

# Dual-frequency shift leads to chaos and beating in an erbium-doped fiber laser

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**Abstract.** We present a dual-frequency shift scheme to produce chaos and beating in an erbium-doped fiber laser while we give this laser physics model including a photo attenuator absorbing the photon, where the absorption frequency of the photo attenuator can be shifted, and a pump modulator shifting the modulation frequency. It can be found that the modulated laser produces chaos, beating, and a lot of quasi-periodic states when both pump level and photo absorption of the laser are modulated by the attenuator and the modulator. Our numerical result illustrates a route to chaos or away from chaos after beating or quasi-period by shifting the pump modulation frequencies and the absorption frequencies of the attenuator while a dual-cycle, a cycle-3, a cycle-4, quasi-cycle, beating and chaos occurs in the modulated laser. We discuss the effects of two frequencies of the attenuator and the modulator on dynamics. And we find the chaos distribution with frequency variations of the absorption. It can be found that two frequencies of the absorption and the modulation guide the dynamics of the modulated laser. The result has great referenced values to laser, optics, and chaos.

**Keywords:** Laser, Chaos, Beating, Quasi-cycle.

## 1 Introduction

Using the nonlinear optics control technology and modulation technology, a lot of optics systems can be motivated to lead to show non-linear dynamics, such as chaos or an irregular movement. And the chaotic optics and the irregular lasing are very interesting for their application in secure communications, radar and random signal generators. A lot of lasers can show chaos and output some chaotic pluses when the lasers are modulated. It is found that a chaotic laser sensitizes to its parameter variation, and its movement trajectory illustrates a chaotic attractor with a high density and non-repetition property in phase space. Recently, the chaotic lasers have used in a lot of fields [1-4]. It can be found that an erbium-doped (Er) fiber laser can show chaos or random dynamics via the modulation technology. Many Er-doped fiber lasers are used in a fiber communication and a fiber sensor [1-4]. The Er-doped fiber laser is very interesting because its wave variation has a unique low speed change characteristic, such as having a few milliseconds, and outputting a

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cycle or a chaotic pulse. This paper will study a dual-frequency shift scheme to how to produce chaos and beating in an erbium-doped fiber laser, and analyze a route to chaos. The paper is organized as follows: first, we give our dual-frequency shift laser physics model. Second, we illustrate our study results. Last, we present our conclusion.

## 2 Dual-frequency shift laser physics model

When the pump is operated on the Er-doped fiber laser, almost all Er ions of the Er-doped fiber can move rapidly to keep in its level-two from its level-three so that its level-two maintains an excited dynamics. The normalized variable  $N$  and the normalized variable  $P$  are used to represent the ion number density and the photon number density. The laser physics model of the modulated laser when the pump and photon absorption are modulated by two sine signals as follows:

$$\frac{dN}{dt} = \frac{P_p [1 + \mu \sin(2\pi f_1 t)]}{h\nu_p A_p} \sigma_1 (1 - N) - \frac{c}{n_c} \sigma_2 (2N - 1)P - \frac{N}{\tau} \quad (1)$$

$$\frac{dP}{dt} = \frac{c}{n_c} Q \sigma_2 (2N - 1)P - P \times \gamma_c [1 + \eta \sin(2\pi f_2 t)] \quad (2)$$

Where the parameters of the laser are:  $P_p$  represents the pumping.  $\mu$  represents the modulation level and  $f_1$  represents the shift frequency of the pumping.  $A_p$  is the effective mode field area.  $\sigma_1$  is the absorption cross section.  $\sigma_2$  is the stimulated emission and absorption cross section.  $h\nu_p$  is the pumping energy.  $c$  is the light velocity in vacuum.  $n_c$  is the refractive index.  $\tau$  is the level-two lifetime. The photon attenuation rate is  $\gamma_c = 1/\tau_c$ , where  $\tau_c$  is the lifetime of photon.  $\eta$  represents the modulation level and  $f_2$  represents the modulation frequency of the photon absorber.  $Q$  is the total ion population density. In our study, we take the laser parameters as  $\sigma_1 = 2 \times 10^{-21} \text{cm}^2$ ,  $\sigma_2 = 7 \times 10^{-21} \text{cm}^2$ ,  $n_c = 1.46$ ,  $\tau = 10 \text{ms}$ ,  $\tau_c = 4 \text{ns}$ ,  $Q = 5 \times 10^{19} / \text{cm}^3$ ,  $A_p = 2 \times 10^{-7} \text{cm}^2$  and  $P_p = 0.1 \text{W}$  [8]. Beating, quasi-period and chaos can be motivated to present at the laser by controlling the frequencies of the photon absorber and the pump modulator. Such dual-frequency shift performance is operated on the laser, the modulated laser will show chaos and beating as well as quasi-period.

