A comparative study of propagation delay applied on varieties carbon nanotubes Bundles interconnects using the FDTD and the ABCD matrix method

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Abstract. Due to their hilarious properties, carbon nanotubes (CNTs) with their different varieties have been suggested as potential alternatives for interconnects in many industrial applications related to Artificial Intelligence. This work made an overview of the various models of the resistive effect inside several CNT based interconnects. In addition, it has studied the propagation delay inside the CNT interconnect varieties by comparing two validated transient analysis methods, namely the FDTD and the ABCD matrix. The work has concluded the superiority of the ABCD-matrix method since it has given a better performance.

Keywords: carbon nanotubes (CNTs); ABCD-matrix; FDTD; interconnects; circuit modeling.

1 Introduction

For purpose of implementing artificial intelligence (AI) on industrial applications and systems such as the Internet Of things (IoT) or the Cyber Physical System (CPS), AI specialists need to look for high-performance Integrated Circuits. Therefore, the manufacturer generally has opted for the miniaturization of semiconductor devices, and the down-scaling of copper interconnects. But, by following this approach, they have increased the side-effects phenomenon such as crosstalk, resistivity [1], propagation delay, and electromigration. The interconnects are classified into three layers according to their length: global, intermediate, and local. The two first layers of interconnects from the substrate are considered local, the intermediate is assumed to be the middle layers, and global are the top layers. For years, the dominant material empoyed for their design was copper but with the current technology node, it start to be limited beyond 14nm [2]. Most of the research introduce carbon nanotubes (CNTs), either single-walled (SWCNTs), multi-walled (MWCNTs), or combination with copper as a future replacement of Cu to counter its weakness [3, 4].

In this work, we study the resistive effect of a variety of carbon nanotubes and also we made a comparative study between the ABCD-parameters method and FDTD one to study the propagation delay of the previous kind of carbon nanotubes.

The remaining part of this paper is classified as follows: in the first one, modeling carbon nanotubes resistance and their composite have been presented. Secondly, the FDTD method and ABCD matrix have been explained. Afterward, the results of the resistive effect and the comparison between the propagation delays value of each method have been discussed. And finally, the paper was concluded.

2 Materials & Methods

2.1 Modeling carbon nanotubes resistance

As shown in fig.1.a, the interconnect either made by copper or carbon nanotubes is assumed as a transmission line with its R, L, G and C parameters [5] which can vary from one type to another according to their intrinsic properties.

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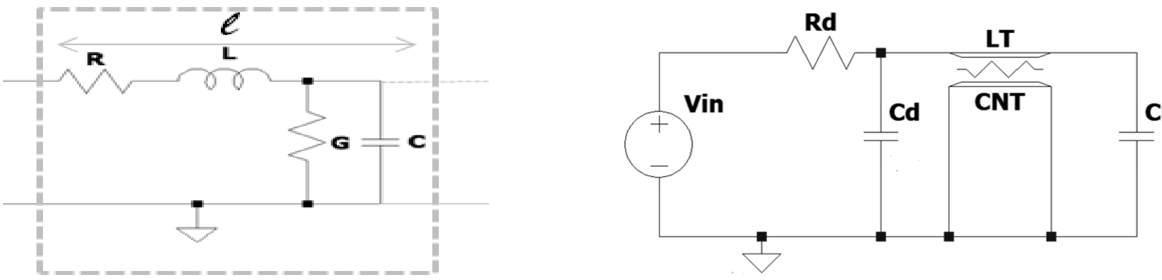

(a) (b)

Fig. 1. (a) Carbon nanotube interconnect's general model (b) Equivalent CMOS terminations in a CNT bundle interconnect circuit.

Since the resistive effect is one of the limitations of the current technology node, this session will resume the state of art of the different models used for evaluating the resistance of the interconnect. Firstly, the Copper resistance can be written as:

$$R = \frac{\rho_{Cu} . l_{Cu}}{\mathrm{W.T}} \tag{1}$$

where ρ_{Cu}, T ,W, and $l_{Cu}$are respectively the resistivity, the thickness, the width, and the the wire's length.

