective of the integrated problem

The objection is total income minus raw material cost, and the specific function is as follows: C^P C^r are product price and raw material price.

$$J = C^P X(N_k) - C^r V \tag{27}$$

$$s.t.\begin{cases}(7)-(18)\ \text{c}onstraints\ at\ scheduling\ level\\(24)-(31)\ explicit\ MPC\end{cases} \tag{28}$$

5 Case studies

The process State Task Network (STN) as shown in figure 3. The reactions price are shown in Table 1. The information of unit capacity and material price is shown in Table 2.

Reaction conversion and concentration as the state $x = [x_1\ x_2]^T$ It can be obtained as follows:

$$\begin{aligned}\dot{x}_1 &= -\vartheta x_1 \kappa(x_2) + q(x_{1f} - x_1)\\ \dot{x}_2 &= \beta\vartheta x_1 \kappa(x_2) - (q+\delta)x_2 + \delta u + q x_{2f}\end{aligned} \tag{29}$$

The system states divide into three areas: $[0,0.35]\times[0,6]$, $[0.35,0.78]\times[0,6]$, $[0.78,1]\times[0,6]$. The discrete time LTI model of CSTR system:

Table 1. Unit capacity and material price.

State	Capacity	Initial amount	Price
raw material A	Unlimited	Unlimited	0.8
raw material B	Unlimited	Unlimited	0.8
raw material C	Unlimited	Unlimited	0.8
Production1	Unlimited	0	4
Production2	Unlimited	0	2

$$\begin{aligned}x(k+1) &= \begin{bmatrix}0.8889 & -0.0123\\0.1254 & 0.9751\end{bmatrix}x(k) + \begin{bmatrix}-0.0002\\0.296\end{bmatrix}u(k) + \begin{bmatrix}0.1060\\-0.0852\end{bmatrix} x\in\Omega_1\\ x(k+1) &= \begin{bmatrix}0.8241 & -0.0340\\0.6365 & 1.1460\end{bmatrix}x(k) + \begin{bmatrix}-0.0005\\0.0322\end{bmatrix}u(k) + \begin{bmatrix}0.1907\\-0.7537\end{bmatrix} x\in\Omega_2\\ x(k+1) &= \begin{bmatrix}0.6002 & -0.0463\\2.4016 & 1.2430\end{bmatrix}x(k) + \begin{bmatrix}-0.0007\\0.0338\end{bmatrix}u(k) + \begin{bmatrix}0.3119\\-1.7083\end{bmatrix} x\in\Omega_3\end{aligned} \tag{30}$$

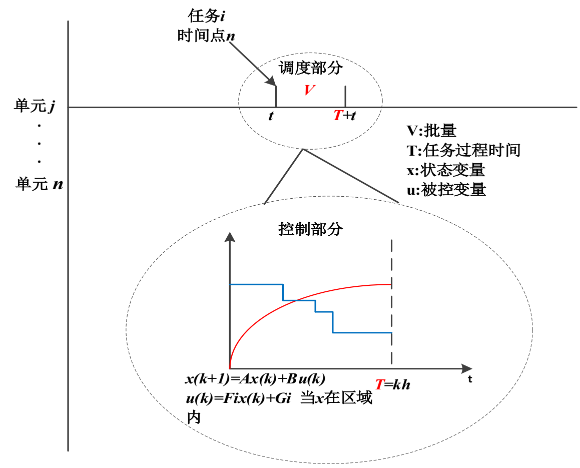

Fig. 2. Integration relationship.

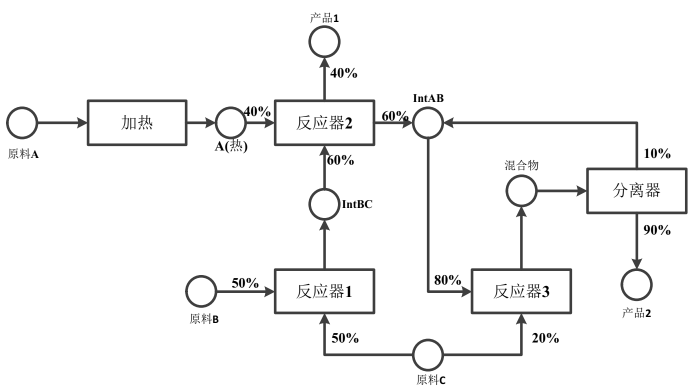

Fig. 3. Flow sheet.

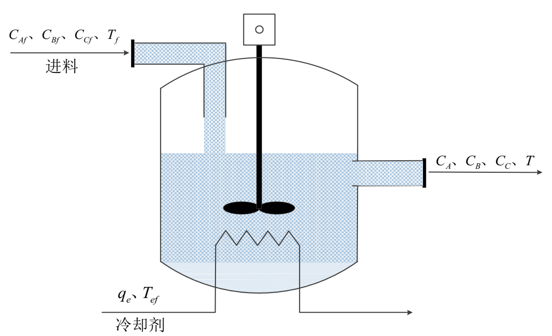

Fig. 4. Diagram of CSTR system.