## 3 Results

We use Eqs. (1) and (2) to simulate study of the dual-frequency shift performance on the lasers. We can obtain our study result: the laser become of chaos, beating, and quasi-cycle states when the dual-frequency shift modulation performing on the laser.

### 3.1 The frequency shift of the pump modulator

When the dual-frequency shift modulation is performed on the laser and the parameters are taken as  $f_2 = 1 \text{kHz}$ ,  $\mu = 0.2$ ,  $\eta = 0.01$ ,  $f_1 = 0.02 \text{kHz}$ , it is found that a beating behaviour of the laser is motivated to show in figure 1. Figure 1 (a) shows a beating movement of laser with a beating frequency of  $0.02 \text{kHz}$ , and figure 1 (b) shows the movement orbit in phase space. The result implies a realization of beating state when the dual-frequency shift modulation is operated on the laser. It can be found that that movement behaviors of the laser can be

motivated to lead to different dynamic states via shifting the frequency  $f_1$ . And there are some cases as following.

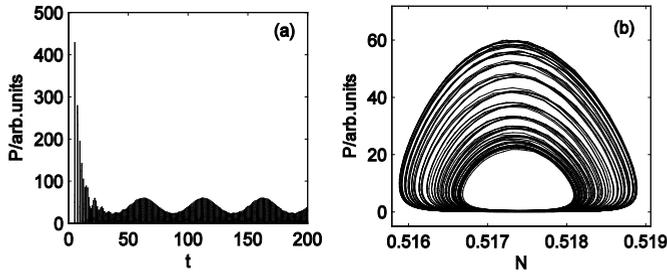


Fig. 1. Beating. (a) output. (b) an orbit in phase space.

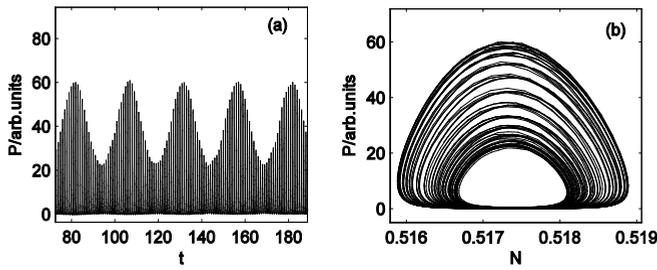


Fig. 2. Different beating. (a) beating. (b) an orbit.

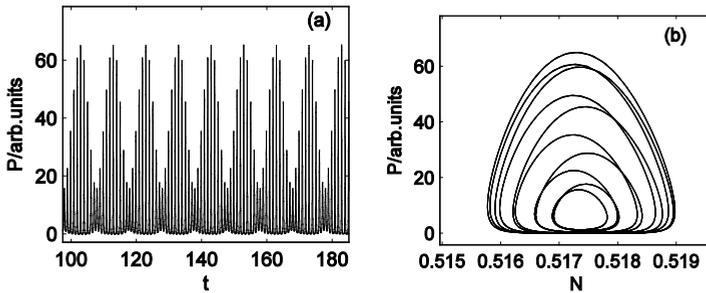


Fig. 3. A cycle-10. (a) a cycle-10 movement. (b) a cycle-10 orbit.

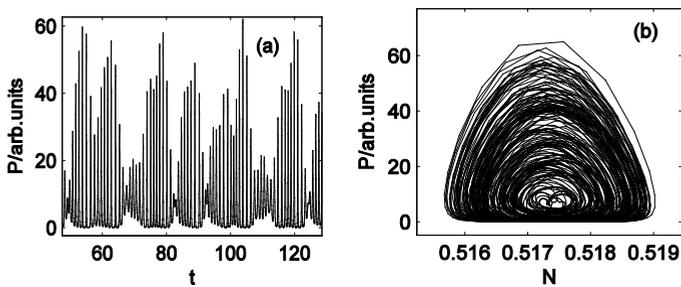


Fig. 4. Chaos. (a) a chaotic movement. (b) a chaotic attractor.

When  $f_1=0.04\text{kHz}$ , the laser is motivated to lead to a different beating state shown in figure 2. Figure 2 (a) shows a different beating movement of laser with a beating frequency of  $0.045\text{kHz}$ , and figure 2 (b) shows the movement orbit in phase space.

When  $f_1=0.1\text{kHz}$ , the laser is motivated to lead to a cycle-10 state shown in figure 3. Figure 3 (a) shows a cycle-10 movement of laser, and figure 3 (b) shows the cycle-10 movement orbit.

