

# TOPOLOGY AND TRADITION: THE KNOT POLYNOMIALS OF KETUPAT NABI

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**Abstract.** This study explores the intersection of mathematics and culinary traditions, focusing on "Ketupat Nabi," a dish from South Sulawesi's Bugis community. By applying knot theory, it seeks to understand the mathematical properties of the dish's knot diagrams. The research began with selecting traditional foods characterized by knots or ties, essential for framing the study's focus. Photographs and cultural histories of these foods were then collected to provide context. The analysis involved comparing these culinary knots with established knot theory literature, leading to the creation of graphical knot diagrams. These diagrams underwent mathematical analysis using the Alexander polynomial, a key tool in knot theory. The ketupat's knot diagram, featuring 24 crossing points and incapable of 3-colorability, was a primary focus. Remarkably, this study identified a unique Alexander polynomial for the Ketupat Nabi's knot diagram, a significant addition to the knowledge of polynomials associated with 24-crossing-point knots. The identified polynomial for the ketupat Nabi's knot diagram is:

$$\Delta_K(t) = -t^{10} + 7t^9 - 21t^8 + 35t^7 - 35t^6 + 26t^5 - 26t^4 + 30t^3 - 37t^2 + 61t^1 - 79 + 61t^{-1} - 37t^{-2} + 30t^{-3} - 26t^{-4} + 26t^{-5} - 35t^{-6} + 35t^{-7} - 21t^{-8} + 7t^{-9} - t^{-10}$$

These findings accentuate the value of interdisciplinary approaches in elucidating the multifaceted relationship between cultural and mathematical paradigms. Moreover, the research illuminates the untapped scholarly potential of traditional foods as avenues for mathematical exploration.

## 1 Introduction

Mathematics has been an integral part of human civilization for millennia, serving as a key tool for solving daily challenges and being employed by influential figures across diverse societies. Notable mathematical contributions emerged from major civilizations such as Ancient Egypt and India, whose legacies are preserved in historically significant structures [1].

One fascinating branch of mathematics, knot theory, studies the intricate configurations of knots in three-dimensional space. In recent decades, this theory has found applications in

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fields as varied as physics, biology, arts, and everyday life. For example, the analysis of traditional tools, including agricultural implements and fishing equipment, reveals forms that can be examined through the lens of knot theory, shedding light on their design and function [2].

Central to knot theory is the Alexander polynomial, a mathematical tool that distinguishes between different knot structures. It reveals the unique properties of knots and aids in the calculation of various knot invariants. Applying knot theory to traditional tools can uncover insights into their physical and mechanical properties. For instance, analyzing knots in fishing nets can help estimate the net's resilience and the strength of its constituent knots, offering potential innovations in industry and technology [3,4].

Ketupat, a traditional rice-based dish in Indonesia, is wrapped in a web of young coconut leaves, or "janur" in Javanese. This dish is boiled extensively, often for around five hours, and is typically infused with flavors from pandanus leaves during its preparation. It is traditionally served during special occasions, particularly with side dishes such as chicken curry, rendang, and satay [6].

## 1.1 History of Ketupat

Ketupat is believed to have been introduced by Sunan Kalijaga, a revered figure in Javanese Islam, during the 15th to 16th century, predominantly in Central Java's Demak regency. Associated closely with the Islamic celebration of Eid, the tradition of making ketupat is an integral part of the post-Ramadan festivities, Bakda Lebaran and Bakda Kupa. Ketupat is also now enjoyed beyond Java, thanks to the spread of Islam, reaching nations including Singapore, Malaysia, and Brunei [8,10].

## 1.2 Philosophies of Ketupat

Ketupat is rich in symbolic meaning. It represents apology and blessing [11]. Its primary components, rice and young coconut leaves, are symbolic: rice is seen as a manifestation of worldly desire, while the leaf signifies conscience or "Jatining nur" in Javanese [12]. In Sundanese culture, "kupa" discourages speaking ill of others and encourages admitting one's wrongs and seeking forgiveness – a tradition upheld during Eid Al-Fitr.