Table 2. Reactor adaptation.

Unit	Capacity	Adaptation
Heater	150	Heating
Reactor 1	200	Reaction 1.2.3
Reactor 2	300	Reaction 1.2.3
Distillation	250	Separation

Table 3. CSTR System parameter value.

Parameter	λ	ϑ	q	β	δ	x_{1f}	x_{2f}
Parameter value	20.0	0.072	1.0	8.0	0.3	1.0	0

Using toolbox MPT to get the explicit control solution, and the distribution diagram was as follows:

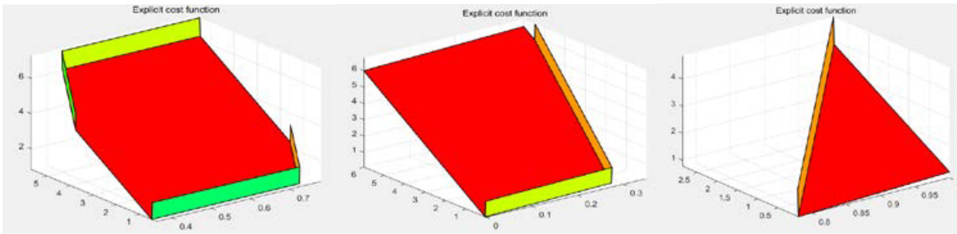

Fig. 5. Explicit control solution partition.

The reaction quantities of product 1 and product 2 are given by $V_1X_1X_2$ and V_3X_3

$$\max_{\substack{w,y,V,d,ST,Tf,Ts\\Trv,x,u,X,y1,y2}} J = C^{P1}V_1X_1(N_k)X_2(N_k)+C^{P2}V_3X_3(N_k)-\sum_I C^{ri}V_i \tag{33}$$

Using LINGO to solve integration problems. The results are shown in Figure 8 and Table 4. Pure scheduling select product status delivery $d(s,n)$ as the target:

$$\max_{w,y,V,d,ST,T^s,T^f} J = C^{P1}*\sum_n d(s='product1',n)+C^{P2}*\sum_n d(s='product2',n)$$
$$s.t.\{7-18 \text{ constraints at scheduling level} \tag{34}$$

The detailed results are shown in Figure 9 and Table 5. The '#' in chart represent task-unit-event point (ijn), the '/' represent conversion rate/inventory.

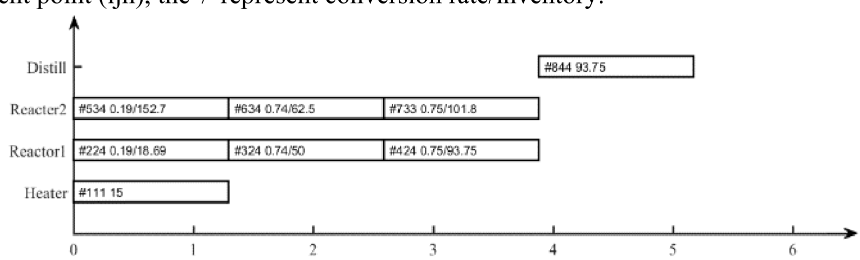

Fig. 8. MINLP problem Gantt charts.

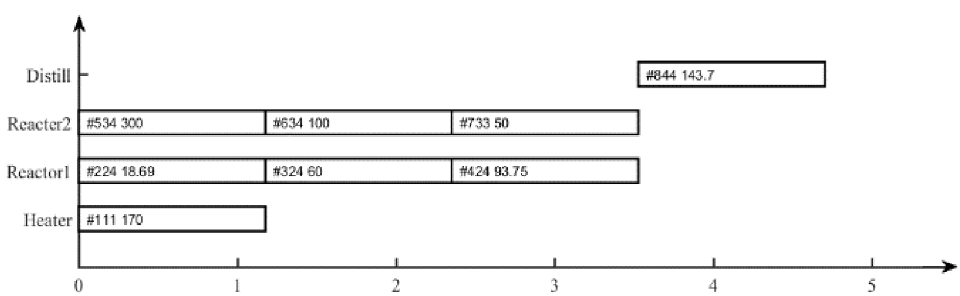

Fig. 9. Sequential scheduling Gantt charts.

Table 4. Integrated MINLP profit results.

Optimization variable	Value
Production 1	143.75
Production 2	84.37
Raw material A	120
Raw material B	135
Raw material C	156.25
Profit	2714.11

Table 5. Sequential scheduling profit results.

Optimization variable	Value
Production 1	133.2
Production 2	44.4
Raw material A	100
Raw material B	136.6
Raw material C	143.7
Profit	1546.75

6 Conclusion

In this paper, It is the possibility and feasibility of exploring e-MPC applications in the field. The main contribution of this study is that we propose an integrated framework that can translate the explicit control solutions generated by e-MPC into explicit linear constraints, combine them with linear constraints at the scheduling level. The result of case study shows that the integrated method has higher benefits than the sequential method.

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