

# Thermal rating probability prediction considering the temporal correlation among the thermal ratings

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**Abstract.** The thermal rating of the overhead transmission line is an important parameter for the operation and control of the power system. In order to further integrate it into the system dispatch decisions, it is necessary to conduct the prediction for the thermal ratings in the look-ahead time horizon of dispatch decisions. At present, the researches on the thermal rating prediction focus on the independent prediction for the thermal rating in each prediction period, without considering the temporal correlation among the thermal ratings. In this paper, a joint probability prediction method for multiperiod thermal ratings considering the temporal correlation among the thermal ratings is proposed. Specifically, based on the temporal correlation among the thermal ratings and the independent probability prediction of thermal rating in each period, the multivariate normal probability density function of multiperiod thermal ratings is generated. The prediction simulation shows that considering the temporal correlation among the thermal ratings in the thermal rating prediction process can improve the prediction results and make full use of the current-carrying capability of overhead lines, which will promote the accommodation of renewable energy and energy saving and emission reduction.

## 1 Introduction

With the continuous increase of renewable energy generation and power consumption, the current-carrying pressure of some critical overhead transmission lines in the power grid is increasing [1]. Taking the traditional static thermal rating (STR) as the current-carrying limit of the overhead transmission lines will lead to the problem of the shortage of current-carrying capability of the lines, which will hinder the economic operation of the power systems and the accommodation of renewable energy [2].

The STR is calculated based on a fixed conservative set of meteorological conditions[3]. In fact, the real current-carrying capability of an overhead conductor changes with the change of meteorological conditions around the conductor. Therefore, replacing the STR

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with the dynamic thermal rating (DTR), which is calculated based on the real meteorological conditions around the conductor[4], as the current-carrying limit of the overhead conductors can make full use of the current-carrying capability of the line. This can alleviate the shortage of line current-carrying capability without power grid expansion and promote the accommodation of renewable energy, which will contribute to realizing energy saving and emission reduction. However, because the DTR is time-varying, to further integrate it into the system dispatch decisions, it is necessary to conduct the prediction for the thermal ratings in the look-ahead time horizon of dispatch decisions. So, the researches on thermal rating prediction have attracted extensive attention in recent years.

In [5], the Artificial Neural Networks model was applied to predict the thermal ratings up to 24 hours ahead based on the historical thermal ratings and NWP. In [6], the thermal ratings in the next 24 hours were predicted employing the integrated factorized Ornstein-Uhlenbeck process.

The above researches belong to the deterministic prediction of the thermal rating, which predicts the expected values of the thermal ratings in the future periods. However, due to the strong randomness and volatility of the thermal rating, it is difficult to be accurately predicted. And the deterministic prediction cannot provide the uncertainty information of the prediction of the thermal rating. In this case, the probability prediction research of the thermal ratings is developed. In [7], the quantile regression (QR) method was used to realize the probability prediction of the thermal ratings by taking the historical meteorological data and thermal ratings as the prediction input. In [8], the quantile regression forest (QRF) was used to achieve the thermal ratings probability prediction of the overhead conductor up to 6 hours ahead. In [9], the machine learning methods based on NWP, historical meteorological data, and historical thermal ratings were employed to produce the prediction intervals of the day-ahead thermal ratings.

It can be found that in the above researches, the thermal rating of each prediction period in the prediction time horizon is predicted independently without considering the temporal correlation among multiperiod thermal ratings in the prediction time horizon. However, the temporal correlation among multiperiod thermal ratings is very important information, which implies the distribution law of the change speed of the thermal ratings. Therefore, based on the independent probability prediction of the thermal rating in each prediction period and the correlation coefficient matrix of the errors between the predicted expectations and the actual values of multiperiod thermal ratings, this paper proposes a joint probability prediction method for multiperiod thermal ratings by generating the multivariate normal probability density function of multiperiod thermal ratings, which can integrate the temporal correlation among the thermal ratings into the probability prediction.

The remainder of this paper is organized as follows. Section 2 introduces the calculation of the thermal rating of the overhead conductor. Section 3 analyzes the correlations between the thermal rating and the corresponding meteorological elements and the correlations among multiperiod thermal ratings. Section 4 introduces the joint probability prediction method. Section 5 analyzes the prediction results obtained by the proposed method, and the conclusions are drawn in section.