The complex woven pattern of ketupat represents human errors, while the white rice symbolizes the purity attained after forgiveness. The diamond shape of the finished ketupat is seen as a symbol of Muslim victory after a month of fasting. Additionally, hanging a cooked ketupat in front of the house is believed to ward off evil spirits [14].

In some preparations, ketupat is boiled in coconut milk instead of water. In Javanese, coconut milk, or "santen," is associated with the word "pangapunten," which means apology. This is depicted in a Javanese poem that translates to "I am sorry, I made a mistake" [15].

Moreover, an Indonesian anthropologist interpreted ketupat as a symbol of social solidarity, embodying the principle of reciprocity. The practice of exchanging ketupat during celebrations fosters a sense of community and interconnectedness, reflecting a deeply ingrained social relationship [16].

Ketupat is also presented in certain life-cycle ceremonies. For instance, during the celebration of a four-month pregnancy, ketupat is offered as a symbol of well wishes for the child's future – prosperity for a boy and beauty and virtue for a girl.

In this way, ketupat serves as more than a culinary tradition; it is a rich tapestry of cultural, philosophical, and social significance that continues to thrive in modern Indonesian society.

### **1.3 Culinary Aspects of Ketupat**

Beyond its symbolic significance, ketupat holds a special place in the culinary landscape of Indonesia and neighboring countries. The cooking process is meticulous. To ensure the glutinous rice expands perfectly within its leafy encasement, the preparation starts with soaking the rice in water infused with pandanus leaves for added flavor [6,7]. Additionally, the young coconut leaves are pre-soaked to prevent tearing during the weaving process.

The result is a unique texture and flavor profile. The ketupat's compact rice grains offer a chewy consistency, complemented by the subtle hints of pandanus and coconut. This serves as a perfect base for robust and spicy dishes that usually accompany it.

### **1.4 Spread and Evolution of ketupat**

Over the years, the popularity of ketupat has transcended the borders of Indonesia. In Malaysia, it's often paired with rendang, a slow-cooked dry curry deeply spiced and rich in coconut. Meanwhile, in Singapore and Brunei, variations of ketupat dishes have emerged, incorporating local flavors and ingredients [17].

The modern era has also seen an evolution in the preparation and presentation of ketupat. Fusion dishes that blend traditional elements with contemporary culinary techniques are gaining traction, providing a fresh twist to this age-old delicacy. This adaptability ensures that ketupat remains relevant and cherished across generations.

### **1.5 Cultural Exchange and the Role of Ketupat**

Given its profound roots in Indonesian culture and its subsequent spread to neighboring countries, ketupat has become a conduit for cultural exchange. It symbolizes the shared history and values of the Malay Archipelago and stands as a testament to the region's rich heritage. Celebrations featuring ketupat often lead to cross-cultural dialogues, allowing communities to exchange stories, traditions, and culinary secrets [18].

In international food festivals and global culinary platforms, ketupat has been introduced to a wider audience. Through this dish, many have gotten their first taste of Indonesia's vibrant culture and traditions.

### **1.6 Knot Theory**

Knot theory is a captivating branch of mathematics that delves into the study of knots in three-dimensional space, examining how these loops intertwine and interact with various mathematical disciplines such as topology, geometry, and graph theory. In knot theory, knots are defined as closed loops in three-dimensional space without any breaks. This definition allows mathematicians to arrange and manipulate these knots, systematically uncovering their unique properties and characteristics.

In "The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots," Adams [14] underscores the significance of knot theory within the broader realm of mathematics. Adams highlights the prevalence of knots in a variety of fields beyond mathematics, including physics, chemistry, biology, and engineering, emphasizing the universal relevance and applicability of knot theory.

Kauffman [17], in his seminal book "Knots," elevates the discussion of knot theory by elucidating its deep connections with group theory and topology. Kauffman's work explores knot theory in three dimensions, offering insight into decomposition theorems, knot invariants, and the intriguing relationships between knots and graph theory.

