SCMA Codebook Design Based on a 16 Star-QAM with MED Maximization

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Abstract. The SCMA codebook design is based on star-QAM signalling constellations with operating several segmentations to generate the different user's codebook while considering the MED maximization as the criterion design. In addition, we constructed the mapping matrix F using Progressive Edge Growth (PEG) algorithm which facilitate iterative decoding performance of the message passing algorithm (MPA). At the end, a comparison with similar studies is conducted on the MEDs of the designed codebooks.

1 Introduction

Non-orthogonal multiple access (NOMA) has been considered as an enabling scheme for massive connectivity and ultra-low-latency in future communication networks [1]. NOMA serves multiple users simultaneously using different power levels or/and codebooks, taken advantage of the non-orthogonal spreading technique, Sparse code Multiple Access (SCMA) is proposed as a multi-dimensional constellation codebook.

In fact, two major research lines are conducted on the SCMA system: the codebook design and the multiuser detection. With the fact that the performance of the SCMA technique remains in the codebook design, we construct these codebooks under multi- stages: First, construct a mother constellation (MC), after assign the MC to different users using specific operators (permutation, rotation …). Finally, applying a mapping matrix which decide on the occupation of users to the resource elements.

To more improve the system performance such as spectral efficiency, bit error rate, and peak-to-average power ratio (PAPR), the codebook design could be optimized considering various key performances, mainly the Minimum Euclidean Distance (MED).

For instance, authors in [2]–[4] focused on optimizing the MED of the MC to generate codebooks with maximized MED, the large MED results in minimizing the complexity of the decoder by facilitating the decision making of the MPA.

Various work considered the maximization of the MC MED thanks to its better Bit to Error Ratio (BER) performance. Therefore, further studies as [5] and [6], are extended to a newer definition of MED: MED of superimposed codewords. Instead of optimizing the Euclidean Distance between the point-to-point within the same dimension of each codebook, the studies are conducted on the user's codewords overlapping on the same subcarrier to establish a larger MED.

While most recent studies as [7], [8], investigated maximization of the two different MEDs, simultaneously. As well as, in this paper we optimize the codebook design using the MED maximization criterion under the point-to-point of same codebook along with superimposed codewords.

In this paper, from a 16 star-QAM we design our codebook with segmentation while always maximizing MED either at the MC parameters construction, the sub-constellation generation, or at the sub-constellations mapping to form the final codebooks. The contributions of this study are:

Design the mapping matrix F using PEG algorithm which impose a respectable minimum distance bound that enhances decoding performances.

Construct an MC with large MED, after that, generates codebooks with large MED between the interfering users.

The rest of this paper is organized as follows. Section II represents the SCMA system model. Section III, the construction of the mapping matrix F using PEG algorithm is discussed. Section IV shows the steps of generating the SCMA codebooks from the constellation type choice of the MC to the final codebooks mapping, while maximizing MEDs. Section V, presents the results of the paper in comparison with other studies. Finally, Section VII presents the conclusion.

2 System model

The SCMA encoder maps a $\log_2(\mathrm{M})$ data/input bits to a K -dimensional complex codeword. The codeword is selected from a pre-defined codebook of size M, and it is a sparse vector with N<K non-zero entries. Finally, the SCMA codewords are transmitted over shared orthogonal resources. The received signal on subcarrier i can be expressed as:

$$y_i = \sum_{j=1}^{J} h_{ij}.x_{ij} + n \quad (1)$$

Here with $x_{ij}(w_j)$ denotes the i-th value within the codeword w_j of user j, h_j denotes the channel coefficient of user j and n is the ambient noise.

3 Constructing the matrix F using PEG

The relationship between users and subcarriers is determined based on a sparse mapping matrix F. Here, we propose to design the matrix F with PEG algorithm. The high flexibility of this algorithm makes it a good candidate to generate codes of any block length [7], hence the SCMA mapping matrix F. Further experimentations with this algorithm will help forge connections of a higher user's number and subcarriers.

A bipartite graph with K check nodes (rows) and J symbol nodes (columns) can be created using F. Such a graph is also called a Tanner graph, Fig. 1. The symbol degree sequence $D_s = \{d_{s1}, d_{s2}, \ldots, d_{sJ}\}$ denotes the number of check nodes connected to each symbol node.

Algorithm 1 Progressive Edge Growth to create F.

1. **Inputs:** Number of check nodes, Number of symbol nodes and symbol degree sequence.
2. **Outputs:** F.
3. **Initialization:** Set the symbol nodes edges to 0.

//*start with the first symbol node.

