The Trend of Geometric Thinking Studies: A Systematic Review

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Abstract. This study focuses on reviewing information from several articles published in educational and social sciences journals worldwide to document the trend of geometric thinking studies in the Indonesian context. The current study shows that the number of publications on geometric thinking has increased in the last four years. The studies conducted in Indonesia and for Indonesian students used a systematic review guided by PRISMA to examine a hundred articles indexed by Scopus, Google Scholar, and Science and Technology Index (SINTA) between 2011 and 2021. Van Hiele's Geometric Thinking Test, developed by Usiskin, is extensively used when examining geometric thinking levels in the intervention. In addition, descriptive analysis and a t-test are also employed. The result shows that the most common type of geometric thinking experimental research is quasi-experimental design, with the qualitative approach found to be the most used. Therefore, educators should increase the quantity of qualitative and quantitative studies from various nations about geometric thinking. Furthermore, it is also essential to examine the correlation between student performance in geometry and the teacher's level of geometric thinking. Finally, combining the van Hiele Geometry test with interviews is strongly advised to provide comprehensive and in-depth data for examining geometric thinking.

1 Introduction

Geometry is the discipline that many students worldwide struggle to master. Many researchers, including [1], contend that many students, particularly secondary school students, needed help comprehending basic geometric concepts and could not enter geometry classes without first mastering the essential information and concepts. TIMSS Report Year 2015 showed that students in countries such as Norway, Turkey, Georgia, South Africa, Thailand, Slovakia, Malaysia, and Indonesia have low achievements in

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It is believed and proven by various studies that there is a relationship between pupils' geometric achievement and their level of geometric thinking [1,3–7]. According to previous studies, Indonesian pupils' geometric thinking levels still need to improve [8–11]. Thus, it is vital to pay attention to the fundamentals of geometry, especially geometric thinking, in the early years of education and to ensure that they are taught well [12]. The teacher must have sufficient knowledge and pedagogical abilities to develop students' geometric thinking [13].

Geometry has long been a difficult and incomprehensible topic for modern students [14,15]. Therefore, teachers are always experimenting with new learning strategies to understand and overcome students' misconceptions in this area [12,16]. The topic of geometric thinking was initially studied in the 1950s by Pierre van Hiele and Dina van Hiele-Geldof [5]. Teachers monitor how their students learn geometry and assess the cognitive level of the learning process. As a result, in 1957, they developed the van Hiele model of geometric thinking. The van Hiele model is currently the most widely used starting point for developing geometry curricula [12]. The thinking levels in geometry have five levels—Visualization, Analysis, Informal Deduction, Formal Deduction, and Rigor—according to the van Hiele model of geometric thinking. They are numbered from 1 to 5, as expounded by [17], although the descriptors of the levels and their labels have varied in different works [1,3,12,17]. Level 1 (Visualization): At this level, students detect geometric shapes using their complex visual awareness, with shape orientation being the most critical factor. Level 2 (Analysis): Students are familiar with geometric shapes' attributes but are unaware of their connections. They define geometric shapes by identifying all their properties, including those that are not required. Level 3 (Informal Deduction): Students understand the relationships between geometric shape attributes and how the different properties are organized and interconnected. They create the appropriate abstract definitions and begin to reason about statements using implication, deduction, and abstraction. Level 4 (Formal Deduction): Students understand the importance of a logical system of geometry, the concepts of deduction, position and postulate tasks, and sentences and definitions. They understand the importance of proving statements and can present simple evidence in secondary school. Level 5 (Rigor): Students can compare axiomatic systems and explain how adding or eliminating postulates in a geometric system affects the system. They are familiar with the standard features of deduction and can apply a variety of proofs.

[18] reviewed 17 articles published from 2009 to 2017 regarding embedding technology with geometric thinking. Their study found that embedding technology into the learning process positively impacts students' achievement and can advance geometric thinking. Furthermore, it indicates that research trends in the field of geometric thinking have attracted the attention of researchers to study their effects, especially if they are integrated with technology to facilitate students' understanding of geometry. A recent review by [19] reviewed 24 articles worldwide, showing how the study of geometric thinking is growing and attractive. Besides, [20] also reviewed 15 articles from 1998 to 2020. Their study showed that the articles' patterns fluctuated from year to year and found that geometric thinking research is based on a separate knowledge area and involves different interventions. In their study, the interventions are divided into two categories: those that use technology (such as GeoGebra, Geometer's Sketchpad, and Dynamic Geometry Software) and those that use manipulative media. Other studies use various theories to examine pupils' progression from one level to another level of geometric thinking.

However, previous studies have conducted literature reviews on the study of geometric thinking worldwide [18–20]. They only examined the numerous interventions, media, and cognitive skills employed to increase the geometric thinking level of students in different regions of the country. The difference between this study and the previous studies is that the current study aims to compile information regarding various studies covering geometric
thinking in the Indonesian context using a systematic review guided by the PRISMA in educational and social science journals published worldwide from 2011 to 2021. Therefore, it is vital to evaluate the numerous studies conducted in Indonesia on how to overcome the difficulties of learning geometry and use them as a guide for providing support in learning geometry based on the level of geometric thinking development of the students and the topic chosen. In addition, this study is essential for researchers since it examines many research methodologies used to conduct the study and the learning approach, instruments, and analysis employed.

