ITM Web of Conferences **48**, 04002 (2022) https://doi.org/10.1051/itmconf/20224804002
ICCWCS 2022

Backstepping control strategy for induction motor with rotor flux and load torque observers

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Abstract. This paper deal with Backstepping control strategy for induction motor drive, using rotor flux and load torque observers. The proposed study uses a modified motor model with new variables, and Luenberger observers for both rotor flux and load torque. This work proposes a precise calculation method for controller parameters based on response time and damping coefficient. Simulation is performed in Simulink, results show that Backstepping works well by adding observers.

1 Introduction

Induction motor is a nonlinear system, so its control cannot be directly designed by classical methods; because it requires some special operations such as decoupling [1], switching table [2]... As a result, the traditional control strategies have complex algorithms and do not always give good results.

Backstepping is an advanced control strategy based on Lyapunov theory [3]. It is performed in several steps, depending on the order of the controlled system, and it can be easily adapted to nonlinear systems of strict feedback form, but also to nonlinear forms of systems by applying appropriate changes of variables.

This paper proposes a change of variables to establish a simple state model in the rotor flux frame for easy use of Backstepping control strategy.

Since the control law requires rotor flux and load torque, this work proposes the design of an observer allowing the calculation of these quantities using Luenberger method.

2 Induction motor modelling

In the two-phase reference frame, electromagnetic state representation of induction motor contains four variables, for this study we choose the two stator currents and two virtual currents replacing rotor flux components [4][1] .

Electromagnetic state-space model is given by (1) and completed by the mechanical equation (2).

$$\frac{d}{dt}\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{\varphi d} \\ i_{\varphi q} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_c} & \frac{d\rho s}{dt} & \frac{1-\sigma}{\sigma Tr} & \frac{1-\sigma}{\sigma} p\Omega \\ -\frac{d\rho s}{dt} & -\frac{1}{T_c} & -\frac{1-\sigma}{\sigma} p\Omega & \frac{1-\sigma}{\sigma Tr} \\ \frac{1}{Tr} & 0 & -\frac{1}{Tr} & \frac{d\rho r}{dt} \\ 0 & \frac{1}{Tr} & -\frac{d\rho r}{dt} & -\frac{1}{Tr} \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{\varphi d} \\ i_{\varphi q} \end{bmatrix} + \frac{1}{\sigma Ls} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} \quad (1)$$

$$\frac{d\Omega}{dt} = -\frac{f}{J}\Omega + \frac{T_M}{J} - \frac{T_L}{J} \quad (2)$$

With:

$$T_M = \frac{3}{2} p(1-\sigma) Ls\left(i_{\varphi d} i_{sq} - i_{\varphi q} i_{sd}\right) \quad (3)$$

$$i_{\varphi d} = \frac{\varphi_{rd}}{Msr};\ i_{\varphi q} = \frac{\varphi_{rq}}{Msr} \quad (4)$$

$$\frac{1}{T_c} = \left(\frac{1}{\sigma Ts} + \frac{1-\sigma}{\sigma Tr}\right) \quad (5)$$

$$\sigma = 1 - \frac{Msr^2}{Ls.Lr} \quad (6)$$

The reference is the same for all variables, then:

$$\frac{d\rho s}{dt} = p\Omega + \frac{d\rho r}{dt} \quad (7)$$

2.1 Induction motor modelling in rotor flux frame

Backstepping can be applied in fixed reference frame, but the equations are longer in this case as well as the resulting control law. This work proposes to apply Backstepping strategy in rotor flux frame [2][1].

In the new reference frame, electromagnetic state representation is defined by the equation (8), completed by the mechanical equation (9).

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$$\frac{d}{dt}\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{\varphi d} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_c} & d\rho s/dt & \frac{1-\sigma}{\sigma Tr} \\ -d\rho s/dt & -\frac{1}{T_c} & -\frac{1-\sigma}{\sigma} p\Omega \\ \frac{1}{Tr} & 0 & -\frac{1}{Tr} \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{\varphi d} \end{bmatrix} + \frac{1}{\sigma Ls}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} \quad (8)$$

$$\frac{d\Omega}{dt} = -\frac{f}{J}\Omega + \frac{T_M}{J} - \frac{T_L}{J} \quad (9)$$

With:

$$T_M = \frac{3}{2}p(1-\sigma)Lsi_{\varphi d}i_{sq} \quad (10)$$

It is clear that the system becomes simpler with only four variables instead of five [5].