4. **If** (Current symbol node has no edges).
5. Select check node with the lowest symbol degree.
6. Establish an edge between the two nodes.

//*For each subsequent edge to be placed, a subtree (subgraph) is expanded from the variable node in question up to a depth l).

7. **Else**
8. **If** (The subgraph expanded from the current symbol node has not established edges with all the check nodes).
9. Select lowest check node from the set of uncovered check nodes.
10. **Else** (The current subgraph node has established edges with all the check nodes).
11. Choose check node farthest from symbol node.

The edge-selection procedure is made edge by edge, and progresses through columns from left to right, according to the Algorithm. 1.

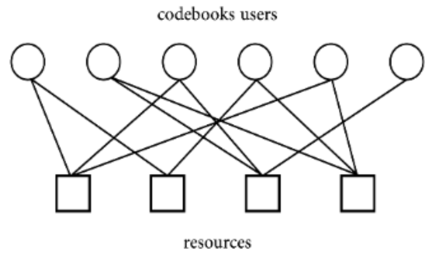

Fig. 1. Factor graph representation of 4 check nodes connected to 6 symbol nodes and $D_s = \{2, 2, 2, 2, 2, 2\}$.

4 SCMA codebook design

4.1 Mother Constellation (MC): Constellation type choice

To construct the multidimensional constellation there are several constellation choices such as: QAM, QPSK…, and for the SCMA choice it has more freedom degrees over previous techniques [3]. Here with, we propose to design the mother constellation using a 16 Star-QAM constellation of 2 radiuses, then we extend the design to four radiuses where the first quadrant points can be connected in a straight line to the third quadrant points, and the remaining constellation points are distributed on the abscissa axis [4], as shown in Fig. 2.

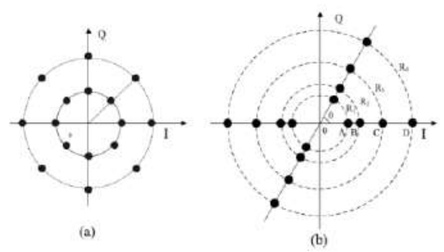

Fig. 2. (a)Constellation of round 16 QAM. (b) Proposed constellation.

Where: |OA| = R1, |OB| = R2, |OC| = R3, |OD| = R4.

The MC parameters can be written as:

$$MC = \begin{pmatrix} x_1 & x_2 & \alpha x_1 & \beta x_2 \\ -x_1 & -x_2 & -\alpha x_1 & -\beta x_2 \\ x_1 e^{i\theta} & x_2 e^{i\theta} & \alpha x_1 e^{i\theta} & \beta x_2 e^{i\theta} \\ -x_1 e^{i\theta} & -x_2 e^{i\theta} & -\alpha x_1 e^{i\theta} & -\beta x_2 e^{i\theta} \end{pmatrix} \quad (2)$$

The length of the sequences is equal to M, with $x_2 = \beta x_1$.. Also, we define:

The ratio between two successive points is: $\frac{R2}{R1} = \frac{R4}{R3} = \beta$, The ratio between non-successive points is: $\frac{R3}{R1} = \frac{R4}{R2} = \alpha$. And $\theta \in [0, 2\pi]$.

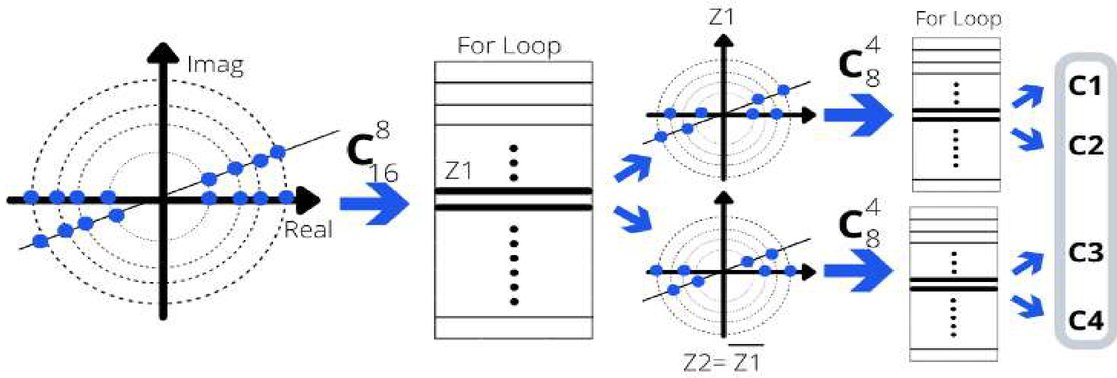

Fig. 3. Sub-constellations generation from the MC.