Cromwell [16] contributes a valuable perspective through his book "Knots and Links," wherein he presents an accessible introduction to knot theory and its integral role in understanding three-dimensional space. Meanwhile, in "On Knots," Kauffman [17] further investigates the multifaceted relationships knot theory shares with other mathematical domains, such as graph theory, topology, and algebraic theory.

The practical applications of knot theory are as diverse and far-reaching as the theory is intricate. For example, in biology, knot theory provides crucial insights into the complex structures of DNA and proteins, offering a unique lens through which scientists can examine enzymatic processes and chemical reactions in knotted biological molecules. In the realm of chemistry, knot theory plays a role in the study of molecular structures and the intricacies of chemical reactions.

Physics is another field where knot theory finds substantial application. Here, it aids in the comprehension of phenomena as varied as wave propagation, relativity theory, and particle physics. The coiling and uncoiling of ties on strings and cables become intricate puzzles that physicists can unravel with the tools of knot theory.

Furthermore, in the world of computer science, knot theory is proving to be a powerful tool for algorithm development. It provides methodologies for detecting knots in network configurations and identifying patterns within complex data sets.

As research in knot theory continues to flourish, it increasingly draws the attention and engagement of mathematicians worldwide. This vibrant field is marked by a steady stream of new discoveries and a wealth of unresolved problems that continue to spark curiosity and inspire fervent research enthusiasm

## 2 Method

This research is anchored in a qualitative case study approach, a design intentionally selected for its capacity to yield in-depth and nuanced insights into specific cases. The central theme of this study focuses on the exploration of knot polynomials, placing a special emphasis on the Alexander polynomial as manifested in traditional Bugis tools.

The subjects under exploration in this research are the 'ketupat nabi' from South Sulawesi, Indonesia. These are intricate woven palm leaf pouches traditionally used in various ceremonies and rituals.

The research procedure has been meticulously structured to ensure a robust and rigorous examination of the study subjects. The initial step involves selecting a 'ketupat' that incorporates knots or ties as fundamental components of its structure. This selection is pivotal, as it sets the stage for the subsequent steps and establishes the primary focus of the research.

Once a 'ketupat' is selected, data collection commences. This phase involves photographing the chosen 'ketupat' and diligently recording relevant details about its origin, history, and purpose. This step is instrumental in furnishing a contextual understanding of the 'ketupat' and its significance.

The third step entails identifying the knots or ties present in the 'ketupat'. This is achieved by visually inspecting the tools, followed by a comparative analysis with scholarly literature

on knot theory. This approach facilitates a more scientific understanding of the knots and ties utilized in these tools.

After identifying the knots, a knot diagram is constructed. This diagram serves as a visual representation of the knots or ties identified in the 'ketupat', aiding in a more detailed examination of their configuration.

Subsequently, a mathematical computation—specifically the calculation of the Alexander polynomial—is performed for each knot or tie depicted in the diagram. This vital step contributes to a mathematical understanding of the knots' characteristics.

The sixth step encompasses a thorough analysis of the calculated polynomials to discern the unique characteristics of the knots present in the 'ketupat'. This examination aids in unraveling the intricacies of the knots and comprehending their functional role within the structure.

To ensure the validity of the findings, a comprehensive literature review is conducted. This process is integral to corroborating the analytical results, thereby reinforcing the credibility and reliability of the research.

Finally, the research concludes by synthesizing the findings and discussing their implications for our understanding of the 'ketupat'. Moreover, the research explores how these findings may potentially contribute to the broader field of knot theory, underscoring their relevance and potential applications in diverse domains.

### 3 Results and Discussion

Ketupat is a traditional cuisine in Indonesia. It is special food for the Ied Fitri celebration. Originally, ketupat was introduced by Sunan Kalijaga. He was a famous Scholar and Saint in the early Islam era in Indonesia. Ketupat comes from java terms “Kupat= Ngaku lepat and Laku Papat. Nagaku lepat means to admit mistakes, and laku papat is four steps. If we observe the model of ketupat, we can see that the ketupat can be represented in the knot diagram. The connection between the model and the philosophy behind the ketupat will be an interesting topic to investigate.