2.2 Induction motor modelling with new variables

For simplification, another state-space representation is proposed in the equations (11), (12), (13) and (14). This representation allows to choose the adequate of variables for easy control design [6][4].

$$\frac{d}{dt}\Omega = -\frac{f}{J}\Omega - \frac{T_L}{J} + \frac{3p}{2J}(1-\sigma)Lsi_{\varphi d}i_{sq} \quad (11)$$

$$\frac{d}{dt}\left(i_{\varphi d}i_{sq}\right) = -\frac{1}{T_e}i_{\varphi d}i_{sq} - p\Omega i_{\varphi d}i_{sd} - \frac{1-\sigma}{\sigma}p\Omega i_{\varphi d}^2 + \frac{1}{\sigma Ls}i_{\varphi d}v_{sq} \quad (12)$$

$$\frac{d}{dt}i_{\varphi d}^2 = -\frac{2}{Tr}i_{\varphi d}^2 + \frac{2}{Tr}i_{\varphi d}i_{sd} \quad (13)$$

$$\frac{d}{dt}\left(i_{\varphi d}i_{sd}\right) = -\frac{1}{T_e}i_{\varphi d}i_{sd} + p\Omega i_{\varphi d}i_{sq} + \frac{1-\sigma}{\sigma Tr}i_{\varphi d}^2 + \frac{1}{Tr}i_s^2 + \frac{1}{\sigma Ls}i_{\varphi d}v_{sd} \quad (14)$$

With:

$$\frac{1}{T_e} = \frac{1}{\sigma}\left(\frac{1}{Tr} + \frac{1}{Ts}\right) \quad (15)$$

It should be noted that the modulus of the stator current is not a separate variable, as it depends on the other variables (16).

$$i_s^2 = \frac{(i_{\varphi d}i_{sq})^2 + (i_{\varphi d}i_{sq})^2}{i_{\varphi d}^2} \quad (16)$$

Proposed new state variables and input variables are defined by equations (17) and (18).

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \Omega \\ \frac{3p}{2J}(1-\sigma)Lsi_{\varphi d}i_{sq} \\ i_{\varphi d}^2 \\ \frac{2}{Tr}i_{\varphi d}i_{sd} \end{bmatrix} \quad (17)$$

$$\begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{3p}{2J}\frac{1-\sigma}{\sigma}i_{\varphi d}v_{sq} \\ \frac{2}{Tr}\frac{1}{\sigma Ls}i_{\varphi d}v_{sd} \end{bmatrix} \quad (18)$$

The resulting state-space representation is given by the four-equations system (19), (20), (21) and (22).

$$\frac{d}{dt}x_1 = -\frac{f}{J}x_1 - \frac{T_L}{J} + x_2 \quad (19)$$

$$\frac{d}{dt}x_2 = -\frac{1}{Te}x_2 - \frac{3p^2}{4J}Tr(1-\sigma)Lsx_1x_4 - \frac{3p^2}{2J}\frac{(1-\sigma)^2Ls}{\sigma}x_1x_3 + \frac{3p}{2J}\frac{1-\sigma}{\sigma}i_{\varphi d}v_{sq} \quad (20)$$

$$\frac{d}{dt}x_3 = -\frac{2}{Tr}x_3 + x_4 \quad (21)$$

$$\frac{d}{dt}x_4 = -\frac{1}{Te}x_4 + \frac{4J}{3Tr}\frac{1}{(1-\sigma)Ls}x_1x_2 + 2\frac{1-\sigma}{\sigma Tr^2}x_3 + \frac{2}{Tr^2}i_s^2 + \frac{2}{Tr}\frac{1}{\sigma Ls}i_{\varphi d}v_{sd} \quad (22)$$

2.3 Vector modelling of induction motor

In order to simplify the synthesis of Backstepping control, this work proposes to group the state and input variables into two-dimensional vector variables [6], thus the state model is given by the vector equations (23) and (24).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_3 \end{bmatrix} = -\begin{bmatrix} \frac{1}{Tm}x_1 + \frac{T_L}{J} \\ \frac{2}{Tr}x_3 \end{bmatrix} + \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} \quad (23)$$

