Stability Analysis of Covid-19 Model Based on Compliance and Carrier Transmission

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Abstract. The Covid-19 pandemic has officially ended with the lifting of the Public Health Emergency of International Concern (PHEIC) status by the World Health Organization (WHO). The world has begun the transition from a pandemic to an endemic period through policy updates such as healthy living habits, wearing masks if sick, vaccination, self-quarantine, contact tracing or testing, increasing understanding or awareness of diseases and treatment. This research aims to analyze the role of individuals in the pandemic transition period and the addition of the Carrier subpopulation to the COVID-19 model. This model produces two equilibrium points: a disease-free equilibrium points and an endemic equilibrium point. Furthermore, stability analysis was carried out around the equilibrium point and obtained three basic reproduction numbers that became the threshold for the spread of disease around the equilibrium point, namely R0 less than one (R0 < 1) and greater than one (R0 > 1). This shows that increasing policies such as disease awareness or understanding, healthy living habits, and vaccination can prevent the spread of COVID-19 so that the pandemic period does not occur and the disease will disappear over time.

1 Introduction

The Coronavirus disease (COVID-19) pandemic which has been running for more than 3 years since its emergence in December 2019 has put a heavy burden on the health sector and affected all aspects of life [1]. COVID-19 statistics in August 2023 estimate that there are 769 thousand cases of infection with deaths reaching 6.9 million since the first case was reported [2]. Wuhan, China marked the beginning of the pandemic with the discovery of cases of infection with the SARS-CoV-2 virus that causes COVID-19. The extremely high intensity of spread of COVID-19 resulted in the whole world experiencing a wave of SARS-CoV-2 virus infections in a short period of time [3,4]. The main method of transmission is an important factor that triggers the rapid spread of the disease, where the SARS-CoV-2 virus spreads between humans through airborne transmission and respiratory droplets from infected individuals [5, 6]. The severity of the symptoms caused by COVID-19 varies and develops over time due to variants of the virus that continue to mutate [7]. The clinical

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symptoms commonly shown by carriers of COVID-19 disease are pneumonia symptoms such as cough, fever, flu, and respiratory problems. Other symptoms include loss of sense of taste or smell, headache, joint or muscle pain and severe conditions that can lead to death especially in individuals with congenital diseases [8, 9].

The incubation period of the SARS-CoV-2 virus before finally showing symptoms is one of the important factors in the spread of the disease. Generally, exposed individuals will experience an incubation period of 5-6 days of the virus and in some cases can occur for 2-14 days [10, 11]. Individuals infected with COVID-19 may be symptomatic or asymptomatic at all but still capable of transmitting the disease (carrier) [12]. The U.S. Canters for Disease Control and Prevention states of reported COVID-19 cases, 35% of infections in individuals are asymptomatic and 40% of transmissions begin even before symptoms begin to appear [13, 14]. For a Carrier individual, it is very difficult to distinguish it from a healthy individual until the individual takes a COVID-19 test. This makes cases of infection in Carrier individuals often neglected and infections continue to occur because individuals are also unaware that they have been infected and continue to have contact with vulnerable individuals, thus aggravating the proportion of disease spread between individuals [15].

Unprecedented policies and approaches continue to be carried out by the government in order to find solutions that can end this pandemic. The main strategy in the absence of vaccines and the crisis is non-pharmaceutical interventions (NPIs) to change individual behaviour in a pandemic situation. The use of masks, physical distancing, quarantine, contact tracing for individual carriers, and maintaining cleanliness are part of nonpharmaceutical measures that have been shown to gradually reduce the spread of COVID-19 [16, 17]. After the discovery of the vaccine, non-pharmaceutical precautions continue to be implemented along with vaccination for each individual. Research studies state that vaccines only minimize symptoms but do not prevent infection with the disease [18]. However, vaccination and the implementation of non-pharmaceutical measures that have been carried out simultaneously have been proven to reduce cases of COVID-19 spread during the pandemic [19]. The implementation of preventive measures that continue to be carried out during the pandemic has finally begun to show results, where at the end of January 2023 cases of COVID-19 infection and deaths due to COVID-19 have decreased significantly compared to the last [20].

The update of new COVID-19 handling guidelines has been issued by WHO in line with the declining cases of infection and death due to COVID-19 worldwide. This guide provides a new platform for each country to be able to assess and adjust appropriate handling measures for COVID-19 [21]. In line with this, WHO on May 5 held press conferences stating that the steady decline in COVID-19 cases is due to increased population immunity due to vaccination and infection, so that COVID-19 no longer qualifies as a global health emergency. Thus, the previously designated Public Health Emergency of International Concern (PHEIC) status was officially revoked by WHO [22]. The lifted state of emergency does not necessarily declare that COVID-19 has completely disappeared from the population, it refers to the transition from a state of emergency to handling the disease because the SARS-CoV-2 virus that causes COVID-19 still exists and new variants can pose a threat to humans. Following this, the renewal of policies against COVID-19 to start the transition period from pandemic to endemic disease has begun to be carried out by each country. Indonesia itself began to update and enact new policies and began a transition period with the lifting of mask use regulations and the issuance of Circular Letter (SE) No.1 of 2023 concerning Health Protocols for the Endemic Corona Virus Disease 2019 (COVID-19) Transition Period in June 2023 [23, 24].