## 2 Thermal rating calculation

IEEE standard [10] proposes a thermal balance model of the overhead conductor, which describes the relationship among the current passing through the overhead conductor, conductor temperature, and meteorological elements around the conductor. According to the model, the thermal rating of the overhead conductor can be expressed by

$$I_{\max} = \sqrt{\frac{q_c(T_{\max}) + q_r(T_{\max}) - q_s}{R(T_{\max})}}, \quad (1)$$

where  $T_{\max}$  is the maximum permissible temperature of the overhead conductor that can ensure the safe operation of the line;  $R(T_{\max})$  is the resistance of the per unit length conductor when the conductor temperature is  $T_{\max}$ ;  $q_r(T_{\max})$  is the radiation heat loss of the per unit length conductor, which is mainly affected by the difference of  $T_{\max}$  and ambient temperature ( $T_a$ ); for the convective heat loss  $q_c(T_{\max})$ , it is not only affected by  $T_a$  and  $T_{\max}$  but also depends on the wind speed ( $V_w$ ) and direction ( $\varphi$ ) around the conductor;  $q_s$  represents the heat gain based on solar radiation ( $Q_{se}$ ). The detailed calculation formulas of  $q_r(T_{\max})$ ,  $q_c(T_{\max})$ ,  $q_s$ , and  $R(T_{\max})$  can be found in [10].

### 3 Data analysis

The section conducts correlation analysis based on the collected meteorological data around a 220 kV overhead conductor ACSR400 that runs from north to south in 2019, with a 15-minute time resolution.

#### 3.1 Cross correlation analysis

It can be found from equation (1) that the main factors influencing the thermal rating of an overhead conductor are the meteorological elements around the conductor, including  $Q_{se}$ ,  $T_a$ ,  $V_w$ , and  $\varphi$ . Therefore, the Pearson correlation coefficients between the thermal rating ( $I_{\max}$ ) and  $Q_{se}$ ,  $T_a$ ,  $V_w$ , and the linear-circular rank correlation coefficient<sup>[11]</sup> between  $I_{\max}$  and  $\varphi$  ( $\varphi$  is a circular variable) are firstly calculated, to investigate and analyze the correlation between thermal ratings and meteorological elements. Pearson correlation coefficient is expressed by

$$\rho_{X,Y} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}}, \quad (2)$$

where  $\rho_{X,Y}$  represents the correlation coefficient between variable  $X$  and variable  $Y$ ;  $n$  is the number of the samples;  $X_i$  represents the  $i$ th sample of variable  $X$ ;  $Y_i$  is the  $i$ th sample of variable  $Y$ ;  $\bar{X}$  and  $\bar{Y}$  represent the average values of the variable  $X$  and variable  $Y$ , respectively. Fig. 1 shows the calculation results of the correlation coefficients between the thermal rating and different meteorological elements under different retardation time windows (0-4 hours with 15-minute time steps).

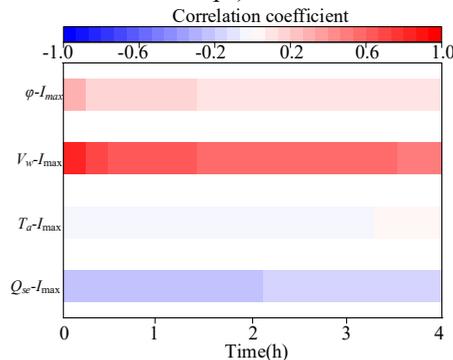


Fig. 1. Correlation coefficients under different retardation time windows.

It can be seen from Fig. 1 that there exists a certain correlation between the thermal rating and meteorological elements, and the correlation becomes weaker with the increase of retardation time. In addition, compared with other meteorological elements, the correlation between  $V_w$  and the thermal rating is relatively stronger.

### 3.2 Temporal correlation analysis

It can be found from equation (1) This subsection analyzes and investigates the temporal correlations among the thermal ratings. In this paper, autocorrelation functions (ACFs) are used to evaluate the temporal correlations among the thermal ratings. The ACF can be expressed as

$$\rho_X(a) = \frac{E[(X_t - \mu)(X_{t-a} - \mu)]}{\sigma^2}, \quad (3)$$

where  $\rho_X(a)$  is the autocorrelation coefficient when the retardation time is  $a$ ;  $X_t$  represents the samples of variable  $X$  at time  $t$ ;  $X_{t-a}$  represents the samples of variable  $X$  at time  $t-a$ ;  $\mu$  and  $\sigma$  represent the average value and standard deviation of the samples sequence, respectively.

Fig. 2 shows the calculation results of the ACFs for the thermal ratings under different retardation time windows (0-24 hours with 15-minute time steps). As shown in Fig. 2, when the retardation time is within 3 hours, the ACFs of the thermal ratings are higher than 0.5, which means that the temporal correlation of the thermal ratings within 3 hours is relatively stronger.