Below is some of the example of Ethnomathematics about ketupat and knot theory. This example is taken from Mandarnese Tribe in West Sulawesi. The ketupat is named Ketupat Nabi or ketupat Bagea.



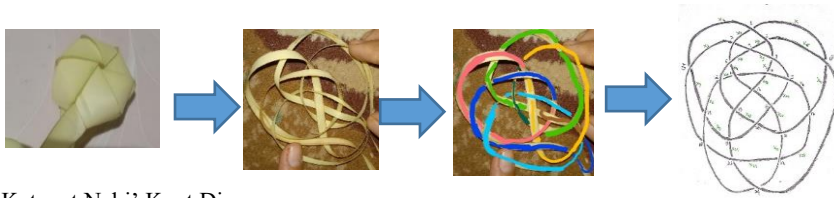
**Fig. 1.** Ketupat Nabi

Ketupat Nabi is one type of ketupat with a hexagonal surface shape, smaller in size than other types of Ketupat and similar to a hexagonal prism. The primary material needed is a coconut leaf to make a Ketupat Nabi.

Ketupat Nabi is one sort of ketupat with a hexagonal surface formed, littler in measure than other Ketupat, and is comparative to the hexagonal prism. The most material required could be a coconut leaf to form a Ketupat Nabi.

Ketupat Nabi has the meaning to do not be unquenchable and eager. The reason is that Ketupat Nabi could be nourishment that has little measure so that the characteristics are not riches, not ravenous, and others. Since most Mandarneses are Muslims, “Nabi” in Ketupat

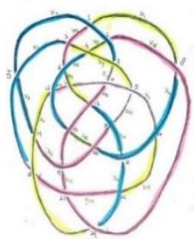
Nabi depends on the prophet Muhammad SAW who demonstrates the leading man and has exceptionally excellent characteristics like not being covetous and more concerned to give charity to charity community.



**Fig. 2.** Ketupat Nabi' Knot Diagram

The knot diagram of ketupat Nabi can be found by following steps:

The knot diagram of the ketupat Nabi is a 24-knot diagram. This is interesting since, in the tabular knot that found up to less than 20 crossings. The knot diagram of the ketupat Nabi is 24 crossing points. If we continue to find the colourability, we get that the diagram is not 3-colourable.



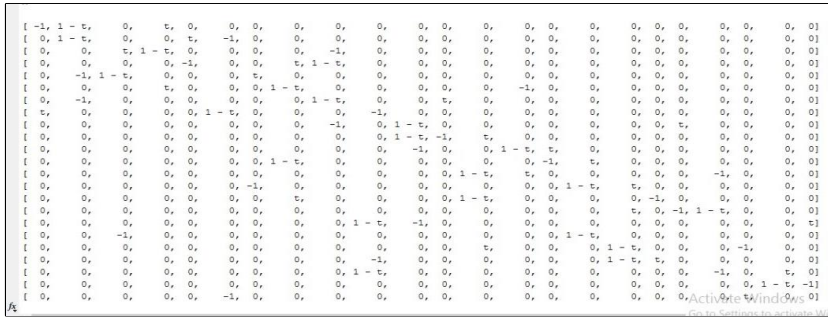
**Fig. 3.** Ketupat Nabi's knot diagram is not colourable

