TDOA –FDOA method for tracking of a moving target in a distributed sensor scenario

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Abstract. Tracking of an underwater target in ocean environment is very important in both military and civilian applications. This paper focuses on the localization and tracking of a moving target by a passive multi-static sonar system using TOA, Time Difference and Frequency Difference of Arrival measurements. An algorithm is developed for an acoustic sensor distributed network of five sensors and an acoustic moving source. TDOA- FDOA method is used to track a moving source usingTime Difference of Arrival (TDOA) and Frequency Difference of Arrival (FDOA) measurements, as a closed form solution. The highly nonlinear measurement equations of TDOA and FDOA are first converted into pseudo linear equations. This is achieved by introducing extra parameters. These equations are solved using a weighted least squares (WLS) method. Five distributed stationary receivers are used to track a moving acoustic target. The moving target parameters estimated from the above WLS method are improved by an Unscented Kalman Filter. The mean square errors of these estimates are compared and the results are analyzed.

1. Introduction

Source localization is the estimation of the location of a source. Tracking can be defined as the continuous localization of a moving target. Passive source localization methods find out the location of an object from the emitted signal. Spatially separated sensors will observe the emitter signal and the signal parameters at the sensors can be obtained through various methods like Time of Arrival(TOA) of signal to the sensors, Received Signal Strength (RSS), Angle of Arrival(AOA), Frequency Difference of Arrival (FDOA) and Time Difference of Arrival(TDOA). From these measurements the location of the emitter can be found. Tracking is very important in many fields such as SONAR, RADAR, Radio astronomy, Seismology, Mobile communications, etc. The measurement data used for tracking can be TOA, TDOA, FDOA, AOA, RSS or a combination of them. The problems of tracking, autopilot or self-driving vehicles, collision avoidance, navigation [1,2, 3, 4] etc. can be solved with these information.

Estimation of position and motion information of targets is very important for multi-static sonar functioning. The noisy measurements and target parameters are related nonlinearly and this nonlinearity is the challenge for Multi-static sonar localization and tracking. The maximum likelihood estimator (MLE) is a nonlinear least-square approach. It is computationally expensive and has a tendency to converge to local minima or diverge. However, the MLE is asymptotically efficient and unbiased. But a closed-form solution is not available. Search algorithms which are numeric and iterative can be used. It is also prone to instability as the maximum-likelihood cost function is non-convex. Such short comings are overcome by converting the nonlinear equations into linear equations. Closed form linear least-squares can be used for the solution. The present paper deals with a closed-form algorithm for passive multi-static tracking.

There are many algorithms available in literature for TDOA and FDOA. Ezzat [1] dealt with the idea of a solution which is closed-form and 3-dimensional, for the time difference of arrival (TDOA) problem. The coordinates of the emitter are provided by the TDOA solution with assumption of no multipath delays. It is also emphasized under the assumption of noise free measurements. It also proposes to use the Least-squares error minimization since it consists of merely 3 simultaneous equations.

Darko Musicki [2] dealt with the hybrid of passive geo-location techniques. Geo-location techniques used include TDOA in combination with FDOA used to locate a stationary emitter with two moving UAVs. TDOA and FDOA measurements are non-linear. For location estimation, multiple non-linear measurements of the emitter need to be fused. Gaussian Mixture Measurement (GMM) filter fuses the non-linear measurements. It incorporates nonlinear measurements from various sensor types, as long as they are represented as the Gaussian Mixture.

Ho and Xu [3] used FDOA and TDOA measurements to estimate the source motion parameters. Sun et al. [4] discussed the moving source localization problem solution by total least squares method. Yu et al. [5] provided a constrained weighted least square method using Newton's iterative method. Most of these methods uses only a single set of TDOA measurements i.e., TDOA measurements made at certain moment. An algorithm named as Multiple-time TDOA makes use of sets of TDOA measurements made at multiple time intervals. This approach can make use of more measurements and hence make the estimate more accurate.

Aous Y Ali.etal [6] used the tracking of the mobile emitters using a sequence of FDOA and TDOA measurement. With only one emitter, the measurement of TDOA is defined by unique hyperbola and then approximated by the Gaussian mixture and tracking results showed better track states. The performance is evaluated and compared with a bank of EKF filters using Matlab simulation.