4.2 Mother constellation parameters

We propose to study the case of six users (J=6) associated with four subcarriers (K=4), (N = 2) entries non-null and M=4 representing the constellation points for occupation, each time single user transmits m bits with $M = 2^m$.

with (J, M, N, K) = (6, 4, 2, 4), to generate the MC defined by equation (2), we need to determine the three parameters (α, β and θ), which help calculate the four radiuses (R1, R2, R3 and R4). We set $x_1 = R_1$ and $x_2 = R_2$.

For a 16 star-QAM, in order to maximize the MED between two adjacent points, as denoted by [9], the following measures are to be taken in consideration:

A degree of $\theta = 22.5°$ is set between the internal ring and the external ring.

The ratio between the internal radius and the external one is set to 0.63, $\beta = \frac{1}{0.63}$.

In order to maximize both the sum distance with l, in equation (3), and the MED with r, in equation (4), of the codewords to ensure a good performance of the SCMA system, we have the following normalized minimum distances [2].

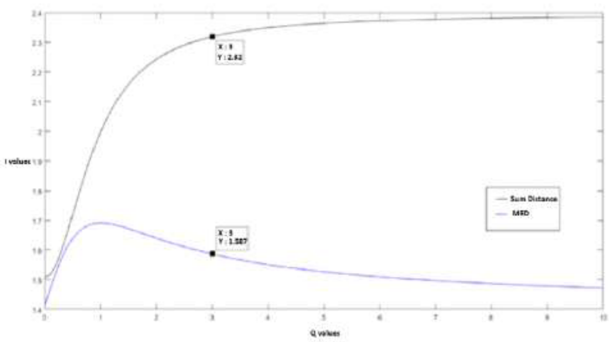

Fig. 4. Performance of Equation (3) and Equation (4).

$$r = \frac{d_{min}}{\sqrt{E}} = \sqrt{2(1 + 2\left(\frac{\beta^2-1}{1+\beta^2}\right)\cdot\left(\frac{1}{\frac{1}{\alpha}+\alpha}\right)} \qquad (3)$$

$$l = \frac{d_{min}}{\sqrt{E}} = \sqrt{8\left(\frac{1}{1+\frac{1}{\beta^2}}\right)\cdot\left(1 - \frac{1-\frac{1}{\beta^2}}{1+\alpha^2}\right)} \qquad (4)$$

The average signal energy is defined as $E = \frac{1}{M}\sum_{i=1}^{M}\|x_i\|^2$; so $E = 0.25\ (R_1^2 + R_2^2 + R_3^2 + R_4^2)$.

The variation of α for equation (3) and equation (4) is represented by Fig.4. Therefore, the maximization of both r and l is achieved by increasing alpha, but at one point r becomes opposite to l where r increases and l decreases, therefore, for a range of alpha variation we found that the perfect value is α=3.

4.3 Sub-constellation generation with MED maximization

With the massive user's connection supported by the SCMA system, the decoder complexity is increased. At the receiver side, to avoid decoding confusion d_f> U is a condition to satisfy [4] with d_f the number of sub-constellations and U the actual number of users on a single subcarrier.

The idea behind the MC segmentation is to extract sub-constellations, with the best and maximum MED, from the multiple choices existing from the MC divisions. We start the divisions of the MC into d_f sub-constellations as shown in Fig. 3.

For a 16-QAM MC there are 16 points to be distributed on d_f sets, with the condition of maximizing the point-to-point MED within the same sub-constellation. The first division gives two sets each with 8 points. The selection of the set's points is accomplished with help of the combination operator C^8_{16} (Mc) which generate several 8 points sets with different points from the MC, and then the loop for go through all the generated sets, noted as Z, searching for the sets with the largest MED. With Z a 36×8 matrix, select Z1=Z(i) as a candidate set and Z2 the complementary points to Z1 from the MC, with $i = [1,36]$.

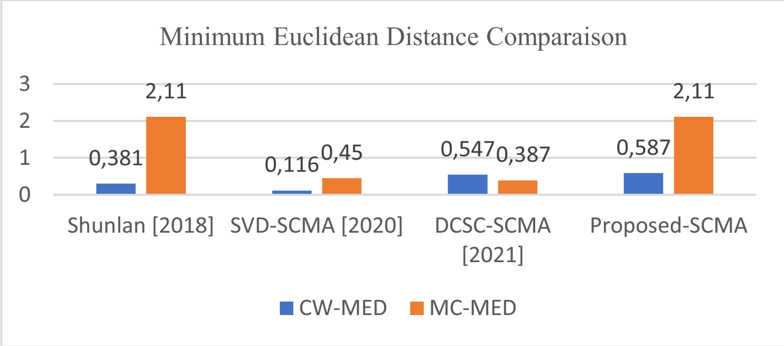

Fig. 5. Comparison of the mother constellation MED (MC-MED) and the superimposed codewords MED (CW-MED).

