A Two-Stage Bi-Objective Data Envelopment Analysis Problem

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Abstract. This paper proposes a novel two-stage bi-objective Data Envelopment Analysis (DEA) model. The objectives considered are maximization of overall efficiency and maximization of labor efficiency. The existing literature has not yet proposed any multi-objective DEA model till date. However, such model is in demand because most of the real life problems consist of multiple stages. The proposed model has also considered dissimilar intermediate weights. The proposed model has been applied to a multi-stage insurance problem with two input variables, two intermediate variables and two output variables in order to establish the effectiveness of the proposed model. The proposed model is an algorithmically complex problem and therefore a hybrid Multi-Objective Genetic Algorithm has been applied in order to solve the proposed model.

Keywords— Data Envelopment Analysis (DEA), Network DEA, Bi-objective problem, Two-stage model, Multi-Objective Genetic Algorithm

1 Introduction

The application of Data Envelopment Analysis (DEA) is a popular way of measuring efficiency of different problems as evident from the existing literature. In particular, Network Data Envelopment Analysis (NDEA) has drawn significant attention of researchers recently. Till date, the existing literature has proposed single objective network DEA problems in general although most of the real life problems where NDEA can be applied have multiple objectives. Thus, the investigation in to multi-objective NDEA problem is in demand. Towards this direction, this paper contributes to the existing literature on DEA in the following ways – 1) by proposing a two-stage DEA with dissimilar intermediate weights, 2) by considering two objectives. Thus, this paper has proposed a multi-objective two-stage DEA problem, emphasizing the internal structure of DEA models. Network structure can be of three types – series structure, parallel structure and a mix of these two structures. This paper has emphasized on NDEA with two stages with series structure whose basic structure is shown in Figure 1.

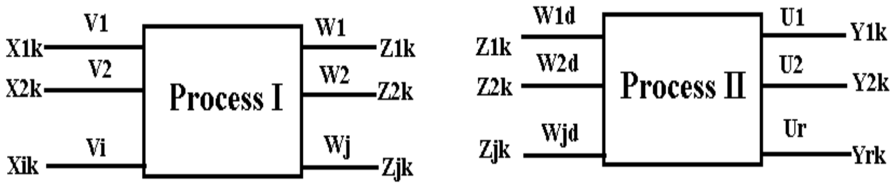

Fig. 1. Basic Structure of Proposed Two-Stage DEA.

2 Literature Review

DEA was first proposed by Charnes et al. [1] in the form of CCR model followed by the BCC model by Banker et al. [2]. These two models were very simple models with no mention of internal structure. Later, Färe and Grosskopf [3] proposed network DEA (NDEA) which considered internal structure of DEA models by considering intermediate stages. There is significant number of two-stage models in the existing literature. Some of them are the research studies of Alder et al. [4], Tsolas [5], Lim and Zhu [6]. There are multiplicative decomposition approaches for DEA such as the research studies of Kao and Hwang [7] and Kao [8]. For a two-stage model, if the first stage has some inputs and the second stage has some additional inputs, then the aggregation approach may be applicable instead of decomposition approach. Such works include the research studies of Lozano [9], Chen and Zhu [10], Lu et al. [11]. However, none of the previously proposed model has considered multi-objective scenario with the DEA models and none has considered dissimilar intermediate weights. This paper has considered both of these approaches, thereby, filling the gap of research in the existing literature. Some DEA researchers have also applied Genetic Algorithm (GA) like the research studies of González et al. [12], Pendharkar [13], Mozaffari et al. [14]. However, since the current has considered Multi-Objective problem, thus, Multi-Objective Genetic Algorithm has been applied.

3 Proposed DEA Model

Based on the DEA model as shown in Figure 1, the mathematical formulation for the proposed two-stage DEA is given in Figure 2. This model, converted to linearized form, is shown in Figure 3. The weights are represented by $V_i, U_r, W_j, \overline{W_j}$ and $W_j \neq \overline{W_j}$, where W_j is the weight of output j from stage 1 and $\overline{W_j}$ is the weight of input to stage 2. The data as shown in Section 4 from a practical example can help to calculate these weights.

4 Case Study and Proposed Multi-Objective Genetic Model

This paper has uses a similar case study as used by Kao and Huang [15]. The data have been simulated with the help of the data in the case study of Kao and Huang [15]. The data is provided in Table 1. This is a case study based on the data as collected from a total of 16 insurance companies. The input for the first stage are number of employees (L), operating expenses (X_1), insurance expenses (X_2); the outputs of the first stage and the inputs of the second stage are direct written premiums (Z_1), reinsurance premiums (Z_2); the outputs of the final (second) stage are underwriting profit (Y_1) and investment profit (Y_2) as shown in Table 1. This proposed model has been solved by a Multi-Objective Genetic

Algorithm which is shown in Figure 4. The structure of each chromosome is shown in Figure 5.