In addition, the SWCNTs bundles resistance is written as [6, 7]:

$$R_{b\_SWCNT} = \frac{R_{CNT}}{\mathrm{n_{SWCNT}}} \tag{2}$$

where $\mathrm{n_{SWCNT}}$ and $\mathrm{R_{CNT}}$ represents the total number of SWCNTs in the bundles and the resistance of single SWCNT respectively. Indeed, it can be written as:

$$\mathrm{n_{SWCNT}} = \begin{cases} \mathrm{n_W . n_H} - \left(\frac{\mathrm{n_H}}{2}\right) & \text{if } \mathrm{n_H} \text{ is even} \\ \mathrm{n_W . n_H} - \left(\frac{\mathrm{n_H} - 1}{2}\right) & \text{if } \mathrm{n_H} \text{ is odd} \end{cases} \tag{3}$$

with: $\mathrm{n_W} = \left[\frac{\mathrm{W-D}}{\mathrm{S_p}}\right]$ and: $\mathrm{n_H} = \left[\frac{\mathrm{H-D}}{\mathrm{S_p}}\right]$; where H, W, $\mathrm{S_p}$ and D are respectively the bundles height, the bundles width, the spacing between two SWCNTs in a bundle, and the diameter of a SWCNT.

The $\mathrm{R_{CNT}}$ is written as [8]:

$$\mathrm{R_{CNT}} = \begin{cases} \mathrm{R_C} + \frac{\mathrm{h}}{4\mathrm{e}^2} . \left(1 + \frac{\mathrm{l_{CNT}}}{\lambda_{\mathrm{mfp}}}\right) & \text{if } \mathrm{l_{CNT}} > \lambda_{\mathrm{mfp}} \\ \mathrm{R_C} + \frac{\mathrm{h}}{4\mathrm{e}^2} & \text{if } \mathrm{l_{CNT}} \leq \lambda_{\mathrm{mfp}} \end{cases} \tag{4}$$

where $\mathrm{R_C}$, $\mathrm{l_{CNT}}$, h , λ_{mfp} and e are respectively the contact resistance, the wire's length, , Planck constant, the mean free path and electron charge.

The MWCNTs bundles resistance can be written as:

$$\mathrm{R_{b\_MWCNT}} = \frac{\mathrm{R_{MWNT}}}{\mathrm{n_{MWCNT}}} \tag{5}$$

where $\mathrm{n_{MWCNT}}$ represents the total number of MWCNTs bundles, it is written as follows:

$$\mathrm{n_{MWCNT}} = \begin{cases} \mathrm{n_W . n_H} - \left(\frac{\mathrm{n_H}}{2}\right) & \text{if } \mathrm{n_H} \text{ is even} \\ \mathrm{n_W . n_H} - \left(\frac{\mathrm{n_H} - 1}{2}\right) & \text{if } \mathrm{n_H} \text{ is odd} \end{cases} \tag{6}$$

with: $\mathrm{n_W} = \left[\frac{\mathrm{W-D_n}}{\mathrm{S_p}}\right]$ and: $\mathrm{n_H} = \left[\frac{\mathrm{H-D_n}}{\mathrm{S_p}}\right]$; where $\mathrm{S_p}$, W , H are the center-to-center distance between neighboring CNTs, the width and the height of the bundles and $\mathrm{D_n}$ is the nth MWCNT diameter.

The $\mathrm{R_{MWNT}}$ represents the resistance of single MWCNTs, it can be written as:

$$R_{MWNT} = R_m + \frac{h}{4.e^2}.\left(\frac{1}{N_i} + \frac{l_{MWNT}}{\lambda_{mfp,i}.N_i}\right) \tag{7}$$

where R_m, N_i, l_{MWNT} and $\lambda_{mfp,i}$ are respectively the contact resistance, the MWCNTs length and the mean free path for i^{th} shell.

Finally, the resistance of hybrid Cu-CNTs bundles can be written as [5]:

$$R_{Cu-CNTs} = \left(\frac{1}{R_{Cu}} + \frac{1}{R_{b\_CNT}}\right)^{-1} \tag{8}$$

where $R_{b\_CNT} = \frac{R_{CNT}}{n_{CNT}}$, $R_{Cu} = \frac{\rho_{Cu}.l_{Cu-CNTs}}{(1-FR_{CNT})W.T}$, $l_{Cu-CNTs}$ and FR_{CNT} represent respectively the length of hybrid Cu-CNT and the filling ratio.