This result is confirmed by computing the determinant of the matrix containing the knot diagram in the figure, which equals 637 and cannot be divided by three.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24
1	-1	2	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	2	0	0	-1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	-1	0	0	-1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	-1	2	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	-1	0	0	0	2	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
7	0	-1	0	0	0	0	0	0	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
8	-1	0	0	0	0	2	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	-1	0	2	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	2	-1	-1	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	-1	0	0	2	-1	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	2	0	0	0	0	0	0	-1	-1	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	-1	0	0	0	0	0	0
14	0	0	0	0	0	-1	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	-1	0	0	0	2	0	0	0	0	-1	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	-1	2	0	0	0	0	0
17	0	0	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
18	0	-1	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	-1
19	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	2	0	0	0	0	-1	0	0	0	0
20	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	2	-1	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	-1	0	-1	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	-1	-1	0	0
23	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	2	0
24	0	0	0	0	2	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	-1	0	0	0

**Fig. 4.** Coefficient of ketupat Nabi's diagram

The knot polynomial is used to ensure that the knot is invariant with other listed knots in appendix 1. By applying the procedure to find The Alexander polynomial for ketupat Nabi by counting the matrix below.



**Fig. 5.** Matrix coefficient of ketupat Nabi’s diagram to find the Alexander

We get the polynomial is below

$$\begin{aligned} \Delta_K(t) &= -t^{21} + 7t^{20} - 21t^{19} + 35t^{18} - 35t^{17} + 26t^{16} - 26t^{15} + 30t^{14} - 37t^{13} \\ &\quad + 61t^{12} - 79t^{11} + 61t^{10} - 37t^9 + 30t^8 - 26t^7 + 26t^6 - 35t^5 \\ &\quad + 35t^4 - 21t^3 + 7t^2 - t \\ \Delta_K(t) &= -t^{11}(-t^{10} + 7t^9 - 21t^8 + 35t^7 - 35t^6 + 26t^5 - 26t^4 + 30t^3 - 37t^2 \\ &\quad + 61t^1 - 79 + 61t^{-1} - 37t^{-2} + 30t^{-3} - 26t^{-4} + 26t^{-5} - 35t^{-6} \\ &\quad + 35t^{-7} - 21t^{-8} + 7t^{-9} - t^{-10}) \\ &\approx -t^{10} + 7t^9 - 21t^8 + 35t^7 - 35t^6 + 26t^5 - 26t^4 + 30t^3 - 37t^2 + 61t^1 - 79 \\ &\quad + 61t^{-1} - 37t^{-2} + 30t^{-3} - 26t^{-4} + 26t^{-5} - 35t^{-6} + 35t^{-7} - 21t^{-8} \\ &\quad + 7t^{-9} - t^{-10} \\ \Delta_K(t) &= -t^{10} + 7t^9 - 21t^8 + 35t^7 - 35t^6 + 26t^5 - 26t^4 + 30t^3 - 37t^2 + 61t^1 - 79 \\ &\quad + 61t^{-1} - 37t^{-2} + 30t^{-3} - 26t^{-4} + 26t^{-5} - 35t^{-6} + 35t^{-7} - 21t^{-8} \\ &\quad + 7t^{-9} - t^{-10} \end{aligned}$$

## 4 Conclusion

Knot theory is not merely an abstract branch of mathematics; it is a dynamic and evolving field with far-reaching implications and applications that extend from the microscopic intricacies of molecular biology to the expansive realms of physics and computing.

Ketupat, while seemingly simple, embodies a multitude of layers, from its intricate preparation to its deep cultural and philosophical underpinnings. It's a testament to the ways food can serve as both sustenance and a repository of collective memory, binding communities together and bridging gaps between generations. As the world becomes increasingly interconnected, dishes like ketupat play a pivotal role in fostering understanding and appreciation of diverse cultures.