Success in achieving a transition period until the COVID-19 disease can disappear completely from the population is certainly not an easy thing and cannot be separated from the shared responsibility between the government and individual communities. This transition period is the most important time that determines whether the disease will disappear or not, because at this time public awareness decreases and the disease can increase again [25]. The role of the community at this time is very necessary, especially following the policies imposed such as the importance of awareness or understanding of disease, healthy living habits (exercise, regular eating and rest, taking medicine and strengthening immunity if sick, and others), using masks if sick, vaccinating, self-quarantine if in direct contact with infected individuals, and taking treatment [26]. In addition to individual communities, the government's role is needed as a controller, especially for individual carriers who are still a major threat in handling COVID-19. The government can conduct testing to control individual carriers and vaccinate evenly for each individual so that the body's immunity increases against COVID-19. Several mathematical models have been formulated to understand the spread of COVID-19 and its preventive measures. There is a SEIQR (Susceptible, Exposed, Infected, Quarantine, Recovered) type COVID-19 model [27] which makes non-pharmaceutical measures the focus of control measures, namely the use of masks and physical distancing to minimize the spread of COVID-19. The development of this research is based on the current condition of COVID-19, the role of government and individual awareness of the community and the role of individual carriers in the spread of COVID-19 in the transition period. This paper developed a COVID-19 model by separating symptomatic individuals and Carrier individuals and preventive measures with the aim of helping to understand how COVID-19 spreads so that the COVID-19 transition period can be achieved.

2 Methods

This research is a follow-up development of the mathematical model of the spread of COVID-19 built by [27] with the basic type of SEIR model which was later modified into the SEIQR model: (*Susceptible, Exposed, Infected, Quarantine, Recovered*), by dividing vulnerable subpopulations and infected subpopulations into two subpopulations each. This case assumed that asymptomatic infected individuals (Carriers) contribute to disease distribution, thus the model was developed by separating symptomatic infected and asymptomatic infected individuals as well as pharmaceutical and non-pharmaceutical preventive measures based on new transitional policies in the form of masks, vaccination, disease awareness or understanding, healthy living habits and contact tracing. Here, the *Infected* subpopulation is divided into three subpopulations bringing the total population to 8 sections: susceptible individuals not using masks (S_{TM}), susceptible individuals using masks (S_M), latent individuals (E), positive infected individuals not using masks (I_{TM}), positive infected individuals using masks (I_M), Carrier infected individuals (I_C), quarantine individuals (Q), and recovered individuals (R). Thus, the total population at time *t* is denoted as:

$$N(t) = S_{TM}(t) + S_M(t) + E(t) + I_{TM}(t) + I_M(t) + I_C(t) + Q(t) + R(t). \tag{1}$$

In the model, the S_{TM} subpopulation is assumed to have increased due to the influx of newborn individuals, individuals who have lost immunity to COVID-19 and S_M individuals who have removed masks. The BC subpopulation increased when the influx of S_{TM} individuals who had worn masks. The same applies to infected positive individuals with symptoms of I_{TM}, if the infected individual has used a mask, it will be included in the I_M subpopulation and vice versa. It is assumed that only S_{TM}-susceptible individuals, symptomatic infected positive individuals do not use I_{TM} masks and I_C Carrier individuals can become infected and infect the disease. Meanwhile, BC-susceptible and symptomatic infected individuals using I_M masks are considered unable to be infected and infect COVID-19 disease. Furthermore, the latent subpopulation E comes from S_{TM} individuals who have been in direct contact with symptomatic infected positive individuals not wearing I_{TM} masks and I_C Carrier individuals, so that these individuals have a chance of infection. Latent individuals E who has awareness or understanding of the disease will be included in subpopulation Q. Quarantine subpopulation Q as a place of quarantine and self-isolation of individuals. Then, individuals in the symptomatic infected Q positive subpopulation will be transferred to the symptomatic infected positive subpopulation using an I_M mask and symptomatic or Carrier uninfected individuals are considered cured by adopting healthy living habits and moving to the recovered R subpopulation. Latent E individuals who test positive for symptomatic and undetected (missed) COVID-19 infection will experience a viral incubation period and move to subpopulation of infected positive individuals symptomatic I_{TM}, I_M and Carrier I_C. Infected individuals with symptoms of I_{TM}, I_M can recover after undergoing treatment and move to the recovered subpopulation R. Carrier I_C individuals who are found positive for infection through testing will be moved into the symptomatic infected positive subpopulation without an I_{TM} mask or remain Carrier and recover from diseases naturally without treatment due to strong body immunity. Finally, we also assume that once a person recovers from illness he develops immunity from the virus, but the immune system of the recovered individual may be lost later in life and the individual again becomes susceptible to COVID-19 disease and will undergo the cycle again. Deaths from COVID-19 disease are negligible, occurring only naturally in each subpopulation.

Based on this, a mathematical model diagram of the spread of Coronavirus Disease 2019 was obtained in Fig. 1.