When adding  $f_1=0.12\text{kHz}$ , the laser is motivated to lead to a chaotic state shown in figure 4. Figure 4 (a) shows a chaotic movement of laser, and figure 4 (b) shows the chaotic attractor. And it is found that the chaotic laser sensitizes to its modulation frequency variation, and its movement trajectory illustrates a chaotic attractor with a high density and non-repetition property in phase space.

When adding  $f_1=0.17\text{kHz}$ , the laser is motivated to lead to another chaotic state shown in figure 5. Figure 5 (a) shows another chaotic movement of laser, and figure 5 (b) shows the chaotic attractor, and it is found that the chaotic laser movement illustrates a high density and non-repetition trajectory.

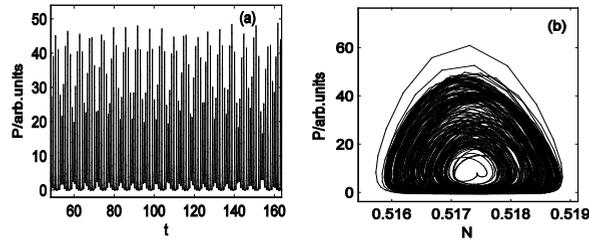


Fig. 5. Chaos. (a) another chaotic movement. (b) another chaotic attractor.

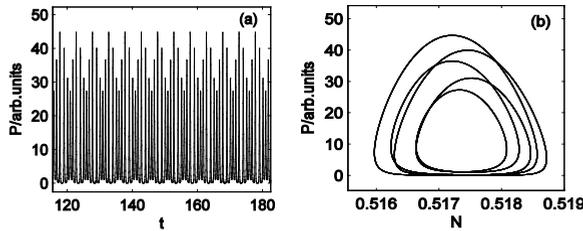


Fig. 6. A cycle-5. (a) a cycle-5 movement. (b) a cycle-5 orbit.

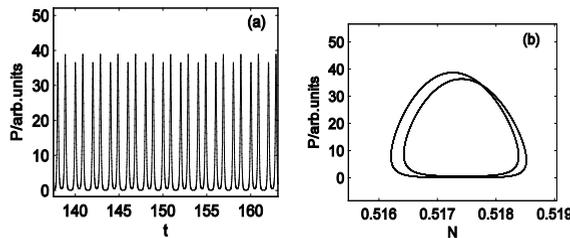
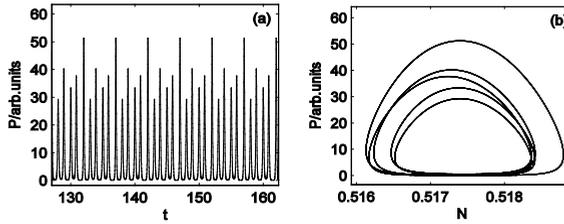


Fig. 7. A cycle-two. (a) a cycle-2 movement. (b) a cycle-2 orbit.

When  $f_1=0.2\text{kHz}$ , the laser is motivated to lead to a cycle-5 state shown in figure 6. Figure 6 (a) shows a cycle-5 movement of laser, and figure 6 (b) shows the cycle-5 movement orbit.

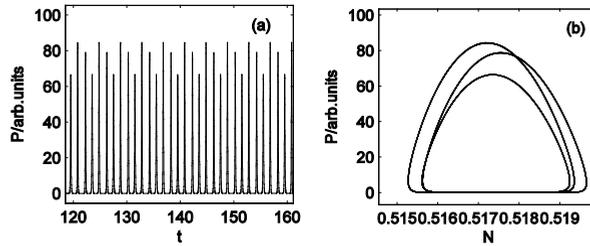
When  $f_1=0.5\text{kHz}$ , the laser is motivated to lead to a cycle-two state shown in figure 7. Figure 7 (a) shows a cycle-2 movement of laser, and figure 7 (b) shows the cycle-2 movement orbit.

When  $f_1=0.6\text{kHz}$ , the laser is motivated to lead to another cycle-5 state shown in figure 8. Figure 8 (a) shows a cycle-5 movement of laser, and figure 8 (b) shows the cycle-5 movement orbit.