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{Te}x_2 - \frac{3p^2}{2J}\frac{(1-\sigma)^2Ls}{\sigma}x_1x_3 - \frac{3p^2}{4J}Tr(1-\sigma)Lsx_1x_4 \\ -\frac{1}{Te}x_4 + \frac{4J}{3Tr}\frac{1}{(1-\sigma)Ls}x_1x_2 + 2\frac{1-\sigma}{\sigma Tr^2}x_3 + \frac{2}{Tr^2}i_s^2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} \quad (24)$$

3 Backstepping control design of induction motor

Vector state-space representation of induction motor can be written compactly in equations (25) and (26), where the state and input variables are two-dimensional vectors [3][7].

$$\dot{\boldsymbol{X}}_1 = \boldsymbol{F}_1(\boldsymbol{X}_1) + \boldsymbol{X}_2 \quad (25)$$

$$\dot{\boldsymbol{X}}_2 = \boldsymbol{F}_2(\boldsymbol{X}_1, \boldsymbol{X}_2) + \boldsymbol{U} \quad (26)$$

Where:

$$\boldsymbol{X}_1 = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix};\ \boldsymbol{X}_2 = \begin{bmatrix} x_2 \\ x_4 \end{bmatrix};\ \boldsymbol{U} = \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} \quad (27)$$

$$\boldsymbol{F}_1(\boldsymbol{X}_1) = -\begin{bmatrix} \frac{1}{Tm}x_1 + \frac{T_L}{J} \\ \frac{2}{Tr}x_3 \end{bmatrix} \quad (28)$$

$$\boldsymbol{F}_2(\boldsymbol{X}_1, \boldsymbol{X}_2) = \begin{bmatrix} -\frac{1}{Te}x_2 - \frac{3p^2}{2J}\frac{(1-\sigma)^2Ls}{\sigma}x_1x_3 - \frac{3p^2}{4J}Tr(1-\sigma)Lsx_1x_4 \\ -\frac{1}{Te}x_4 + \frac{4J}{3Tr}\frac{1}{(1-\sigma)Ls}x_1x_2 + 2\frac{1-\sigma}{\sigma Tr^2}x_3 + \frac{2}{Tr^2}i_s^2 \end{bmatrix} \quad (29)$$

It is a second-order vector system of strict feedback form which is easily amenable to Backstepping control design. Only two steps are necessary to establish the control law [8].

3.1 First step Backstepping design

In this step, variable X_1 representing flux and speed is controlled by the intermediate variable X_2. The aim is to find the desired control law X_{2d}.
We first define the first error (30), then we introduce the stabilizing control term in the error derivative (31).

$$\boldsymbol{E_1} = \boldsymbol{X_1} - \boldsymbol{X_{1ref}} \quad (30)$$

$$\dot{\boldsymbol{E}}_1 = \boldsymbol{F_1}(\boldsymbol{X_1}) + \boldsymbol{X_{2d}} - \dot{\boldsymbol{X}}_{1ref} = -\boldsymbol{K_1}\boldsymbol{E_1} \quad (31)$$

Lyapunov candidate function is positive and its derivative is negative when the coefficients of the diagonal matrix K_1 are positive; this ensures stability in this step (32).

$$V_1 = \frac{1}{2}\boldsymbol{E_1^T}\boldsymbol{E_1} > 0 \rightarrow \dot{V}_1 = \boldsymbol{E_1^T}\dot{\boldsymbol{E}}_1 = -\boldsymbol{E_1^T}\boldsymbol{K_1}\boldsymbol{E_1} < 0 \quad (32)$$

Thus, intermediate control law is given by equation (33).

$$\boldsymbol{X_{2d}} = -\boldsymbol{K_1}\boldsymbol{E_1} - \boldsymbol{F_1}(\boldsymbol{X_1}) + \dot{\boldsymbol{X}}_{1ref} \quad (33)$$

3.2 Second step Backstepping design

In this step the intermediate error (34) is defined in order to calculate the main control law [9].

$$\boldsymbol{E_2} = \boldsymbol{X_2} - \boldsymbol{X_{2d}} = \boldsymbol{X_2} + \boldsymbol{K_1}\boldsymbol{E_1} + \boldsymbol{F_1}(\boldsymbol{X_1}) - \dot{\boldsymbol{X}}_{1ref} \quad (34)$$