2.2 Transient analysis using FDTD

By applying the Kirchhoff law on the general model of interconnects (fig.1.a) we get the following system of differential partial equations, known as Telegrapher Equations (TE), their expression is the following:

$$\frac{\partial V(z,t)}{\partial z} + L.\frac{\partial I(z,t)}{\partial t} + R.I(z,t) = 0 \tag{9}$$

$$\frac{\partial I(z,t)}{\partial z} + C.\frac{\partial V(z,t)}{\partial t} + G.V(z,t) = 0 \tag{10}$$

where I, V, R, L, C and G represent the current, voltage, per unit length resistance, inductance, capacitance and conductance. As per the ITRS reports [5], due to the presence of inter-level dielectric (ILD) the conductance G becomes negligible at all interconnect levels. Thus we update the previous equations and we get:

$$\frac{\partial V(z,t)}{\partial z} + L.\frac{\partial I(z,t)}{\partial t} + R.I(z,t) = 0 \tag{11}$$

$$\frac{\partial I(z,t)}{\partial z} + C.\frac{\partial V(z,t)}{\partial t} = 0 \tag{12}$$

We begin by discretizing the previous equations, utilizing the grid (fig.2). Indeed, in the Yees' grid the voltage position is materialized by a rectangle and the current position by a circle which are time and space separated respectively by a step of Δt and Δz. We notice that a half step in time and space separate each voltage and current position.

The previous TEs become:

$$\frac{V_{k+1}^{n} - V_{k}^{n}}{\Delta z} + L.\frac{I_{k+\frac{1}{2}}^{n+\frac{1}{2}} - I_{k+\frac{1}{2}}^{n-\frac{1}{2}}}{\Delta t} + R.I_{k+\frac{1}{2}}^{n} = 0 \tag{13}$$

$$\frac{I_{k+\frac{1}{2}}^{n+\frac{1}{2}} - I_{k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} + C.\frac{V_{k}^{n+1} - V_{k}^{n}}{\Delta t} = 0 \tag{14}$$

By using the Cranck-Nickolson approximation we write:

$$I_{k+\frac{1}{2}}^{n} = \frac{I_{k+\frac{1}{2}}^{n+\frac{1}{2}} + I_{k+\frac{1}{2}}^{n-\frac{1}{2}}}{2}$$

Finally we get:

$$I_{k+\frac{1}{2}}^{n+\frac{1}{2}} = c_1.(V_{k+1}^{n} - V_{k}^{n}) + c_2.I_{k+\frac{1}{2}}^{n-\frac{1}{2}} \tag{15}$$

$$V_{k}^{n+1} = c_3.(I_{k+\frac{1}{2}}^{n+\frac{1}{2}} - I_{k-\frac{1}{2}}^{n+\frac{1}{2}}) + c_4.V_{k}^{n} \tag{16}$$

where $c_1 = -\left(\Delta z.\left(\frac{L}{\Delta t} + \frac{R}{2}\right)\right)^{-1}$; $c_2 = \left(\frac{L}{\Delta t} - \frac{R}{2}\right).\left(\frac{L}{\Delta t} + \frac{R}{2}\right)^{-1}$; $c_3 = -\frac{\Delta t}{C.\Delta z}$; and $c_4 = 1$.

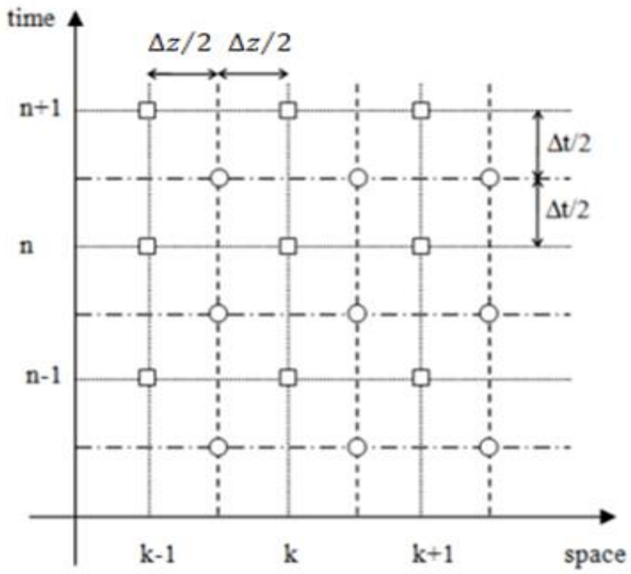

Fig. 2. Yee's grid used for the Telegrapher Equation discretization.