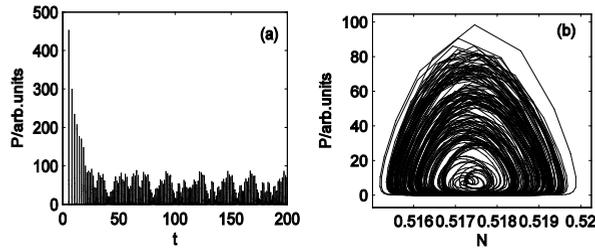


**Fig. 8.** Another cycle-5. (a) another cycle-5 movement. (b) another cycle-5 orbit.

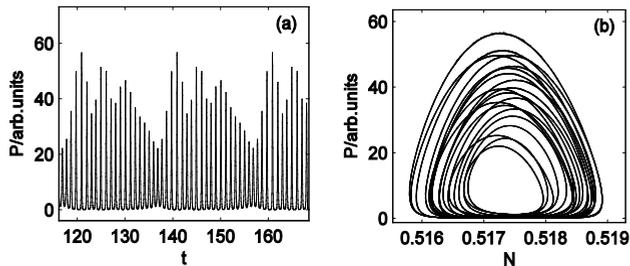
When  $f_1=0.75\text{kHz}$ , the laser is motivated to lead to a cycle-3 state shown in figure 9. Figure 9 (a) shows a cycle-3 movement of laser, and figure 9 (b) shows the cycle-3 movement orbit.



**Fig. 9.** A cycle-3. (a) a cycle-3 movement. (b) a cycle-3 orbit.



**Fig. 10.** Chaos. (a) a chaotic movement. (b) a chaotic attractor.



**Fig. 11.** A quasi-period. (a) a quasi-cycle movement. (b) a quasi-cycle orbit.

Adding  $f_1=0.8\text{kHz}$ , the laser is motivated to lead to a chaotic state shown in figure 10. Figure 10 (a) shows another chaotic movement of laser, and figure 10 (b) shows the chaotic attractor, and it is found that the chaotic laser movement illustrates a high density and non-repetition trajectory.

When  $f_1=0.95\text{kHz}$ , the laser is motivated to lead to a cycle-one state shown in figure 11. When  $f_1=1\text{kHz}$ , the laser is motivated to lead to a cycle-one state shown in figure 12.

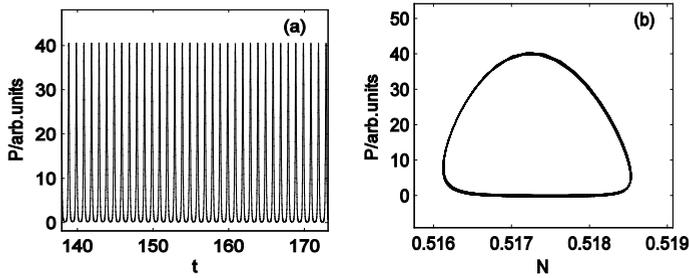


Fig. 12. A period-one. (a) a cycle-one movement. (b) a cycle-one orbit.

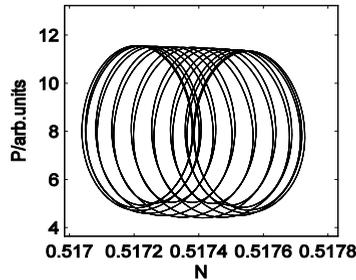


Fig. 13. Another quasi-period.

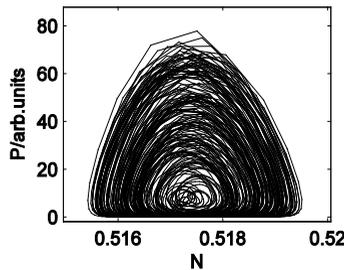


Fig. 14. Chaos.

### 3.2 The frequency shift of the photon absorber

When the dual-frequency shift modulation is performed on the laser and the parameters are taken as  $\mu=0.2$ ,  $\eta=0.01$ ,  $f_1=0.85\text{kHz}$  and  $f_2=0.1\text{kHz}$ , it is found that a quasi-cycle behaviour of the laser is motivated to show in figure 13.

When the frequency shifts in  $f_2=1.1\text{kHz}$ , it is found that a chaotic behaviour of the laser is motivated to show in figure 14. And it is found that the chaotic laser movement illustrates a high density and non-repetition trajectory and a chaotic attractor. And we find the chaos distribution with frequency variations of the absorption and modulation between 0.6kHz and 1.2kHz.

## 4 Conclusion

We studied a dual-frequency shift scheme to how to produce chaos and beating in an erbium-doped fiber laser. It can be found that the modulated laser produces chaos, beating, and a lot of quasi-periodic states. Our numerical result illustrates a route to chaos or away

from chaos after beating or quasi-period by shifting the pump modulation frequencies and the absorption frequencies of the attenuator. And we find the chaos distribution with frequency variations of the absorption and modulation. It can be found that two frequencies of the absorption and the modulation guide the dynamics of the modulated laser. The result has great referenced values to laser, optics, and chaos.

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