In this case we have to rewrite the error derivative of the first step considering the error on the secondary variable. This results in the equation (35).

$$\dot{\boldsymbol{E}}_1 = \boldsymbol{F_1}(\boldsymbol{X_1}) + \boldsymbol{E_2} + \boldsymbol{X_{2d}} - \dot{\boldsymbol{X}}_{1ref} = -\boldsymbol{K_1}\boldsymbol{E_1} + \boldsymbol{E_2} \quad (35)$$

A second positive candidate Lyapunov function must be defined on the basis of the two error vectors (36). The control law must guarantee the stability of the system, so it must be able to make the derivative (37) of this new Lyapunov function negative.

$$V_2 = \frac{1}{2}\boldsymbol{E_1^T}\boldsymbol{E_1} + \frac{1}{2}\boldsymbol{E_2^T}\boldsymbol{E_2} > 0 \quad (36)$$

$$\dot{V}_2 = \boldsymbol{E_1^T}\dot{\boldsymbol{E}}_1 + \boldsymbol{E_2^T}\dot{\boldsymbol{E}}_2 = \boldsymbol{E_1^T}(-\boldsymbol{K_1}\boldsymbol{E_1} + \boldsymbol{E_2}) + \boldsymbol{E_2^T}\dot{\boldsymbol{E}}_2 = -\boldsymbol{E_1^T}\boldsymbol{K_1}\boldsymbol{E_1} + \boldsymbol{E_1^T}\boldsymbol{E_2} + \boldsymbol{E_2^T}\dot{\boldsymbol{E}}_2 = -\boldsymbol{E_1^T}\boldsymbol{K_1}\boldsymbol{E_1} + \boldsymbol{E_2^T}(\boldsymbol{E_1} + \dot{\boldsymbol{E}}_2) \quad (37)$$

It is necessary to introduce a stabilizing term (38) with positive coefficients control matrix K_2

$$\boldsymbol{E_1} + \dot{\boldsymbol{E}}_2 = -\boldsymbol{K_2}\boldsymbol{E_2} \quad (38)$$

This leads to (39).

$$\dot{V}_2 = -\boldsymbol{E_1^T}\boldsymbol{K_1}\boldsymbol{E_1} - \boldsymbol{E_2^T}\boldsymbol{K_2}\boldsymbol{E_2} < \boldsymbol{0} \quad (39)$$

To find the main control law it is necessary to calculate the derivative of the second error (40).

$$\dot{\boldsymbol{E}}_2 = \dot{\boldsymbol{X}}_2 - \dot{\boldsymbol{X}}_{2d} = \boldsymbol{F_2}(\boldsymbol{X_1},\boldsymbol{X_2}) + \boldsymbol{U} - \dot{\boldsymbol{X}}_{2d} \quad (40)$$

Finally, the control law is given by the equations (41), (42), (43), (44), (45), (46), and (47). The order of equations is reversed when implementing control algorithm.

$$\boldsymbol{U} = \dot{\boldsymbol{E}}_2 - \boldsymbol{F_2}(\boldsymbol{X_1},\boldsymbol{X_2}) + \dot{\boldsymbol{X}}_{2d} \quad (41)$$

$$\dot{\boldsymbol{E}}_2 = -\boldsymbol{E_1} - \boldsymbol{K_2}\boldsymbol{E_2} \quad (42)$$

$$\dot{\boldsymbol{X}}_{2d} = -\boldsymbol{K_1}\dot{\boldsymbol{E}}_1 - \dot{\boldsymbol{F}}_1(\boldsymbol{X_1}) + \ddot{\boldsymbol{X}}_{1ref} \quad (43)$$

$$\dot{\boldsymbol{E}}_1 = -\boldsymbol{K_1}\boldsymbol{E_1} + \boldsymbol{E_2} \quad (44)$$

$$\boldsymbol{E_2} = \boldsymbol{X_2} - \boldsymbol{X_{2d}} \quad (45)$$

$$\boldsymbol{X_{2d}} = -\boldsymbol{K_1}\boldsymbol{E_1} - \boldsymbol{F_1}(\boldsymbol{X_1}) + \dot{\boldsymbol{X}}_{1ref} \quad (46)$$

$$\boldsymbol{E_1} = \boldsymbol{X_1} - \boldsymbol{X_{1ref}} \quad (47)$$

3.3 Calculation of the Backstepping controller parameters.

According to Lyapunov method, the system is stable when parameters positive. However, it is necessary to choose values that guarantee the specific performance. First, error vectors correspond to the equations (48).