TDOA approach can be used for tracking moving

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source or emitter i.e., finding instantaneous position and velocity. Tracking is the continuous localization of a moving target. Tracking of a single source moving with constant velocity can be done from TDOA measurements alone and from TDOA/FDOA measurements.

In the present paper, the performance of a 3D solution for the TDOA – FDOA problem for a distributed sensor scenario consisting of an emitter and four receivers ,is analyzed. An algorithm to find the motion parameters of a constant velocity source using TDOA and FDOA method is discussed. Towards this end, the rest of the paper is organized as follows; Section 2 describes the TDOA method and Section 3 the FDOA method. Section 4 describes the one method of tracking using TDOA/FDOA. Section 5 describes TDOA-FDOA tracking algorithm, problem formulation and the theory and section 6 discusses performance analysis. Simulation results are discussed in section 7. The algorithm performance is compared with CRLB. Section 8 concludes the paper.

2 Time Difference of Arrival

Difference in time arrivals from different locations can be used to estimate position and velocity. A signal is transmitted from a source and received by multiple sensors. The arrival times are estimated and location can be estimated using measurements from three or more receivers or multiple measurements. The accuracy of these measurements are affected by the observer relative positions. For an accurate estimate accurate clock synchronization is an essential requirement. Optimal geometry of sensors, accurate synchronization and noise free transmission of data are essential for more accurate estimate.

3 Frequency Difference of Arrival

Position and velocity can be estimated using frequency difference of arrivals of signals at different locations. The doppler shift causes frequency difference when targets are moving. In FDOA, multiple observers can share data or a single observer can take multiple measurements.

Relative positions and velocities of observers, affect the accuracy of this method.

4 Tracking

TDOA approach can be used for tracking moving source or emitter i.e., finding instantaneous position and velocity. Tracking is the continuous localization of a moving target. Tracking of a single source moving with constant velocity can be done from multiple TDOA measurements alone and from TDOA/FDOA measurements. These algorithms are Multiple-time TDOA algorithm & TDOA-FDOA algorithm.

4.1 Multiple Time TDOA Algorithm

A moving source localization solution in closed form is developed using only TDOA measurements, taken at different or multiple time instants. The solution is derived first by converting the highly nonlinear TDOA measurement equations into pseudo linear equations by introducing extra parameters and secondly by solving those equations via a weighted least squares (WLS)method.

5 TDOA- FDOA Tracking Algorithm

This is a closed form solution obtained by the minimization of the Weighted Least Square (WLS) criterion. Advantages of this method are low computational complexity and its closed form solution.

Problem Formulation: Consider a scenario with N stationary sensors at positions $s_i = [x_i, y_i, z_i]^T$ where $i = 1,2, \dots, N$. A source, at initial position$u_0 = [x_0, y_0, z_0]^T$, is moving with constant velocity$v = [\dot{x}, \dot{y}, \dot{z}]^T$.

The TDOA between reference sensor and other sensors multiplied by the speed of propagation is called the range difference of arrival (RDOA). The TDOAs are measured and the corresponding RDOAs are calculated. RDOAs are

$$r_{i1} = r_i - r_1 , \quad i = 2,3, \dots, N \tag{1}$$

where $r_i = \|u_0 - s_i\|, \quad i = 1, \dots, N.$ Similarly the FDOA between first sensor and other sensors multiplied by the speed of signal propagation is called range difference rate(RDR),is

$$\dot{r_{i,1}} = \dot{r_i} - \dot{r_1} , \quad i = 2,3, \dots, N \tag{2}$$

$$\dot{r_i} = \frac{v^T(u_0 - s_i)}{r_i}, \quad i = 1, \dots, N$$

The two possible errors are RDOA measurement noise and RDR measurement noise.