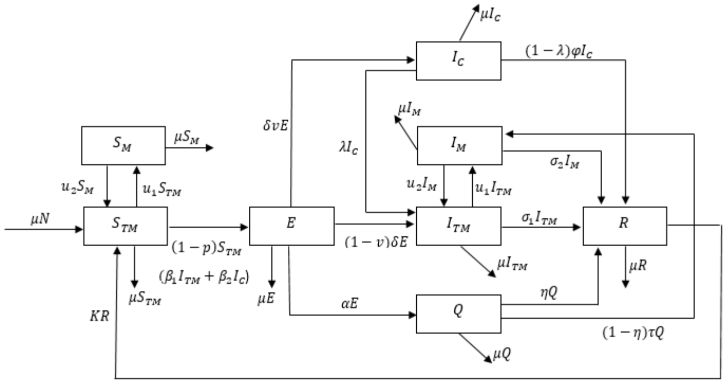

Fig. 1. COVID-19 spread diagram

Based on Fig. 1, a system of non-linier differential equations is obtained from the COVID-19 model as follows

$$\frac{dS_{TM}}{dt} = \mu N + KR + u_2 S_M - u_1 S_{TM} - ((1-p)S_{TM}(\beta_1 I_{TM} + \beta_2 I_C)) - \mu S_{TM} \quad (2)$$

$$\frac{dS_M}{dt} = u_1 S_{TM} - u_2 S_M - \mu S_M \quad (3)$$

$$\frac{dE}{dt} = (1-p)S_{TM}(\beta_1 I_{TM} + \beta_2 I_C) - \delta v E - (1-v)\delta E - \alpha E - \mu E \quad (4)$$

$$\frac{dI_{TM}}{dt} = (1-v)\delta E + \lambda I_C + u_2 I_M - u_1 I_{TM} - \sigma I_{TM} - \mu I_{TM} \quad (5)$$

$$\frac{dI_M}{dt} = (1-\eta)\tau Q + u_1 I_{TM} - u_2 I_M - \sigma I_M - \mu I_M \quad (6)$$

$$\frac{dI_C}{dt} = \delta v E - \lambda I_C - (1-\lambda)\varphi I_C - \mu I_C \quad (7)$$

$$\frac{dQ}{dt} = \alpha E - \eta Q - (1-\eta)\tau Q - \mu Q \quad (8)$$

$$\frac{dR}{dt} = \sigma I_{TM} + \sigma I_M + (1-\lambda)\varphi I_C + \eta Q - (K+\mu)R \quad (9)$$

The use of masks during the pandemic has become an alternative in preventing the spread of disease to vulnerable and infected individuals. The assumption is that susceptible individuals who have been in direct contact with infected individuals either symptomatic or carrier are likely to be infected or uninfected and will become latent E individuals. Because vaccination is readily available, vulnerable individuals who vaccinate have a smaller chance of becoming infected than those who do not. Thus, the proportion of individuals susceptible to vaccination and disease transmission to latent E is $((1\text{-}p)S_{Tm}\,(\beta_1\, I_{TM} + \beta_2\, I_C))$ with the rate of transmission of the disease from symptomatic infected positive individuals I_{TM} and Carrier I_C each β_1 and β_2, respectively. The p parameter indicates the vaccination rate with the proportion of individuals vaccinated $0< p <1$ with vaccine efficiency ε, with $p= (p\text{x}\varepsilon)$. Proportion of $0< v <1$ is a latent individual being a Carrier individual I_C and the remaining proportions 1-v indicates latent individuals to be symptomatic infected positives I_{TM}. The rate of transfer of latent individuals to infected is expressed as δ. The relationship between the spread of diseases and human behavior is fundamentally interactive. During the COVID-19 pandemic, individual communities were bound by government regulations and policies to prevent the spread of COVID-19, resulting in each community diligently adhering to and implementing the regulations, leading to successful control of COVID-19. However, with the lifting of COVID-19 regulations during this transition period, as stated in Ministerial Regulation No. 1 of 2023, the government no longer has control over community behavior, and the responsibility for preventing COVID-19 has shifted to the awareness of each individual. Individuals in society are becoming more indifferent, and awareness of COVID-19 is starting to decrease. This is, of course, very dangerous because the COVID-19 virus is still present in the community and continues to mutate every day. So, the role of the community takes precedence in preventing the spread, making the parameter of understanding or awareness of the disease crucial. Furthermore, the parameter α as the level of latent individual E who moves to quarantine individual Q because they are aware or understand that they have had direct contact with an infected individual even though they have not been proven to be positionally infected and undergo quarantine or self-isolation. Symptomatic infected individuals $I_{TM,}$ I_M will be cured with the level of treatment of the disease expressed as σ. Then, the individual level of the Carrier I_C be a positive infected individual symptomatic not wearing a mask I_{TM} after contact tracing is carried out of λ. Untracked or missed individuals will recover in proportion to $(1\text{-}\lambda)\varphi$, where φ the natural healing rate of the individual Carrier. Scientific understanding indicates that adopting a healthy lifestyle can offer overall health benefits and contribute to maintaining a robust immune system, reducing susceptibility to diseases. Healthy living practices, coupled with individual awareness, have become integral preventive measures during the COVID-19 pandemic. In the case of a mild COVID-19 infection, adhering to healthy living habits such as regular exercise, maintaining cleanliness, following a balanced diet, staying hydrated, and more, can expedite the recovery process and prevent the transmission of COVID-19 to the surrounding environment. In addition, quarantine Q individuals who are not positionally infected will recover with a η rate, while quarantine Q individuals become positive for symptomatic infected using masks I_M is $(1\text{-}\eta)$. Assuming that the immune immunity of recovered individuals can be lost and return to vulnerability, then the level of vulnerability without health protocols for the use of masks is stated as K and enters vulnerable subpopulations S_{TM}.

Simplification of Eqs.2 to Eqs.9 can be done by forming the proportion of the numberof individuals in a subpopulation to the total population, can be expressed as follows :

$$s_{TM} = \frac{S_{TM}}{N}, s_M = \frac{S_M}{N}, e = \frac{E}{N}, i_{TM} = \frac{I_{TM}}{N}, i_M = \frac{I_M}{N}, i_C = \frac{I_C}{N}, q = \frac{Q}{N}, r = \frac{R}{N} \tag{10}$$

where, Eq.10 is substituted into Eqs.2 to 9.