$$\boldsymbol{E_1} = \begin{bmatrix} e_1 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_{1ref} \\ x_3 - x_{3ref} \end{bmatrix};\ \boldsymbol{E_2} = \begin{bmatrix} e_2 \\ e_4 \end{bmatrix} = \begin{bmatrix} x_2 - x_{2d} \\ x_4 - x_{4d} \end{bmatrix} \quad (48)$$

On the other hand, coefficients of the controller are given by the equations (49).

$$\boldsymbol{K_1} = \begin{bmatrix} k_1 & 0 \\ 0 & k_3 \end{bmatrix};\ \boldsymbol{K_2} = \begin{bmatrix} k_2 & 0 \\ 0 & k_4 \end{bmatrix} \quad (49)$$

Thus, it is possible to write the error state-space equation (50) from the error vector equations. It is a fourth order system.

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 & 0 & 0 \\ -1 & -k_2 & 0 & 0 \\ 0 & 0 & -k_3 & 1 \\ 0 & 0 & -1 & -k_4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \quad (50)$$

This system (50) is separable into two subsystems of second order each. This results in two equations (51) and (52).

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 \\ -1 & -k_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \quad (51)$$

$$\begin{bmatrix} \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} = \begin{bmatrix} -k_3 & 1 \\ -1 & -k_4 \end{bmatrix} \begin{bmatrix} e_3 \\ e_4 \end{bmatrix} \quad (52)$$

From characteristic polynomial (53), coefficients are calculated to ensure a unitary damping coefficient and a response time $tr = 200ms$.

$$det \begin{bmatrix} \lambda + k_1 & -1 \\ 1 & \lambda + k_2 \end{bmatrix} = \lambda^2 + (k_1 + k_2)\lambda + k_1k_2 + 1 = \lambda^2 + 2m\omega_n\lambda + \omega_n^2 \quad (53)$$

$$m = 1 \rightarrow tr = \frac{4.75}{\omega_n} \rightarrow \omega_n = \frac{4.75}{tr} = \frac{4.75}{0.2} = 23.75 \quad (54)$$

So, controller parameters are given by (55) and (56).

$$k_3 = k_1 = \omega_n + 1 = 24.75 \quad (55)$$

$$k_4 = k_2 = \omega_n - 1 = 22.75 \quad (56)$$

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3.4 Synoptic diagram of Backstepping control

Control scheme contain a Backstepping controller and observers for rotor flux and load torque.

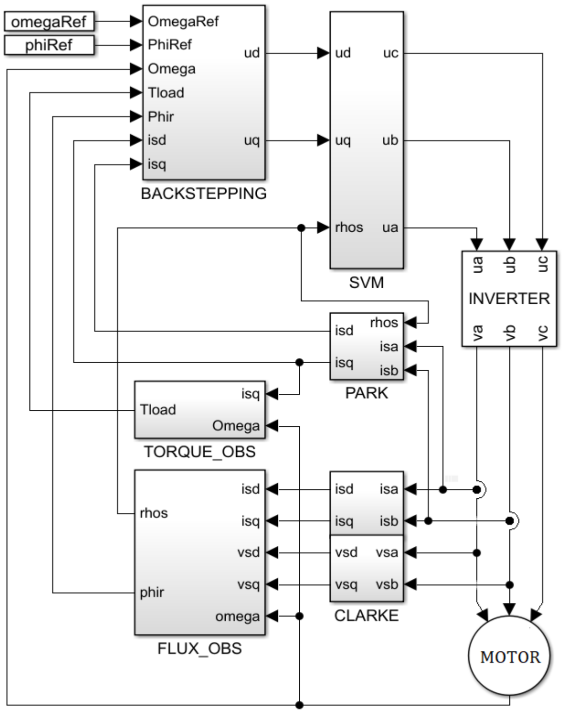

Fig. 1. Synoptic diagram of Backstepping control

4 Rotor flux and load torque observers

In this work, measured quantities are voltages, currents and speed. Rotor flux components are not measured but they are necessary for the control design, so it is necessary to estimate them using an observer [10][8][11].