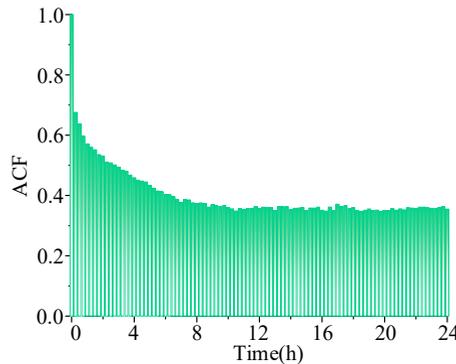


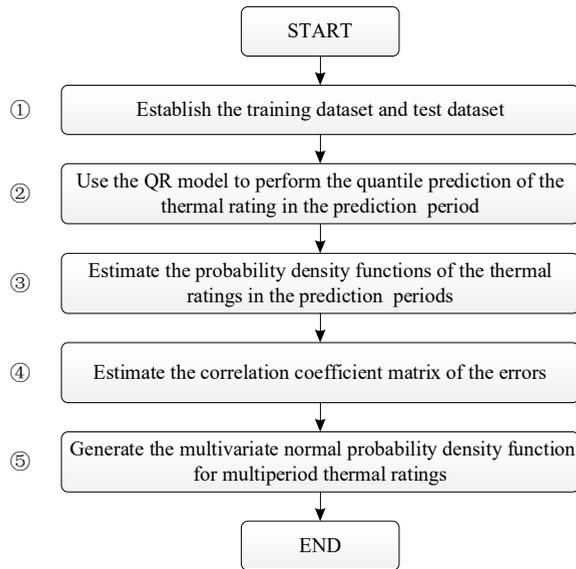
Fig. 2. ACFs for the thermal ratings under different retardation time windows.

### 4 Prediction method

In this paper, the probability prediction process of the thermal rating is divided into two steps. Specifically, the quantile regression (QR) method is firstly used to conduct the quantile prediction for the thermal rating in each prediction period over the prediction time horizon. Next, based on the predicted quantiles, the probability density functions of the thermal ratings in the corresponding periods are estimated to complete the first step of the prediction, the independent prediction of the thermal rating. Then, the multivariate normal probability density function of multiperiod thermal ratings in the prediction time horizon is generated based on the independent prediction results of each period, thus completing the second step of the prediction and realizing the joint probability prediction of multiperiod thermal ratings considering the temporal correlation among the thermal ratings.

### 4.1 Prediction process

The flowchart of the probability prediction method proposed in the paper is shown in Fig. 3.



**Fig. 3.** Flowchart of the prediction process.

In step ①, the training dataset and test dataset are established to make preparations for the thermal rating prediction.

In step ②, the QR model is used to perform quantile prediction for the thermal rating of each period in the prediction time horizon. The variables which have a strong correlation with the thermal rating in the prediction period are selected as the input parameters of the prediction model.

In step ③, based on the predicted quantiles of the thermal rating in each period over the prediction time horizon (see step ②), the probability density functions of the thermal ratings in the corresponding periods are estimated. Through research and analysis, this paper believes that the prediction results of the thermal rating in each prediction period based on the QR model obey the normal distribution.

In step ④, the correlation coefficient matrix of the errors between the predicted expectations and actual values of multiperiod thermal ratings in the prediction time horizon is estimated based on the training dataset.

In step ⑤, based on the correlation coefficient matrix estimated (see step ④) and the mean and standard deviation of the independent prediction of the thermal rating in each prediction period (see step ③), the multivariate normal probability density function of multiperiod thermal ratings in the prediction time horizon is generated to realize the joint probability prediction, which integrates the temporal correlation of the thermal ratings into the thermal rating prediction process.

## 4.2 Independent probability prediction of the thermal rating

In this subsection, the QR model is used to perform quantile prediction for the thermal rating of each period in the prediction time horizon. Referring to [7,12], the  $\tau$  quantile of the thermal rating in the  $k$ th period ( $Q_k(\tau)$ ) over the prediction time horizon can be expressed by

$$Q^k(\tau) = \beta_0^k(\tau) + \beta_1^k(\tau)x_1 + \beta_2^k(\tau)x_2 + \cdots + \beta_n^k(\tau)x_n, \quad k=1, \dots, K \quad (4)$$