The following is a table that explains the parameters used to determine the model solution. Some parameters are assumptions as have been clearly explained in the modeling process.

Table 1. Variable symbol of COVID-19 spread model

Symbol	Indicator
N	Number of individuals in a population
S_{TM}	The number of subpopulations of COVID-19 vulnerable individuals not wearing masks
S_M	Number of subpopulations of COVID-19 vulnerable individuals wearing masks
E	Number of populations of COVID-19 latent individuals
I_{TM}	Number of subpopulations of infected positive individuals with symptoms of COVID-19 not wearing masks
I_M	Number of subpopulations of infected positive individuals with symptoms of COVID-19 wearing masks
I_C	Number of individual populations of COVID-19 Carriers
Q	Number of populations of COVID-19 quarantined individuals
R	Number of individual population recovered from COVID-19

Table 2. COVID-19 spread model parameters

Parameters	Indicator	Value	Source
μ	Birth or death rates in individual populations	0.002116268	[27]
p	Proportion of individuals who have been vaccinated	0.81	[28]
ε	Vaccine efficiency	0.634	[29]
u_1	Mask usage rate	0.8	[27]
u_2	Rate of not wearing a mask	0.2	[27]
β_1	Rate of disease transmission with symptomatic infected individuals	0. 5465	[30]
β_2	Rate of Disease Transmission with Individual *Carrier*	0. 5926	[30]
α	Level of awareness or understanding of the disease	[0.1;0.3;0.5]	Assumption
δ	Rate of latent individuals to infected individuals	1/5.84	[11]
v	Proportion of latent individuals to be *Carrier*	0.8079	[31]
σ	Cure rate of symptomatic infected individuals with treatment	1/7.5	[32]
λ	Carrier individual testing rate	0.6968	[31]
φ	Carrier*'s individual cure rate* naturally	1/7	[33]
η	Adherence rate for healthy living habits	[0.04;0.15;0.45]	Assumption
τ	Rate of quarantined individuals to infected individuals	1/14	[34]
K	The rate of recured individuals susceptible to disease	1/120	[35, 36]

3 Results and discussions

The results obtained are presented in several main points, namely equilibrium points, basic reproductive numbers, and model stability analysis.

3.1 Equilibrium points

The equilibrium point is a point that will not change its value for any variable of state and the first derivative for each variable is equal to zero [27]. If there is a function f over independent variables x and y, we will look for values of x and y that satisfy the equilibrium condition, where f'(x, y) = 0. Thus, the solutions to the equation are the points (x, y) that indicate the function has reached the equilibrium point. To obtain the equilibrium point, linearization is carried out, so that a Jacobian matrix is obtained. There are two equilibrium points, the first is a disease-free equilibrium point, where in this condition no disease infection occurs. Points are obtained from Eq. 11.

$$E_0({s_{TM}}^*, {s_M}^*, 0,0,0,0,0,0) \tag{11}$$

where, ${s_{TM}}^* = \frac{\mu+u_2}{\mu+u_1+u_2}$ and ${s_M}^* = \frac{u_1}{\mu+u_1+u_2}$.

Another equilibrium point is the point at which infection will continue to exist in a system called the endemic equilibrium point. Point obtained from Eq. 12

$$E_1({s_{TM}}^*, {s_M}^*, e^*, {i_{TM}}^*, {i_M}^*, {i_C}^*, q^*, r^*) \tag{12}$$

where, ${s_{TM}}^* = \frac{\mu+Kr^*+u_2 {s_M}^*}{\mu+u_1+(1-p)(\beta_1 {i_{TM}}^*+\beta_2 {i_C}^*)}$, ${s_M}^* = \frac{u_1 {s_{TM}}^*}{\mu+u_2}$, $e^* = \frac{(1-p){s_{TM}}^*(\beta_1 {i_{TM}}^*+\beta_2 {i_C}^*)}{(v+(1-v)\delta+\alpha+\mu}$, ${i_{TM}}^* = \frac{(1-v)\delta e^*+\lambda {i_C}^*+u_2 {i_M}^*}{\sigma+u_1+\mu}$, ${i_M}^* = \frac{(1-\eta)\tau q^*+u_1 {i_{TM}}^*}{\sigma+u_2+\mu}$, ${i_C}^* = \frac{\delta v e^*}{\lambda+(1-\lambda)+\mu}$, $q^* = \frac{\alpha e^*}{\eta+(1-\eta)\tau+\mu}$, and $r^* = \frac{({i_{TM}}^*+{i_M}^*)\sigma+(1-\lambda)\varphi {i_C}^*+\eta q^*}{K+\mu}$. The asterisk (star) in each state variable is referred to as the optimal value obtained from the first derivative which is equated to zero.