The load torque also appears in control law, so it must be observed because its value is assumed to be unknown.

4.1 Rotor flux observer

This paper proposes to estimate rotor flux components in fixed reference frame, in order to calculate rotor flux modulus and its angle necessary [12][13]. The complex state-space representation of the motor is used: (57) and (58).

$$\frac{d}{dt}\underline{Is} = -\frac{1}{T_c}\underline{Is} + \frac{1-\sigma}{\sigma}\left(\frac{1}{Tr} - jp\Omega\right)\underline{I\varphi} + \frac{1}{\sigma Ls}\underline{Vs} \quad (57)$$

$$\frac{d}{dt}\underline{I\varphi} = \frac{1}{Tr}\underline{Is} - \left(\frac{1}{Tr} - jp\Omega\right)\underline{I\varphi} \quad (58)$$

From this complex representation, complex equations (59) and (60) of rotor flux observer are proposed.

$$\frac{d}{dt}\underline{\tilde{I}s} = -\frac{1}{T_c}\underline{Is} + \frac{1-\sigma}{\sigma}\left(\frac{1}{Tr} - jp\Omega\right)\underline{\tilde{I}\varphi} + \frac{1}{\sigma Ls}\underline{Vs} - C1(\underline{\tilde{I}s} - \underline{Is}) \quad (59)$$

$$\frac{d}{dt}\underline{\tilde{I}\varphi} = \frac{1}{Tr}\underline{Is} - \left(\frac{1}{Tr} - jp\Omega\right)\underline{\tilde{I}\varphi} - C2(\underline{\tilde{I}s} - \underline{Is}) \quad (60)$$

This results in the complex state-space equation of the observation errors (61).

$$\frac{d}{dt}\begin{bmatrix}\underline{\tilde{I}s} - \underline{Is}\\ \underline{\tilde{I}\varphi} - \underline{I\varphi}\end{bmatrix} = \begin{bmatrix}-C1 & \frac{1-\sigma}{\sigma}C\\ -C2 & -C\end{bmatrix}\begin{bmatrix}\underline{\tilde{I}s} - \underline{Is}\\ \underline{\tilde{I}\varphi} - \underline{I\varphi}\end{bmatrix} \quad (61)$$

Where:

$$C = \left(\frac{1}{Tr} - jp\Omega\right) \quad (62)$$

Complex coefficients $C1$ and $C2$ guarantee stability and optimal dynamics of the observer. We need to choose a response time $tr = 5ms$.

The characteristic polynomial (63) of error state-space matrix must be compared to a desired polynomial to determine complex gains.

$$det\begin{bmatrix}\lambda + C1 & -\frac{1-\sigma}{\sigma}C\\ C2 & \lambda + C\end{bmatrix} =$$

$$\lambda^2 + (C1 + C)\lambda + \left(C1 + \frac{1-\sigma}{\sigma}C2\right)C =$$

$$\lambda^2 + 2m\omega_n\lambda + \omega_n^2 \quad (63)$$

So:

$$m = 1 \rightarrow \omega_n = \frac{4.75}{tr} = \frac{4.75}{0.005} = 950 \quad (64)$$

$$C1 = 2\omega_n - C = 2\omega_n - \left(\frac{1}{Tr} - jp\Omega\right) \approx 2\omega_n + jp\Omega \quad (65)$$

$$C2 = \frac{\sigma}{1-\sigma}\left(\frac{\omega_n^2}{C} - C1\right) \approx \frac{\sigma}{1-\sigma}\omega_n^2 Tr^2\left(\frac{1}{Tr} + jp\Omega\right) \quad (66)$$

The approximate value of $C2$ is obtained for low speed; at high speed real parts of the eigenvalues of the error state-space matrix remain negative and do not change much, this guarantees stability and fast dynamics of the observer.

4.2 Load torque observer

In general, load torque is an unknown variable and independent of the system state variables. In the proposed Backstepping control strategy, this torque must be estimated and then injected into the control law.