$$\textbf{Maximize} \quad A = \frac{\sum_r U_r Y_{rk}}{\sum_j \overline{W_j} Z_{jk}} \times \frac{\sum_j W_j Z_{jk}}{\sum_i V_i X_{ik}}$$

$$\textbf{Maximize} \quad B = \frac{\sum_r U_r Y_{rk}}{\delta L}$$

$$\textbf{Subject to the constraints:}$$

$$\frac{\sum_r U_r Y_{rk}}{\sum_j \overline{W_j} Z_{jk}} \times \frac{\sum_j W_j Z_{jk}}{\sum_i V_i X_{ik}} \leq 1$$

$$\frac{\sum_r U_r Y_{rk}}{\delta L} \leq 1$$

$$\frac{\sum_r U_r Y_{rk}}{\sum_i V_i X_{ik}} \leq 1$$

$$\frac{\sum_r U_r Y_{rk}}{\sum_j \overline{W_j} Z_{jk}} \leq 1$$

$$\frac{\sum_j W_j Z_{jk}}{\sum_i V_i X_{ik}} \leq 1$$

$$V_i, U_r, W_j, \overline{W_j} \geq \varepsilon, \quad \forall i, r, j$$

Fig. 2. Proposed DEA Model.

5 Results and Discussion

The Multi-Objective GA has been implemented in Matlab 2014b. The crossover and mutation probabilities are 0.7 and 0.3 respectively. The population size and the number of iterations have been taken to be 100 and 100 respectively. The final results of the proposed model and that obtained from the model of Kao and Huang [15] are shown in Table 2. Table 2 shows different rankings for the two methods. However, the results for the proposed method are similar to those for the approach of Kao and Huang [15] In order to verify whether the proposed method is providing any similar results, Table 3 shows the results for Spearman's rank correlations which can find associations between two sets of ranks. Table 3 shows high positive association between the proposed approach and the approach by Kao and Huang [15].

$$\textit{Maximize } A = \frac{\sum_r U_r Y_{rk}}{\sum_j \overline{W}_j Z_{jk}} \times \frac{\sum_j W_j Z_{jk}}{\sum_i V_i X_{ik}}$$

$$\textit{Maximize } B = \frac{\sum_r U_r Y_{rk}}{\delta L}$$

Subject to the constraints:

$$\frac{\sum_r U_r Y_{rk}}{\sum_j \overline{W}_j Z_{jk}} \times \frac{\sum_j W_j Z_{jk}}{\sum_i V_i X_{ik}} \leq 1$$

$$\sum_r U_r Y_{rk} - \delta L \leq 0$$

$$\sum_r U_r Y_{rk} - \sum_i V_i X_{ik} \leq 0$$

$$\sum_r U_r Y_{rk} - \sum_j \overline{W}_j Z_{jk} \leq 0$$

$$\sum_j W_j Z_{jk} - \sum_i V_i X_{ik} \leq 0$$

$$V_i, U_r, W_j, \overline{W}_j \geq \varepsilon, \ \forall i, r, j$$

Fig. 3. Linearized Proposed DEA Model

Table 1. Data on Case Study.

Company	Employees	X1	X2	Z1	Z2	Y1	Y2
E1	8221	1298746	553613	6451768	8467354	884148	782688
E2	726	11808721	20408989	20130275	2802890	23850998	936784
E3	58438	1068482	499900	3677555	460257	293613	778829
E4	560	6889164	3271508	17362863	2035498	7851229	2929279
E5	628	2526708	759364	8757906	963427	1713598	515057
E6	620	21401802	243116	12683452	743513	7239594	5439036
E7	450	2889503	973534	16167245	1304659	3499576	523866
E8	1528	2303138	2297475	9222381	514526	21687943	594888
E9	7234	1802549	572316	6233287	604276	2486915	28258
E10	16300	1608842	1358707	10822465	711342	4809236	323067
E11	980	2096052	855543	6996397	364510	2401209	432396
E12	2084	1984842	551869	13522296	6944897	2356198	656483
E13	900	1121699	516078	6606112	502990	954553	208941
E14	5606	9302	24125	325899	42543	61956	16492
E15	750	16894	11505	42464	24571	96142	14141
E16	26753	16428	215396	576517	546813	1902971	18977

1. Initialize the Chromosomes each of which consists of the values of the weights for the proposed DEA
2. Evaluate objective values
3. Find the aggregate of the normalized objective values
4. Sort the chromosomes in descending order of aggregate values
5. Randomly select a set of chromosomes
6. Perform crossover or mutation based on crossover probability in order to generate offsprings
7. Combine the main population of chromosomes with the offsprings
8. Sort these combined population of chromosomes based on their aggregate objective values
9. Choose the best chromosomes of population of original size.

Fig. 4. Multi-Objective GA.

1	2	3	4	5	6	7	8	9
U1	U2	V1	V2	W1	W2	W1d	W2d	Delta

Fig. 5. Structure of Chromosome.

Table 2. Results of the Proposed Model and the Model of Kao and Huang [15].