3.2 Basic reproductive numbers

The baseline reproduction number is the expected value of the number of infections per unit of time caused by a single case of infected individuals occurring in a population over a given period of time. The base reproduction number is denoted by R_0. There are several methods used to determine R_0, but we use a next-generation matrix as done by [37]. If $R_0 < 1$ the disease will disappear, $R_0 = 1$ the disease will remain (endemic) both called stable, and unstable if $R_0 > 1$, so the disease will escalate to plague [38]. The subpopulation to be used is the subpopulation that causes the spread of disease (infection), namely latent subpopulation (E), symptomatic infected positive subpopulation not using a mask (I_{TM}), symptomatic infected positive subpopulation using a mask (I_M), carrier subpopulation (I_C) and quarantine subpopulation (Q). The equation used is Eqs. 4-8. These equation can be written as $x' = \varphi(x) - \zeta(x)$, wherex' is a derivative of the infection equation with φ as a matrix of new infection rates and ζ as a matrix of infection transition rates. So that

$$\varphi = \begin{bmatrix} (1-p)S_{TM}(\beta_1 I_{TM} + \beta_2 I_C) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \zeta = \begin{bmatrix} E((\delta(v+(1-v)) + \alpha + \mu) \\ -(1-v)\delta E - \lambda I_C - u_2 I_M + I_{TM}(\sigma + u_1 + \mu) \\ -(1-\eta)\tau Q - u_1 I_{TM} + I_M(\sigma + u_2 + \mu) \\ -\delta v E + I_C((1-v)\varphi + \lambda + \mu) \\ -\alpha E + Q(\eta + (1-\eta)\tau + \mu) \end{bmatrix} \tag{13}$$

By substituting disease-free equilibrium points $E_0({s_{TM}}^*, {s_M}^*, 0,0,0,0,0,0)$ Jacobian matrices F and V are obtained from φ and ζ. Then, determine the matrix $K=FV^{-1}$ (Where K is defined as the next-generation matrix obtained by multiplying the matrices F and V^{-1} from matrix V This is then associated with the basic reproduction number, which is the largest eigenvalue of the K matrix) to be used to obtain the largest eigenvalue with the characteristic equation. So that the basic reproduction number is obtained (R_0) as follows:

$$R_0 = \frac{(1-p)(\mu+u_2)\beta_1 A}{(u_1(\mu+\sigma)+u_2(\mu+\sigma)+\mu(\mu+\sigma))B} - \frac{(1-p)(\mu+u_2)\beta_2 \delta v}{(u_1+u_2+\mu)(\lambda\varphi-\lambda-\varphi-\mu)(\alpha+\delta+\mu)} \tag{14}$$

where

$$
\begin{aligned}
A = & -\delta\eta\lambda\mu\tau\nu\varphi - \delta\eta\lambda\sigma\tau\nu\varphi - \delta\eta\lambda\tau\nu\varphi u_2 + \alpha\eta\lambda\tau\varphi u_2 + \delta\eta\lambda\mu\tau\varphi + \delta\eta\lambda\mu\nu\varphi + \delta\eta\lambda\sigma\tau\varphi + \delta\eta\lambda\sigma\nu\varphi + \delta\eta\lambda\tau\varphi u_2 + \delta\eta\lambda\nu\varphi u_2 + \delta\eta\mu^2\tau\nu \\
& +\delta\eta\mu\sigma\tau\nu + \delta\eta\mu\tau\nu\varphi + \delta\eta\mu\tau\nu u_2 + \delta\eta\sigma\tau\nu\varphi + \delta\eta\tau\nu\varphi u_2 + \delta\lambda\mu^2\nu\varphi + \delta\lambda\mu\sigma\nu\varphi + \delta\lambda\mu\tau\nu\varphi + \delta\lambda\mu\nu\varphi u_2 + \delta\lambda\sigma\tau\nu\varphi + \delta\lambda\tau\nu\varphi u_2 \\
& -\alpha\eta\lambda\tau u_2 - \alpha\eta\mu\tau u_2 - \alpha\eta\tau\varphi u_2 - \alpha\lambda\tau\varphi u_2 - \delta\eta\lambda\mu\tau - \delta\eta\lambda\mu\varphi - \delta\eta\lambda\sigma\tau - \delta\eta\lambda\sigma\varphi - \delta\eta\lambda\tau u_2 - \delta\eta\lambda\varphi u_2 - \delta\eta\mu^2\tau - \delta\eta\mu^2\nu - \delta\eta\mu\sigma\tau \\
& -\delta\eta\mu\sigma\nu - \delta\eta\mu\tau\varphi - \delta\eta\mu\tau u_2 - \delta\eta\mu\nu\varphi - \delta\eta\mu\nu u_2 - \delta\eta\sigma\tau\varphi - \delta\eta\sigma\nu\varphi - \delta\eta\tau\varphi u_2 - \delta\eta\nu\varphi u_2 - \delta\lambda\mu^2\varphi - \delta\eta\mu\sigma\varphi - \delta\lambda\mu\tau\varphi \\
& -\delta\lambda\mu\varphi u_2 - \delta\lambda\sigma\tau\varphi - \delta\lambda\tau\varphi u_2 - \delta\mu^3\nu - \delta\mu^2\sigma\nu - \delta\mu^2\tau\nu - \delta\mu^2\nu\varphi - \delta\mu^2\nu u_2 - \delta\mu\sigma\tau\nu - \delta\mu\sigma\nu\varphi - \delta\mu\tau\nu\varphi - \delta\mu\tau\nu u_2 - \delta\mu\nu\varphi u_2 - \delta\sigma\tau\nu\varphi \\
& -\delta\tau\nu\varphi u_2 + \alpha\lambda\tau u_2 + \alpha\mu\tau u_2 + \alpha\tau\varphi u_2 + \delta\eta\lambda\mu + \delta\eta\lambda\sigma + \delta\eta\lambda u_2 + \delta\eta\mu^2 + \delta\eta\mu\sigma + \delta\eta\mu\varphi + \delta\eta\mu u_2 + \delta\eta\sigma\varphi + \delta\eta\varphi u_2 + \delta\lambda\mu^2 + \delta\lambda\mu\sigma \\
& +\delta\lambda\mu\tau + \delta\lambda\mu u_2 + \delta\lambda\tau u_2 + \delta\mu^3 + \delta\mu^2\sigma + \delta\mu^2\tau + \delta\mu^2\varphi + \delta\mu^2 u_2 + \delta\mu\sigma\tau + \delta\mu\sigma\varphi + \delta\mu\tau\varphi + \delta\mu\tau u_2 + \delta\mu\varphi u_2 + \delta\sigma\tau\varphi + \delta\tau\varphi u_2
\end{aligned}
$$

$$B = (u_1 + u_2 + \sigma + \mu)(\lambda\varphi - \lambda - \varphi - \mu)(\eta\tau - \eta - \mu - \tau)(\alpha + \delta + \mu)$$