This work proposes a Luenberger observer in the rotor flux frame from the state-space equation (67). In this equation load torque is assumed to have a zero derivative [14].

$$\frac{d}{dt}\begin{bmatrix}\Omega\\ T_L\end{bmatrix} = \begin{bmatrix}-\frac{f}{J} & -\frac{1}{J}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Omega\\ T_L\end{bmatrix} + \begin{bmatrix}\frac{3p}{2J}(1-\sigma)Ls\\ 0\end{bmatrix} i_{\varphi d} i_{sq} \quad (67)$$

State-space equation of load torque observer is given by (68).

$$\frac{d}{dt}\begin{bmatrix}\tilde{\Omega}\\ \tilde{T}_L\end{bmatrix} = \begin{bmatrix}-\frac{f}{J} & -\frac{1}{J}\\ 0 & 0\end{bmatrix}\begin{bmatrix}\tilde{\Omega}\\ \tilde{T}_L\end{bmatrix} + \begin{bmatrix}\frac{3p}{2J}(1-\sigma)Ls\\ 0\end{bmatrix} i_{\varphi d} i_{sq} - \begin{bmatrix}k_1\\ k_2\end{bmatrix}(\tilde{\Omega} - \Omega) \quad (68)$$

This gives state-space equation of observation error (69).

 https://doi.org/10.1051/itmconf/20224804002

$$\frac{d}{dt}\begin{bmatrix}\tilde{\Omega}-\Omega\\ \tilde{T}_L-T_L\end{bmatrix}=\begin{bmatrix}-\frac{f}{J}-k_1 & -\frac{1}{J}\\ -k_2 & 0\end{bmatrix}\begin{bmatrix}\tilde{\Omega}-\Omega\\ \tilde{T}_L-T_L\end{bmatrix} \quad (69)$$

The observer must have a very large dynamic compared to the system. Since sampling period of the control algorithm is $T_{SMP}=500\mu s$, the response time chosen for the observer is $tr=5ms$.

From the characteristic polynomial of error state-space matrix (x), observer parameters can be determined.

$$det\begin{bmatrix}\lambda+\frac{f}{J}+k_1 & \frac{1}{J}\\ k_2 & \lambda\end{bmatrix}=\lambda^2+\left(\frac{f}{J}+k_1\right)\lambda-\frac{k_2}{J}=\lambda^2+2m\omega_n\lambda+\omega_n^2 \quad (70)$$

$$m=1\rightarrow\omega_n=\frac{4.75}{tr}=\frac{4.75}{0.005}=950 \quad (71)$$

$$k_1=2\omega_n-\frac{f}{J}\approx 1900 \quad (72)$$

$$k_2=-J\omega_n^2=-45125 \quad (73)$$

5 Validation

Proposed scheme is verified using Simulink, frequency of the inverter carrier is $f_{SVM}=2000\ Hz$.

Thus, we validate Backstepping controller by examining the speed and rotor flux responses, and we verify the rotor flux and load torque observers by comparing the estimated values with the actual values.

The simulation results are shown in Fig.2, Fig.3, Fig.4, Fig.5 et Fig.6.