where the  $\beta_i^k(\tau)$  ( $i=0, \dots, n; k=1, \dots, K$ ) are the regression parameters for the  $\tau$  quantile of the thermal rating in the  $k$ th prediction period;  $K$  is the number of the periods over the prediction time horizon;  $x_i$  ( $i=1, \dots, n$ ) are the input parameters. According to the correlation analysis in section 3, the length of the prediction time horizon is set as 1 hour, which consists of four 15-minute prediction periods ( $K=4$ ), and the historical 8-period thermal ratings and  $V_w$  from the previous 2 hours, which have a strong correlation with the thermal ratings in the prediction time horizon, are selected as the input of the QR model. And the regression parameter vector  $\beta^k(\tau)$  can be estimated by

$$\hat{\beta}^k(\tau) = \arg \min_{\beta^k \in R} \sum_{i=1}^{n_{sz}} \eta_{\tau}^k(y_i^k - Q^k(\tau)), \quad k = 1 \dots K \quad (5)$$

where  $y_i^k$  represents the actual values of the thermal rating;  $n_{sz}$  is the number of the samples; and  $\eta_{\tau}^k(\cdot)$  is a test function, which can be expressed by

$$\eta_{\tau}^k(a) = \begin{cases} \tau \cdot a & a \geq 0 \\ (\tau - 1) \cdot a & a < 0 \end{cases}, \quad k = 1 \dots K \quad (6)$$

After estimating the regression parameters of the QR model, quantile prediction can be conducted. In this paper, 99 quantiles ( $\tau=0.01, 0.02, \dots, 0.99$ ) are selected. Then, the probability density function of the thermal rating in the corresponding period can be estimated based on the predicted quantiles of the thermal rating in each prediction period. Through research and analysis, it is found that the prediction probability distribution of the thermal rating in each period based on the QR model is very close to the normal distribution. Therefore, considering that the thermal rating in each prediction period obeys the normal distribution, the probability density function can be estimated based on the predicted quantiles, thus realizing the independent probability prediction of the thermal rating in each prediction period.

## 4.3 Joint probability prediction for multiperiod thermal ratings

Taking the independent probability predictions of the thermal ratings in four consecutive periods over the prediction time horizon as the marginal distributions, the multivariate normal probability density function of the multiperiod thermal ratings, which considers the temporal correlation among the thermal ratings in the prediction time horizon, can be generated. It can be expressed as

$$f_y = \frac{1}{(2\pi)^2 |\mathbf{B}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{y} - \boldsymbol{\lambda})^T \mathbf{B}^{-1}(\mathbf{y} - \boldsymbol{\lambda})\right), \quad (7)$$

where  $\mathbf{y}$  is a 4 dimension random vector, which represents the thermal ratings in 4 periods over the prediction time horizon;  $\boldsymbol{\lambda}$  is the 4-dimension predicted expectation vector of the thermal ratings;  $\mathbf{B}$  is the predicted  $4 \times 4$  covariance matrix.

It is difficult to realize a direct prediction of the covariance matrix. Referring to [13], the prediction of the covariance matrix can be decomposed into two independent subproblems: constant correlation coefficient matrix estimation and standard deviation prediction for each marginal distribution, which can greatly simplify the prediction problem of the covariance matrix and has great practical value. Therefore, the predicted covariance matrix can be expressed by

$$B = DRD \tag{8}$$

where  $D$  is a diagonal matrix, whose diagonal elements are the predicted standard deviations of the thermal ratings in four periods over the prediction time horizon in the paper;  $R$  is the correlation coefficient matrix of the errors between the predicted expectations and actual values of thermal ratings in four consecutive periods. The correlation coefficient matrix of the errors can be calculated using equation (2) based on the historical data in the training dataset. The mean and standard deviation can be obtained from the marginal normal distribution probability density function constructed in subsection 4.2. Then, substituting the estimated correlation coefficient matrix and the mean values and standard deviations of the independent probability prediction of the thermal ratings in four prediction periods into equation (7) and equation (8), the joint probability density function of the multiperiod thermal ratings can be generated, thus integrating the temporal correlation of the thermal ratings into the probability prediction and realizing the joint probability prediction of multiperiod thermal ratings.

## 5 Case studies

The section conducts case studies based on the collected meteorological data around a 220 kV overhead conductor ACSR400 that runs from north to south in March 2020, with a 15-minute time resolution. The thermal ratings were firstly calculated using equation (1). Then, the thermal ratings and the meteorological data in the first 28 days were used as the training dataset to estimate the regression parameters of the QR model and the correlation coefficient matrix. And the data in the remaining 3 days were used as the test dataset to verify the performance of the proposed prediction method.