3.3 Stability analysis

In stability analysis, an analysis of the equilibrium point contained in the model will be carried out using eigenvalues obtained from the Jacobian matrix. Stability tests were performed by linearization Eqs. 2-9. then substitute each disease-free and endemic equilibrium point into the Jacobian matrix Eq.15.

$$
J(E) = \begin{bmatrix}
-u_1 - (1-p)(\beta_1 i_{TM} + \beta_2 i_C) - \mu & u_2 & 0 & -(1-p)s_{TM}\beta_1 & 0 & -(1-p)s_{TM}\beta_2 & 0 & K \\
u_1 & u_2 - \mu & 0 & 0 & 0 & 0 & 0 & 0 \\
(1-p)(\beta_1 i_{TM} + \beta_2 i_C) & 0 & \delta v - (1-v)\delta - \alpha - \mu & (1-p)s_{TM}\beta_1 & 0 & (1-p)s_{TM}\beta_2 & 0 & 0 \\
0 & 0 & (1-v)\delta & -u_1 - \sigma - \mu & u_2 & \lambda & 0 & 0 \\
0 & 0 & 0 & u_1 & -u_2 - \sigma - \mu & 0 & (1-\eta)\tau & 0 \\
0 & 0 & \delta v & 0 & 0 & -(1-\lambda)\varphi - \lambda - \mu & 0 & 0 \\
0 & 0 & \alpha & 0 & 0 & 0 & -\eta - (1-\eta)\tau - \mu & 0 \\
0 & 0 & 0 & \sigma & \sigma & (1-\lambda)\varphi & \eta & -K - \mu
\end{bmatrix} \quad (15)
$$

Substitute the parameter values of Table 2 and Eqs 11 into the matrix to obtain the eigenvalues of the disease-free equilibrium point. The eigenvalue of the matrix J(E) has a nonzero solution if and only if det(J(E) − λI) is called the characteristic equation. Next by substituting the value of the Table 2 to Eqs. 14 will obtain the base reproduction number (R_0) as follows:

Table 3. The eigenvalue and base reproduction number (R_0) of the disease-free equilibrium point

1.	α=0.1 and η=0.04		α=0.1 and η=0.15		α=0.1 and η=0.45	
	Eigen Value	R_0	Eigen Value	R_0	Eigen Value	R_0
	λ_1 = -0.0021	1.2134	λ_1 = -0.0021	1.0731	λ_1 = -0.0021	0.9869
	λ_2 = -1.0021		λ_2 = -1.0021		λ_2 = -1.0021	
	λ_3 = -1.1218		λ_3 = -1.1218		λ_3 = -1.1217	
	λ_4 = -0.7781		λ_4 = -0.7780		λ_4 = -0.7779	
	λ_5 = -0.3605		λ_5 = -0.3504		λ_5 = -0.4706	
	λ_6= +0.0156		λ_6 = -0.2484		λ_6 = -0.3996	
	λ_7 = -0.0104		λ_7 = +0.0062		λ_7 = -0.0012	
	λ_8 = -0.1455		λ_8 = -0.0104		λ_8 = -0.0104	
	Unstable		Unstable		Asymptotically stable	
4.	α=0.3 and η=0.04		α=0.3 and η=0.15		α=0.3 and η=0.45	
	Eigen Value	R_0	Eigen Value	R_0	Eigen Value	R_0

	λ_1 = -0.0021		λ_1 = -0.0021		λ_1 = -0.0021 +0.0000i	
	λ_2 = -1.0021		λ_2 = -1.0021		λ_2 = -1.0021 +0.0000i	
	λ_3 = -1.1223		λ_3 = -1.1222		λ_3 = -1.1218 +0.0000i	
	λ_4 = -0.7995	1.0007	λ_4 = -0.7994	0.7577	λ_4= -0.7984 +0.0000i	0.6085
	λ_5 = -0.4796		λ_5 = -0.4717		λ_5 = -0.5036 +0.0755i	
	λ_6 = -0.1889		λ_6 = -0.2760		λ_6 = -0.5036 - 0.0755i	
	λ_7 = +0.0000		λ_7 = -0.0104		λ_7 = -0.0104 +0.0000i	
	λ_8 = -0.0104		λ_8 = -0.0232		λ_8 = -0.0435 +0.0000i	
	Unstable		Asymptotically stable		Asymptotically stable	
7.	α=0.5 and η=0.04		α=0.5 and η=0.15		α=0.5 and η=0.45	
	Eigen Value	R_0	Eigen Value	R_0	Eigen Value	R_0
	λ_1 = -0.0021		λ_1 = -0.0021		λ_1 = -0.0021 + 0.0000i	
	λ_2 =-1.0021		λ_2 = -1.0021		λ_2 = -1.0021 + 0.0000i	
	λ_3 =-1.1233		λ_3 = -1.1231		λ_3 = -1.1220 + 0.0000i	
	λ_4 =-0.8531	0.9144	λ_4 = -0.8526	0.6297	λ_4= -0.8499 + 0.0000i	0.4549
	λ_5 =-0.6011		λ_5 = -0.5978		λ_5 = -0.5672 +0.0549i	
	λ_6 =-0.2068		λ_6 = -0.2825		λ_6 = -0.5672 - 0.0549i	
	λ_7 =-0.0059		λ_7 = -0.0104		λ_7 = -0.0104 + 0.0000i	
	λ_8 =-0.0104		λ_8 = -0.0364		λ_8 = -0.0646 + 0.0000i	
	Asymptotically stable		Asymptotically stable		Asymptotically stable	