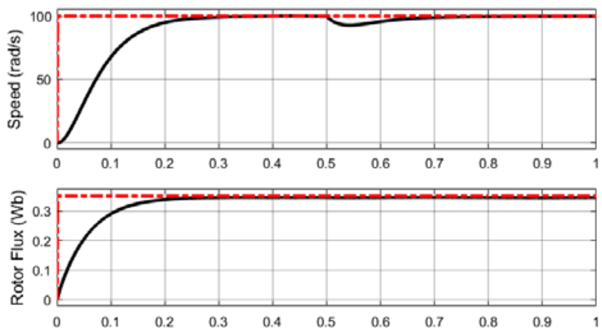

Fig. 2. Speed and rotor flux responses

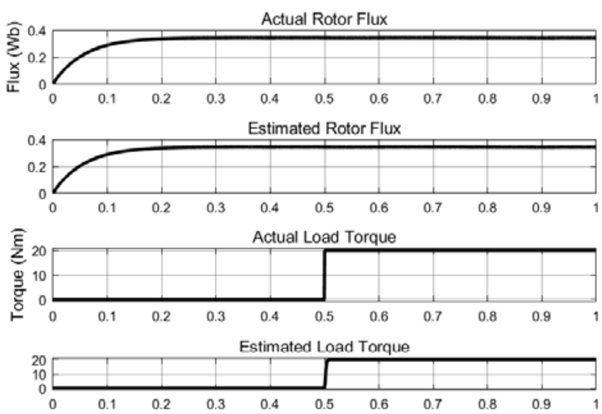

Fig. 3. Estimated rotor flux and load torque

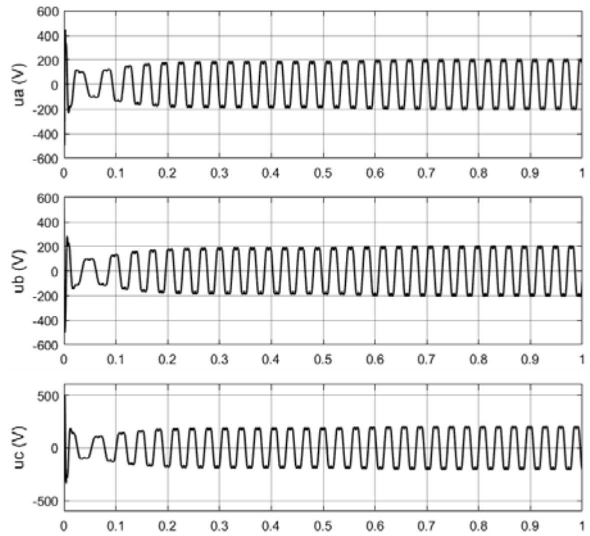

Fig. 4. Input signals for inverter

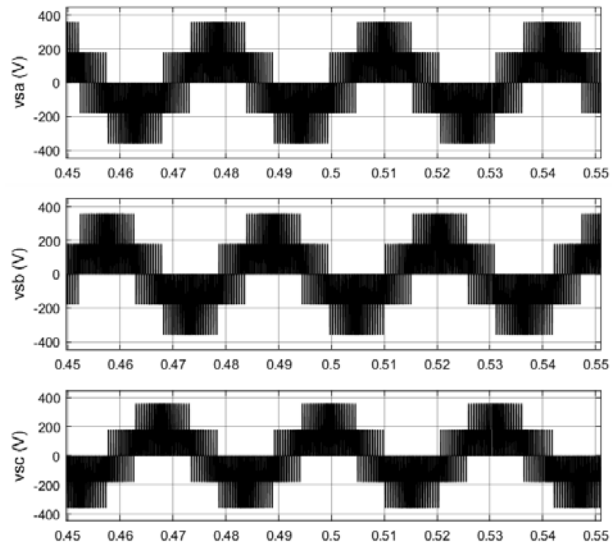

Fig. 5. Stator voltages of induction motor

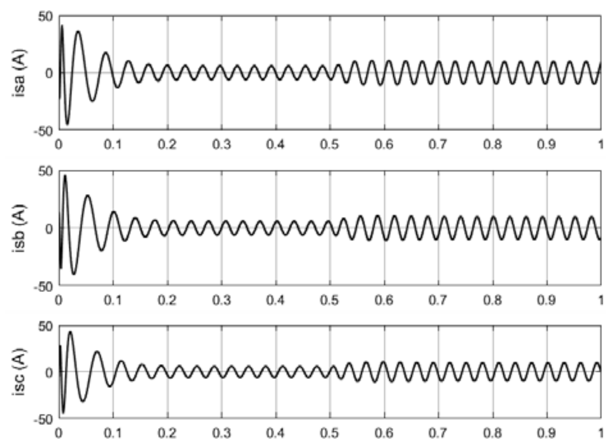

Fig. 6. Stator currents

Fig.2 shows that speed and rotor flux responses respect the dynamics fixed in the controller design. From Fig.3 the responses of the observer are satisfactory because the estimated values quickly follow the real values.

Signals in Fig.4 are not sinusoidal waveforms since they drive a space vector modulation inverter.

Waveforms in Fig.5 and Fig.6 are typical. Visually starting currents are acceptable.

6 Conclusion

Controller design by the Backstepping technique is direct and structured even when the system is non-linear. In this work, this strategy is applied to the induction motor with the use of observers of rotor flux and load torque. Simulation results are conclusive.

Compared to IRFOC and classical strategies, Backstepping is easily adapted to non-linear systems such as the induction motor.

Unlike many works, this paper proposes the numerical calculation of the controller and observer parameters according to the requirements already set.

In general, Backstepping does not require integral action, which makes it vulnerable to changes in machine and inverter parameters. Robustness of Backstepping and its improvement will be examined in next works.

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