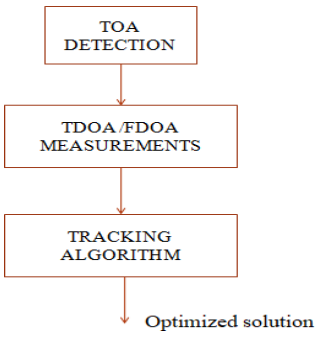

Fig. 1. Tracking Methodology

Both RDOA and RDR error with assumption of Gaussian zero mean with covariance matrices

Q_t and Q_f respectively. Rearrange the RDOA equation as

$$r_{i,1} + r_1 = r_i \tag{3}$$

Squaring on both sides,

$$(s_i - s_1)^T u_0 + r_{i,1} r_1 = 0.5(s_i^T s_i - s_1^T s_1 - r_{i,1}^2) \tag{4}$$

In matrix form

$$[A_t]_i = \left[(s_i - s_1)^T r_{i,1}\ 0_{1\times 3} \quad 0\right] \tag{5}$$

$$[b_t]_i = \quad 0.5(s_i^T s_i - s_1^T s_1 - r_{i,1}^2) \tag{6}$$

Taking time derivative,

$$(s_i - s_1)^T v + \dot{r}_{i,1} r_1 + r_{i,1}\dot{r}_1 = (-\dot{r}_{i,1} r_{i,1}) \tag{7}$$

In matrix firm,

$$[A_f]_i = \left[0_{13} \quad r_{i,1}(s_i - s_1)^T r_{i,1}\right] \tag{8}$$

$$[b_f]_i = \quad (-\dot{r}_{i,1} r_{i,1}) \tag{9}$$

then

$$A = \left[A_t^T, A_f^T\right]^T b = \left[b_t^T, b_f^T\right]^T \tag{10}$$

The unknown vector

$$\theta = \quad [u_0^T r_1 \ v^T \dot{r}_1]^T \tag{11}$$

Let

$$\varepsilon \quad = A\theta - b \tag{12}$$

Applying WLS minimization

$$\theta = \ (A^T W A)^{-1} A^T W b \tag{13}$$

where $W = Q^{-1}$ is called the weighting matrix and $Q = \begin{bmatrix} Q_t & 0 \\ 0 & Q_f \end{bmatrix}$ is the total error covariance.

6 Performance Analysis

The performance of the TDOA-FDOA estimator is analyzed with CRLB[7]. For the normal distribution total measurement vector, CRLB becomes

$$CRLB = trace \ \ (G^0 W G^0)^{-1} \tag{14}$$

G^0 is the G matrix without considering TDOA-FDOA measurement errors.

Cramer Rao Lower Bound (CRLB) is the performance benchmark of an estimator as per estimation theory. An estimator with variance closest to the CRLB is regarded as a good estimator. The mean square error (MSE) of the estimator is compared against CRLB under various noise conditions.

$$MSE(x) = \frac{\sum_{l=1}^{L} \left\|\hat{x}^{(l)} - x\right\|^2}{L} \tag{15}$$

$\hat{x}$ is the position estimate and x is the actual position and L is number of ensemble runs.

7 Simulation and Results

Consider five stationary sensors at locations $s_1 = [250,200,10]$, $s_2 = [1300,1750,20] s_3 = [3250,1750,20], s_4 = [3300,700,10]$, $s_5 = [2100,650,120]$ and a moving source with initial position $u_0 = [2000,2500,3000]^T \ m$ and constant velocity $v = [200,-150,150]^T m/s$. The RDR and RDOA measurements are generated by adding zero mean Gaussian noise with the covariance matrix $Q = \begin{bmatrix} J & 0_{N-1\times N-1} \\ 0_{N-1\times N-1} & J \end{bmatrix}$,where J is an $(N-1)\times(N-1)$ matrix with diagonal elements equal to 1 and all other elements 0.5.Here total error variance $\sigma^2 = 0.001$ The mean square error (MSE) of the estimator is compared against CRLB under various noise conditions. UKF is implemented to improve the accuracy of estimate.