Table 4. The eigenvalue and basic reproduction number (R_0) of the endemic equilibrium

1.	α=0.1 and η=0.04		α=0.1 and η=0.15		α=0.1 and η=0.45	
	Eigenvalue	R_0	Eigenvalue	R_0	Eigenvalue	R_0
	λ_1 = -1.0027 + 0.0000i		λ_1 = -1.0023 + 0.0000i		λ_1 = +0.0011	
	λ_2= -1.1237 + 0.0000i		λ_2 = -1.1225 + 0.0000i		λ_2 = -0.0021	
	λ_3 = -0.7714 + 0.0000i		λ_3 = -0.0055 + 0.0060i		λ_3 = -0.0114	
	λ_4 = -0.3501 +0.0000i	1.2134	λ_4 = -0.0055 - 0.0060i	1.0731	λ_4 = -1.0021	0.9869
	λ_5 = -0.0060 +0.0115i		λ_5 = -0.0021 +0.0000i		λ_5 = -1.1215	
	λ_6 = -0.0060 - 0.0115i		λ_6 = -0.3464 +0.0000i		λ_6 = -0.4010	
	λ_7 = -0.0021 + 0.0000i		λ_7 = -0.2482 + 0.0000i		λ_7 = -0.4701	

	$\lambda_8 = -0.1452 + 0.0000i$		$\lambda_8 = -0.7754 + 0.0000i$		$\lambda_8 = -0.7784$	
	Asymptotically stable		Asymptotically stable		Unstable	
4.	α=0.3 and η=0.04		α=0.3 and η=0.15		α=0.3 and η=0.45	
	Eigenvalue	R_0	Eigenvalue	R_0	Eigenvalue	R_0
	$\lambda_1 = -1.0021$	1.0007	$\lambda_1 = +0.0115$	0.7577	$\lambda_1 = +0.0173 + 0.0000i$	0.6085
	$\lambda_2 = -1.1223$		$\lambda_2 = -0.0021$		$\lambda_2 = -0.0021 + 0.0000i$	
	$\lambda_3 = -0.4796$		$\lambda_3 = -0.0200$		$\lambda_3 = -0.0244 + 0.0000i$	
	$\lambda_4 = -0.0021$		$\lambda_4 = -0.2850$		$\lambda_4 = -1.0005 +0.0000i$	
	$\lambda_5 = -0.0000$		$\lambda_5 = -0.4713$		$\lambda_5 = -1.1137 +0.0000i$	
	$\lambda_6 = -0.0104$		$\lambda_6 = -1.0012$		$\lambda_6 = -0.5117 +0.0868i$	
	$\lambda_7 = -0.1889$		$\lambda_7 = -1.1185$		$\lambda_7 = -0.5117 - 0.0868i$	
	$\lambda_8 = -0.7995$		$\lambda_8 = -0.8174$		$\lambda_8 = -0.8341 + 0.0000i$	
	Asymptotically stable		Unstable		Unstable	
7.	α=0.5 and η=0.04		α=0.5 and η=0.15		α=0.5 and η=0.45	
	Eigenvalue	R_0	Eigenvalue	R_0	Eigenvalue	R_0
	$\lambda_1 = -1.0018$	0.9144	$\lambda_1 = -1.1161$	0.6297	$\lambda_1 = +0.0222 + 0.0000i$	0.4549
	$\lambda_2 = -1.1223$		$\lambda_2 = -1.0002$		$\lambda_2 = -0.0021 + 0.0000i$	
	$\lambda_3 = -0.8605$		$\lambda_3 = -0.8971$		$\lambda_3 = -0.0278 + 0.0000i$	
	$\lambda_4 = -0.5975$		$\lambda_4 = -0.5755$		$\lambda_4 = -1.1034 +0.0000i$	
	$\lambda_5 = -0.2099$		$\lambda_5 = -0.3042$		$\lambda_5 = -0.9968 +0.0000i$	
	$\lambda_6 = +0.0043$		$\lambda_6 = +0.0154$		$\lambda_6 = -0.9393 +0.0000i$	
	$\lambda_7 = -0.0021$		$\lambda_7 = -0.0021$		$\lambda_7 = -0.5658 + 0.0911i$	
	$\lambda_8 = -0.0141$		$\lambda_8 = -0.0228$		$\lambda_8 = -0.5658 - 0.0911i$	
	Unstable		Unstable		Unstable	

Based on Table 3 and Table 4 for each given α and η value obtained the base reproduction numbers $R_0 > 1$ and $R_0 < 1$. The disease-free and endemic equilibrium points of parameter values α=0.1 with η=0.04, η=0.15, and η=0.45 obtained base reproduction number values are respectively $R_0 > 1$, $R_0>0$, and $R_0 < 1$. The first case of $R_0 > 1$ indicates that an infected individual will continue to infect more than one other individual. It occurs when only 10% of individuals who have awareness or understanding of the disease then take preventive measures and 4% of individuals who adopt healthy living habits. The disease-free and endemic equilibrium points in these conditions are unstable and asymptotic stable respectively, indicating that the disease continues to spread exponentially in the population and will not reach a disease-free state but towards an endemic stable state.