2.3 Transient analysis using the ABCD matrix method

Applying the ABCD transmission parameter matrix for a uniform RLC on the circuit presented in fig.1, we may express the total ABCD transmission parameter matrix of the configuration as:

$$TL = \begin{bmatrix} 1 & R_d \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 \\ sC_d & 1 \end{bmatrix} . \begin{bmatrix} \cosh(\theta . l) & Z_0 . \sinh(\theta . l) \\ {Z_0}^{-1} . \sinh(\theta . l) & \cosh(\theta . l) \end{bmatrix} \tag{17}$$

where $Z_0 = \sqrt{(R + sL)/(sC)}$, $\theta = \sqrt{(R + sL).(sC)}$ and $s = j.\omega$. The matrix TL can be written as:

$$TL = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{18}$$

where:

$A = (1 + s.R_d.C_d).\cosh(\theta.l) + R_d {Z_0}^{-1}.\sinh(\theta.l)$,
$B = R_d.\cosh(\theta.l) + \big(Z_0.(1 + s.R_d.C_d)\big).\sinh(\theta.l)$,
$C = sC_d\ .\cosh(\theta.l) + {Z_0}^{-1}.\sinh(\theta.l)$,
$D = \cosh(\theta.l) + sC_d Z_0 .\sinh(\theta.l)$.

As done in [9], by approximating ${Z_0}^{-1}.\sinh(\theta.l)$ and $\cosh(\theta.l)$ we can obtain an equivalent transfer function which can be written as:

$$H(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1}{A + sC_l B} = \frac{1}{1 + a_1 s + a_2 s^2} \tag{19}$$

where $a_1 = R_d(C_d + C.l + C_l) + R.l.\left(\frac{C.l}{2!} + C_l\right)$,

and $a_2 = R_d.R.C.l^2\left(\frac{C_d.C.l}{2!} + \frac{(C.l)^2}{3!} + C_d.C_l + \frac{C.l.C_l}{2!}\right) + \frac{R^2.C^2.l^4}{4!} + \frac{L.C.l^2}{2!} + \frac{R^2.C.l^3.C_l}{3!} + l.L.\mathrm{C_l}$.

3 Results and discussion

This section presents the result found by comparing the resistance values of different CNTs configurations (fig.3).

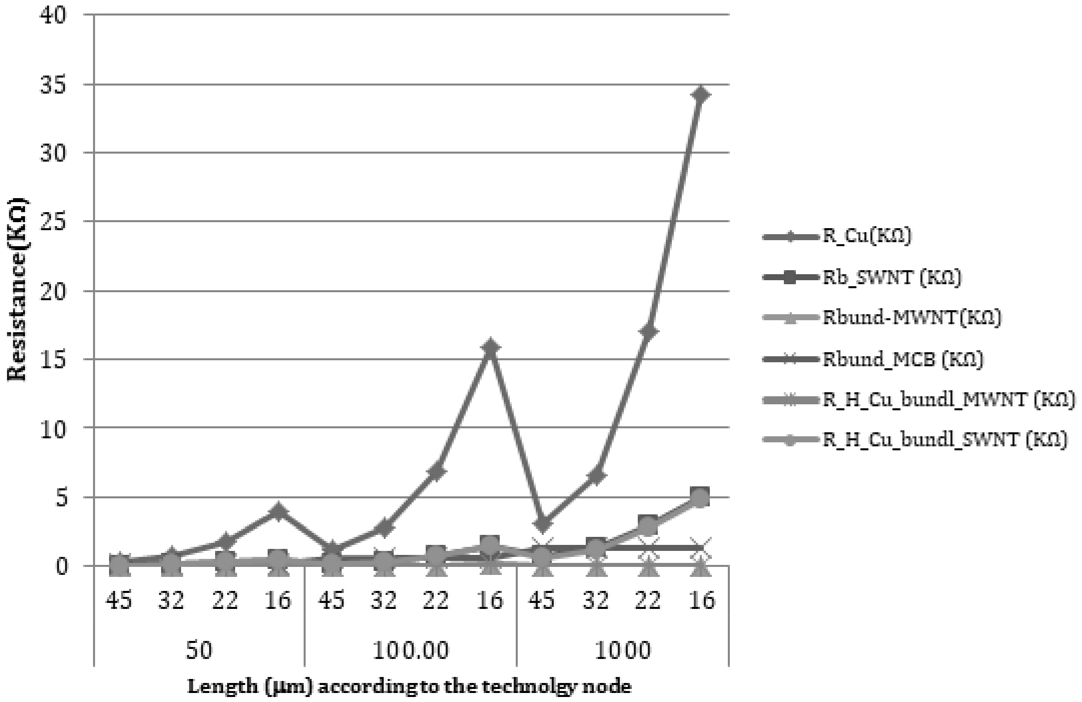

Fig. 3. Comparison of different CNT varieties resistance according to ITRS [10].

Indeed, the pure CNT or its composite has a resistance value less than copper interconnect. In fact, they show almost the same value of resistance. Thus, the CNT-composite is an alternative replacement of the copper interconnects in term of resistive effect decrease.

Furthermore, in order to validate our FDTD and ABCD matrix model we begin by comparing their transient analysis simulation to LTSPICE one. The results are shown in fig.4.