Operate another division to both sets Z1 and Z2 with the same pervious procedures: Generate all possible combinations of 4 points with C_8^4 from Z1 and Z2, each of the two sets (Z1, Z2) generate another two sets each of 4 points. Arriving at the end of the divisions with 4 sets, we calculate the MED of every set, after we decide whether it's the best 4 sets with the largest MED or not.

4.4 SCMA codebook generation by sub-constellation mapping with MED maximization

We have d_f sub-constellations generated from the MC $\{C_1, C_2 \ldots . C_{df}\}$, such as each subset contains Q/d_f points and Q is the number of the MC QAM constellation points. These sub-constellations are the base to the codebook, we map the sub-constellations to the user's codebook as:

$$CB(i,j) = \begin{cases} C'_{i(n)} & \text{if } f_{ij} = 1 \\ 0 & \text{if } f_{ij} = 0 \end{cases} \qquad (5)$$

With $C'_{i(n)}$ is the n-th value of a specific reordering of the set $\{C_1, C_2 \ldots . C_{df}\}$. Using the mapping matrix F, we replace the elements "1" with $C'_{i(n)}$.

In addition to maximize MED of the sub-constellations point-to-point within the same codeword, we aim to maximize the MED between superimposed codewords of conflicting users on each subcarrier.

Therefore, we project the sub-constellations on the overlapping subcarrier, after we calculate the distances between the superimposed codewords. The best sets achieving the largest MED are to be taking in consideration to form our desired codebook. The codebook of the six users is represented in Table. 1.

5 Numerical Results and Discussions

Fig. 5. shows the comparison of MC-MED and superimposed CW-MED of the codebooks. The comparison shows the better performance of the proposed codebooks within the two different MEDs. The better performance of the proposed codebooks returns to the main focus on the MED maximization criterion. However, these papers investigated further other factors such as:

with DCSC-SCMA [8], authors took the codebook design under power constraints, such as balancing the MC power, in addition to the maximization of both MEDs. Also, with SVD-SCMA [7] authors first construct the MC based on singular value decomposition (SVD) to guarantee a higher MED, then applying a rotation operator to generate different user's codebooks in which MED of the different users are equal. With [4], the main focus of the authors was on maximizing the MC MED but no optimization was on the MED of superimposed codewords. Finally, here comes the necessity of BER comparisons to be taken in consideration to more value the codebook performance.

	Codebook
1	$\begin{bmatrix} 1 & 2.781+1.123i & -1 & -2.781-1.123i \\ 1.587 & 4.414+1.783i & -1.587 & -4.414-1.783i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
2	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.587 & 4.414+1.783i & -1.587 & -4.414-1.783i \\ 1 & 2.781+1.123i & -1 & -2.781-1.123i \end{bmatrix}$
3	$\begin{bmatrix} 1.587 & 4.414+1.783i & -1.587 & -4.414-1.783i \\ 0 & 0 & 0 & 0 \\ 1.471+0.594i & 4.761 & -1.471-0.594i & -4.761 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
4	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.927+0.374i & 3 & -0.927-0.374i & -3 \\ 0 & 0 & 0 & 0 \\ 1.471+0.594i & 4.761 & -1.471-0.594i & -4.761 \end{bmatrix}$
5	$\begin{bmatrix} 1.471+0.594i & 4.761 & -1.471-0.594i & -4.761 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.587 & 4.414+1.783i & -1.587 & -4.414-1.783i \end{bmatrix}$
6	$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1.471+0.594i & 4.761 & -1.471-0.594i & -4.761 \\ 1 & 2.781+1.123i & -1 & -2.781-1.123i \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Table 1. SCMA codebook of an optimized round 16-QAM modulation after division with (J, M, N, K) = (6, 4, 2, 4).

6 Conclusion

For the codebook design we covered the construction of the mapping matrix F with PEG algorithm, then constructed the MC from a 16 Star-QAM constellation. After generated the user's codebooks with segmentation of the MC while keeping the MED maximization as the main criterion. Finally, we had acceptable results compared to the other studies at the MC-MED and CW-MED, keeping in mind the better performances of these studies in other fields.

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