The second case of $R_0 > 1$, this indicates that infected individuals will continue to infect more than one other individual even though the R_0 value obtained is smaller than the first case, where the spread of the disease has the potential to spread smaller than before and because the value of $R_0 > 1$ the infection will continue to occur. It occurs when only 10% of individuals who have awareness or understanding of the disease then take preventive measures and 15% of individuals adopt healthy living habits. The disease-free and endemic equilibrium points in these conditions are unstable and asymptotic stable respectively, indicating that the disease continues to spread exponentially in the population and will not reach a disease-free state but towards an endemic stable state.

The third case of $R_0 < 1$ indicates that an infected individual infects less than one other individual. It occurs when only 10% of individuals who have awareness or understanding of the disease then take preventive measures and 45% of individuals who adopt healthy living habits. The disease-free and endemic equilibrium points in these conditions are successively stable asymptotic and unstable, indicating that the disease will have reduced potential and will not spread exponentially in the population and will reach a disease-free state. Furthermore, the parameter value was increased to $\alpha=0.3$ with $\eta=0.04$, $\eta=0.15$, and $\eta=0.45$ obtained the basic reproduction values were respectively $R_0 > 1$, $R_0 < 1$, and $R_0 < 1$. The first case of $R_0 > 1$, the R_0 value obtained is almost close to one but the value is still greater than one, this indicates that infected individuals will infect more than one other individual with slight differences. This condition has a small rate of spread of infection cases at $\alpha=0.1$, but the possibility of disease spread still exists. It occurs when there are 30% of individuals who have awareness or understanding of the disease then take preventive measures and 4% of individuals who adopt healthy living habits. The disease-free and endemic equilibrium points in these conditions are unstable and asymptotic stable respectively, indicating that the disease spreads with smaller spread in the population and will not or will take less time than the previous case to reach a disease-free state and move towards an endemic stable state. The second case is $R_0 < 1$, this indicates that the infected individual infected less than one other individual. The R_0 value obtained is lower than in the case of infection at $\alpha=0.1$. It occurs when there are 30% of individuals who have awareness or understanding of the disease then take preventive measures and 15% of individuals who adopt healthy living habits.

The disease-free and endemic equilibrium points in these conditions are respectively asymptotic stable and unstable, indicating that the disease has been significantly reduced and is not spreading exponentially in the population and will reach a disease-free state. The third case of $R_0 < 1$ indicates that an infected individual infected less than one other individual. The obtained R_0 value is smaller than in the first case. It occurs when there are 30% of individuals who have awareness or understanding of the disease then take preventive measures and 45% of individuals who adopt healthy living habits. The free and endemic equilibrium points in these conditions are asymptotic stable and unstable respectively, indicating that the disease is diminishing and not spreading exponentially in the population and will reach a disease-free state. Then for parameter values $\alpha=0.5$ with $\eta=0.04$, $\eta=0.15$, and $\eta=0.45$, the basic reproduction number values are all $R_0 < 1$. In this condition there has been a decrease in the spread of infection and an increase in disease control capabilities. The first case of $R_0 < 1$, there are 50% of individuals who have awareness or understanding of the disease then take preventive measures and 4% of individuals who adopt healthy living habits. The disease-free and endemic equilibrium points in these conditions are asymptotic and unstable stable, indicating that disease is diminishing and disease control is improving in the population and will achieve a stable disease-free state. The second case $R_0 < 1$, there are 50% of individuals who have awareness or understanding of the disease then take preventive measures and 15% of individuals who adopt healthy living habits. The disease-free and endemic equilibrium points in these conditions are successively stable asymptotic and unstable, indicating that disease is diminishing and disease control is improving more than ever in the population and will achieve a stable disease-free state. The third case $R_0 < 1$, there are 50% of individuals who have awareness or understanding of the disease then take preventive measures and 45% of individuals who apply healthy living habits. The disease-free and endemic equilibrium points in these conditions are successively stable asymptotic and unstable, this indicates that the disease is decreasing and will disappear and controlling the spread of disease in the population and will reach a stable state with the ability to maintain a high level of health and contain the spread of disease.

4 Conclusion

This research proposes the development of a mathematical model of the spread of Covid-19 with carrier transmission and individual awareness and compliance in the transition period. The parameters of concern are awareness or understanding of disease and healthy living habits in accordance with applicable policies. This model also includes the role of the government with vaccination parameters and testing on individual carriers. The analysis was performed for two different equilibrium points, namely the disease-free equilibrium point and the endemic equilibrium point that obtained an asymptotic stable state when the base reproduction number values $R_0 < 1$ and $R_0 > 1$. The assumption of two parameter values tested shows that the parameters of awareness or understanding of disease and healthy living habits affect the value of the base reproduction number (R_0). The greater the increase in both parameter values, the smaller the R_0 value produced so that the infection can decrease and disappear from the population within a certain period of time. Therefore, the role of individuals is very necessary in this transition period, especially increasing awareness or understanding of diseases and healthy living habits. The implementation of other policies such as the use of masks for the sick, the provision of vaccinations, and contact tracing or testing of individual carriers which are the role of the government can help minimize the spread of disease so that the pandemic period does not return and internal diseases disappear completely from the population.

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