### 5.1 Probability prediction results of the thermal rating

In this subsection, the joint probability prediction for the multiperiod thermal ratings is conducted and the corresponding prediction intervals obtained are shown in Fig. 4 (The prediction intervals can be obtained based on the sampling results of the joint probability density function). The prediction time horizon is 1 hour, including four 15-minute prediction periods. The probability predictions are conducted hour by hour for 3 days. As a comparison, the prediction intervals based on the independent probability prediction for the thermal rating are also shown in Fig. 4.

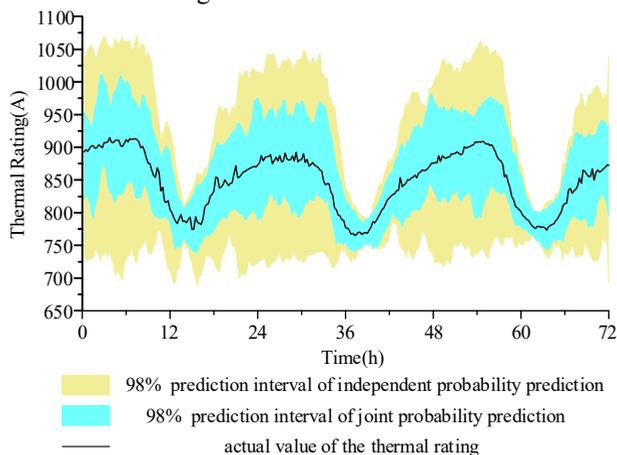


Fig. 4. The prediction intervals of two kinds of probability prediction methods.

It can be seen from Fig. 4 that the prediction intervals obtained based on the two kinds of prediction methods can both contain the actual values of the thermal ratings, which means that the two probability prediction results have good reliability. However, the prediction intervals obtained from the joint probability prediction for multiperiod thermal ratings are narrower, which indicates that it has better sharpness. To ensure the safety of power system operation, the lower boundaries of the probability prediction intervals of the thermal ratings are usually applied to the dispatch decision. The lower boundaries of the prediction intervals obtained from the joint probability prediction for multiperiod thermal ratings are relatively higher, which implies prediction considering the temporal correlation among the thermal ratings is conducive to making full use of the current-carrying capability of the overhead conductor. Therefore, it can contribute to the accommodation of renewable energy and energy saving and emission reduction.

## 5.2 Evaluation of probability prediction results

Reliability and sharpness are generally used to reflect the performance of the probability prediction results. The continuous ranked probability score (CRPS)<sup>[9]</sup> is a common evaluation criterion that can comprehensively evaluate the reliability and sharpness of probability prediction results. The CRPS of the probability prediction results for the thermal rating in the  $i$ th period can be expressed by

$$CRPS_i(f_i, I_{\max i}) = \int_{-\infty}^{+\infty} \left( \int_{-\infty}^I f_i(x) dx - H(I - I_{\max i}) \right)^2 dI \quad (9)$$

where the  $f_i$  is the predicted probability density function for the thermal rating in the  $i$ th period;  $I_{\max i}$  is the actual value of the thermal rating in the  $i$ th period;  $H(\cdot)$  is the Heaviside function. The smaller CRPS indicates better reliability and sharpness of the probability prediction results. The average CRPS of probability prediction results for thermal ratings in  $n_s$  periods can be expressed by

$$\text{Average CRPS} = \frac{1}{n_s} \sum_{i=1}^{n_s} CRPS_i \quad (10)$$

where  $n_s$  is the number of the prediction periods considered in the evaluation and it equals 72 in the paper. Table 1 shows the average CRPS of the prediction results based on two kinds of prediction methods.

**Table 1.** Evaluation criterion of two kinds of probability prediction results.

| Evaluation Criterion | Independent probability prediction | Joint probability prediction |
|----------------------|------------------------------------|------------------------------|
| Average CRPS         | 12.02                              | 8.51                         |

It can be found from Fig. 4 and table 1 that the joint probability prediction for multiperiod thermal ratings which considers the temporal correlation among the thermal ratings can improve the probability prediction results.

## 6 Conclusion

In this paper, a joint probability prediction method for multiperiod thermal ratings considering the temporal correlation among the thermal ratings is proposed. Based on the independent probability prediction of the thermal rating in each prediction period and the temporal correlation among the thermal ratings, the multivariate normal probability density function of multiperiod thermal ratings in the prediction time horizon can be generated. Compared with the independent probability prediction, the average CRPS of the joint probability prediction for multiperiod thermal ratings is reduced by 29.2%, which implies

that considering the temporal correlation can improve the prediction results. In addition, the joint probability prediction is also conducive to making full use of the current-carrying capability of overhead lines, thus promoting the accommodation of renewable energy and energy saving and emission reduction.

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