Company	Ek1	EK2	EK	OBJ2	Rank Based on Ek	Rank Based on Obj2	EK1	EK2	EK	Rank Based on Ek
E1	0.239	1.98	0.474	0.93	8	6	0.538	0.977	0.526	9
E2	0.562	0.964	0.542	0.711	7	14	0.55	0.971	0.534	8
E3	0.414	0.546	0.226	0.691	10	15	0.69	0.99	0.683	5
E4	0.935	0.922	0.862	0.564	2	16	0.823	0.892	0.734	4
E5	0.902	0.676	0.61	0.907	6	10	0.913	0.901	0.823	2
E6	0.85	0.97	0.824	0.916	3	9	0.76	0.857	0.651	7
E7	0.612	0.245	0.15	0.934	13	5	0.961	0.359	0.345	11
E8	0.621	0.755	0.469	0.927	9	8	0.613	0.798	0.489	10
E9	0.107	0.985	0.105	0.929	16	7	0.182	0.829	0.151	13
E10	0.241	0.693	0.167	0.779	12	12	0.241	0.61	0.147	14
E11	0.349	0.332	0.116	0.9	15	11	0.332	0.395	0.131	15
E12	0.182	0.671	0.122	0.978	14	2	0.192	0.646	0.124	16
E13	0.988	0.982	0.97	0.978	1	2	0.991	0.974	0.965	1
E14	0.438	0.47	0.206	0.774	11	13	0.428	0.649	0.278	12
E15	0.784	0.992	0.777	0.954	4	4	0.874	0.871	0.761	3
E16	0.624	0.984	0.614	0.983	5	1	0.816	0.822	0.671	6

Table 3. Comparison of Result.

	Rank Obtained from Proposed Approach	Rank Based on Objective 2
Rank Obtained Kao & Huang [15]	0.870588	0.004412

6 Conclusion

This paper has proposed two-stage bi-objective Data Envelopment Analysis (DEA) model with dissimilar intermediate weights. Such approach considering multi-objective scenario with dissimilar intermediate weights have not yet been considered till date, as evident from the existing literature. The two objectives as considered are maximization of overall efficiency and maximization of labor efficiency. The proposed approach has been applied to a case study on different insurance companies. The results of the proposed approach have been compared with those of an existing approach. The association among these two shows that the two approaches provide similar results.

References

1. A. Charnes, W. W. Cooper, E. Rhodes, Measuring the efficiency of decision making Units, European Journal of Operational Research, **2**, 429–444, (1978).
2. R. D. Banker, A. Charnes, W. W. Cooper, Some models for estimating technical and scale efficiencies in data envelopment analysis, Management Science, **30**, 1078–1092, (1984).
3. R. Färe, S. Grosskopf, Network DEA", Socio-Economic Planning Sciences, **34**, 35–49, (2000).
4. N. Alder, V. Liebert, F. Yazhemsky, Benchmarking airports from a managerial Perspective, Omega, **41**, 442-458, (2013).
5. I. E. Tsolas, Modeling Portability and stock market performance of listed construction firms on the Athens Exchange. Two stage DEA approach, Journal of Construction Engineering and Management, **139**, 111-119, (2013).
6. S. Lim, J. Zhu, Primal-dual correspondence and frontier projections in two-stage network DEA models, Omega, **83**, 236-248, (2019).
7. C. Kao, S. N. Hwang, Decomposition of technical and scale efficiencies in two-stage production systems, European Journal of Operational Research, **211**, 515-519, (2011).
8. C. Kao, Decomposition of slacks-based efficiency measures in network data envelopment Analysis, European Journal of Operational Research, **283**, 588-600, (2020).
9. S. Lozano, Technical and environmental efficiency of a two-stage production and abatement system, Annals of Operations Research, **255**, 199-219, (2017).
10. K. Chen, J. Zhu, Scale efficiency in two-stage network DEA, Journal of the Operational Research Society, **70**, 101-110, (2019).
11. C. Lu, J. Tao, A. Qiuxian, X. Lai, A second-order cone programming based robust data envelopment analysis model for the new-energy vehicle industry, Annals of Operational Research, **292**, 321-339, (2020).
12. M. González, J. J. López-Espín, J. Aparicio, D. Giménez, A Parallel Application of Matheuristics in Data Envelopment Analysis, International Symposium on Distributed Computing and Artificial Intelligence, **800**, 172–179, (2018).
13. P. C. Pendharkar, A hybrid genetic algorithm and DEA approach for multi-criteria fixed cost allocation, Soft Computing, **22**, 7315–7324, (2018).
14. M. R. Mozaffari, S. Ostovan, P. Fernandes Wanke, A Hybrid Genetic Algorithm-Ratio DEA Approach for Assessing Sustainable Efficiency in Two-Echelon Supply Chains, Sustainability, **12**, 1 – 17, (2020).
15. C. Kao, S.-N. Hwang, Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan, European Journal of Operational Research, **185**, 418 – 429, (2008).