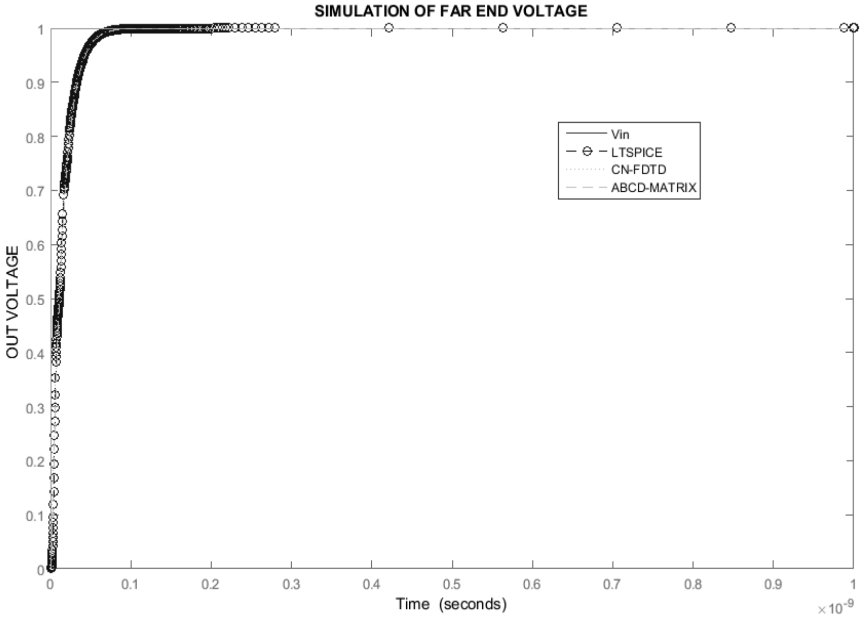

Fig. 4. Comparison of the simulation between CN-FDTD, ABCD-MATRIX and LTSPICE methods.

Comparing the three results obtained by MATLAB and LTSPICE tools, we observe good agreement between them. Therefore, the proposed models of ABCD-matrix and FDTD interconnects are verified.

Finally, the result of the comparison of CPU time and the propagation delay required for the simulation in conformity with different technology nodes is resumed in Table 1. The simulation was based on a material length of 50μm, a rise time of 1ps, a stop time value of 1ns, and an inverted CMOS model with $R_d = 13{,}8\Omega$, $C_d = 1pF$, and $C_l = 1fF$. Indeed, the table shows a better accuracy of the ABCD-matrix method over the FDTD one; however, the CPU time needed for the FDTD is less since it is considered almost the third of the ABCD-matrix. In addition, we notice, that using the hybrid or CNT interconnect ensures the decrease of the propagation delay.

Table 1. Comparison of propagation delay and CPU time of different numerical method.

	Technology node (nm)	LTSPICE	ABCD			CN-FDTD		
		Delay (ps)	Delay (ps)	CPU time (s)	Error (%)	Delay (ps)	CPU time (s)	Error (%)
Cu	45	12.9	12.0	7.0	7.0	11.5	2.0	10.9
	32	17.0	14.0	18.7	17.6	13.0	1.6	23.5
	22	21.0	18.0	6.9	14.3	16.2	2.4	22.9
	16	29.7	25.0	7.1	15.9	22.8	2.4	23.3
CNT	45	11.0	11.0	17.4	0.0	10.2	7.7	7.3
	32	12.0	11.0	7.3	8.3	10.2	2.4	15.0
	22	12.0	11.0	6.9	8.3	10.3	2.3	14.2
	16	15.4	12.0	6.9	22.1	10.2	1.7	33.8
HYBRID	45	11.0	11.0	6.9	0.0	10.0	1.0	9.1
	32	11.0	11.0	6.9	0.0	10.0	1.0	9.1
	22	12.0	11.0	7.1	8.3	9.9	1.7	17.5
	16	13.0	11.0	6.9	15.4	10.0	1.4	23.1

4 Conclusion

This paper has presented a comparison of the resistive effect on CNT varieties versus copper ones. Also, it has presented the results of the comparison between the ABCD-matrix method and the FDTD for the transient analysis. It is observed that the hybrid Copper-carbon nanotubes provides effectively interesting properties in term of resistive effect and propagation delay which may be used for the interconnect design. In addition, we concluded the superiority in terms of accuracy of the ABCD-matrix method over the FDTD one. For a perspective it may be interesting the study of the coupling of carbon nanotubes varieties considering the method already presented in this paper.

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