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## References

1. Kelter P, Carr J, Scott A. Chemistry: A world of Choices. (McGraw-Hill, New York, 2003)
2. Jatmiko D. Jurnal Konservasi Cagar Budaya Borobudur. **18**, 1, 1-12 (2019)
3. Kusuma A, Dewi AS, Sari AP. Journal of Mathematics Education, Science, and

- Technology. **5**, 1, 29-36 (2019)
4. Hajiaghayi M and Hajiaghayi R. *Journal of Applied Environmental and Biological Sciences*, **3**, 9, 95-99 (2013)
  5. Tan B. Ketupat [Internet]. 2015 [cited 2017 Oct 12]. Available from: [http://eresources.nlb.gov.sg/infopedia/articles/SIP\\_2015-10-26\\_111827.html](http://eresources.nlb.gov.sg/infopedia/articles/SIP_2015-10-26_111827.html).
  6. Mulyawati W and Ilse H. *Resep andalan resto Indonesia lontong dan ketupat*. (PT Gramedia Pustaka Utama, Jakarta, 2008)
  7. Sutari T. Tips Memasak Ketupat agar Pulen & Enak ala Chef Yuda Bustara [Internet]. 2017 [cited 2017 Oct 12]. Available from: <https://www.cnnindonesia.com/gaya-hidup/20170624170722-262-223965/tips-memasakketupat-agar-pulen-enak-ala-chef-yuda-bustara/>.
  8. Yusuf and Toet. *Indonesia has stories*. (Jakarta, Penerbit Cerdas Interaktif, 2014)
  9. Tan HP. Hari raya lebaran, hari toleransi. *Reformata* 2011:143.
  10. Kiple KF and Ornelas KC. *The Cambridge world history of food*, (USA, Cambridge University Press, 2001)
  11. Pakri M. *Strengthening local knowledge towards globalization*. (Penerbit Universiti Sains Malaysia, Pulau Pinang, 2015)
  12. Dewabrata DP. *Tatanan baru rangkaian janur gaya Indonesia*. (PT Gramedia Pustaka Utama, Jakarta, 2009)
  13. Tim PISS-KTB. *Kumpulan tanya jawab keagamaan*. Yogyakarta. 2015, [www.piss-ktb.com](http://www.piss-ktb.com).
  14. Prawiro AMB. *Reception through selection-modification: antropologi hukum Islam di Indonesia*. (Deepublish, Yogyakarta, 2016)
  15. Sigalingging IW. Ternyata ketupat punya filosofi hidup yang sarat makna, lho! [Internet]. 2017 [cited 2017 Jun 29]. Available from: <https://life.idntimes.com/inspiration/irma-wulandriani/filosofi-hidup-ketupat-c1c2>.
  16. Tim Koordinasi Siaran Direktorat Jenderal Kebudayaan. *Aneka ragam khasanah budaya nusantara VI*. (Departemen Pendidikan dan Kebudayaan, Jakarta, 1995)
  17. Adam M, ZDM, **43**, 2, 161-174, (2011)
  18. Chen WH, Ja'faruddin, *Journal of Mathematics and Mathematics Science*. **12**, 1, 1-14 (2019)
  19. Cromwell PR. *Knots and Links*. (Cambridge University Press, 2004)
  20. Kauffman LH. *On Knots*. (Princeton University Press, 1987)
  21. Rolfsen D. *Knots and Links*. (Publish or Perish, 1976)
  22. Hoste J, et al. *Inventiones Mathematicae*, **84**, 1, 1-54 (1986)
  23. Livingston C. *Knot Theory*. (The Mathematical Association of America, 1993)
  24. Adams C. *The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots* (American Mathematical Society, 2004)
  25. D'Ambrosio U. *For the Lear of Math*. **5**, 3, 44-48 (1985)
  26. D'Ambrosio U. *Etnomathematics: A multicultural view of mathematical ideas*. World Scientific Publishing Co. Inc, (2016)
  27. Holme A. *Geometry: Our Cultural Heritage*. (Springer, Berlin, 2010)
  28. Zhang Q, Zhang Y, ZDM, **42**, 6, 671-675 (2010)
  29. Murasugi K. *Knot Theory and Its Applications (English Version)*. (Boston, USA,



- Birkhauser, 1996)
30. Adams NA. Weaving Culture and Mathematics. (Auckland, The University of Auckland, 2011)
  31. Suleiman AS. Traditional Crafts and Skills of the Bugis. (Yayasan Cipta Loka Caraka, Jakarta, 2002)