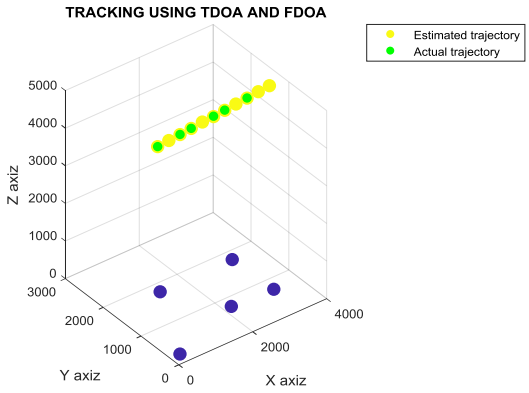

Fig. 2. Scatter Plot

$\hat{x}$ is the estimate of position and x is the actual position and L is the number of ensemble runs. The estimator whose variance closest to CRLB is the more efficient estimator.

Fig 2. Shows the scatter plot showing sensor positions, actual and estimated source positions. Fig 3 shows the plot showing source trajectory. From fig.4 the position estimate variance follows CRLB at all noise conditions but deviates more at high noise condition. So position estimate by TDOA-FDOA algorithms is efficient and accurate with noise conditions low to moderate.

From fig 5. the velocity estimate variance follows CRLB with noise conditions low to moderate.

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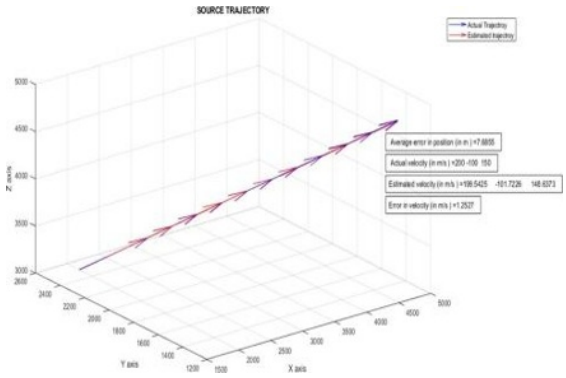

Fig. 3. Source Trajectory

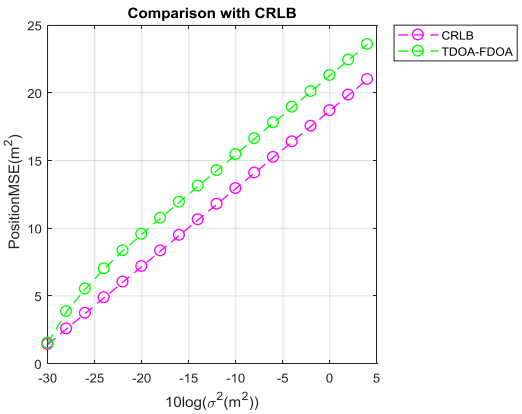

Fig. 4. Position estimate variance with CRLB

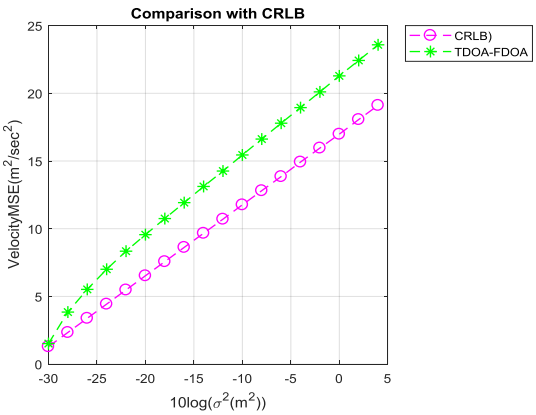

Fig. 5. Velocity estimate variance with CRLB

8 Conclusion

Analyzing the tracking algorithm we find that it is good in performance in terms of CRLB. The position estimate of the algorithm follows CRLB with noise conditions low to moderate, and deviates from CRLB as the noise power increases. The velocity estimate of the algorithm follows CRLB for all the observed noise conditions similar to position estimate. The algorithm is efficient, with small noise assumption.

Estimation of position and motion information of targets is very important for multi static sonar functioning. It is also an important part of intelligent surveillance for the Internet of Vehicles (IoV) [8]. The problems of tracking, autopilot or self-driving vehicles, collision avoidance, navigation [9, 10, 11,12] etc. can be solved with these information.

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