This study, therefore, intends to address the following questions in detail: (1) What was the trend in the number of geometric thinking studies in Indonesia? (2) What research methods were used to explore geometric thinking in Indonesia? (3) What was the most common topic used to examine geometric thinking in Indonesia? (4) What interventions did the researchers use to help students develop geometric thinking in Indonesia? (5) What instruments did the researchers employ to evaluate geometric thinking in Indonesia? (6) What data analysis techniques did the researchers use to analyze geometric thinking in Indonesia?

2 Method

2.1 Research Approach

This study was a systematic review. We referred to the PRISMA principles [21], which consists of identification, screening, eligibility, and inclusion, to systematically filter and collect pertinent information. It directed to identify information from several articles published in educational and social sciences journals worldwide and to view and document the trend of geometric thinking studies in the Indonesian context, in which the studies were conducted in Indonesia or for Indonesian students.

2.2 Instrument

The instrument employed the PRISMA guideline that included related aspects under investigation (Figure 1). There were seven key aspects to investigate. Those aspects consisted of (1) the number of studies; (2) the type of research; (3) participants; (4) mathematics topics selected; (5) intervention; (6) data collection techniques or instruments; and (7) data analysis methods. Exceptionally, categories on aspects (1), (4), and (5) were not defined at the early stage due to a lack of previous studies. They could be compared to determine what should be included in the categories and the risk of oversimplified categories arising from the review of some articles. Aspect (2), (3), (6), and (7) categories were also defined before data collection.

2.3 Data Collection Techniques and Sources

The information was derived from the review results of mathematics education studies. The articles were taken from educational and social sciences journals indexed by Scopus, Google Scholar, and Science and Technology Index (SINTA). The articles in this study were all published online as of December 2021. The keywords used in the search for related articles were "geometric thinking," "geometry," "Indonesia," and "van Hiele." We used the software assistance, Publish or Perish, developed by Harzing.com, to search the articles. We established inclusion and exclusion criteria to examine the relevancy of each article returned from the search. The following were the inclusion criteria: (1) the article studied
geometric thinking in the context of Indonesia; (2) the type of article was a journal article; and (3) the language used was English.

Fig. 1. PRISMA flow chart.

Similarly, the following criteria were used to exclude articles from the search: (1) the article studied geometry in general; (2) the type of article was a conference and review article, thesis, and books; and (3) printed articles. In total, 33 journal articles matched the inclusion criteria described above among the hundreds of articles returned by the initial search. Using the PRISMA Flow Chart, Figure 1 describes the search and screening procedure.

2.4 Data Analysis

Each article was assigned to a specific category based on the characteristics that fit the criteria. The authors of the articles reviewed presented the information in the abstract, methods, findings, and conclusion sections, which contributed to the decision. Furthermore, the collected data was presented as a bar chart. Several articles used more than one category for the data collection techniques or instruments and the data analysis aspects. Therefore, we counted the article more than once if it fulfilled these criteria.

3 Results

3.1 Number of Articles

The number of articles published in this period reflected how frequently the articles were undertaken. Since 2013, 33 articles on geometric thinking have been published, as shown in Figure 2. There was no clear pattern in the number of articles produced from year to year. However, beginning in 2018, the number of articles climbed significantly compared to previous years. It implies that the trend of the topic of geometry, particularly geometric thinking, is in high demand to be studied in Indonesia since it has a significant impact on students' comprehension of geometry concepts and how teachers teach geometry following
students' geometric thinking level. Furthermore, many recent articles on geometric thinking demonstrated that geometric thinking studies had earned the recognition of many different journals as an important topic area.

![Fig. 2. The number of articles in Indonesia emphasized geometric thinking as the primary concern in the last 10 years.](image)

### 3.2 Type of Research

The approach and research design determine the focus of a study. The trend of the qualitative approach decreased from 2018 to 2020 and then increased in 2021 (Figure 3). In 2018, the qualitative approach reached an all-time high in contrast to the trend of the quantitative research approach, which fluctuated annually. In 2020, the quantitative research approach was utilized most frequently. Similarly, Research and Development (R&D) fluctuated frequently.

Meanwhile, using the mixed method approach experienced ups and downs but was stable in the last two years (2020-2021). It indicates that the use of approaches in studying geometrical thinking is varied. Furthermore, the qualitative approach was the most common method used by researchers to study geometric thinking (Figure 4). However, qualitative approach trends were rising, focusing on social sciences and some educational concerns. Such circumstances are closely related to the advantages of a qualitative approach to defining a phenomenon precisely and completely [23].

![Fig. 3. The trend of research approach in geometric thinking studies.](image)
As a result, the scarcity of qualitative approaches allowed future researchers to employ a qualitative approach and concentrate their studies on geometric thinking. Although the quantitative research approach is used a little in geometric thinking research, in fact, in the mixed method and R & D, they used a quantitative research approach in designing their research.

Fig. 5. The trend of research design in geometric thinking studies.

Fig. 6. The distribution of research design used in geometric thinking studies.
Most articles did not conduct intervention studies or were unidentified (Figure 6). However, quasi-experimental design is used most frequently in experimental research. True experimental research design and descriptive qualitative were the most uncommon, appearing in only one article [24,25]. Twelve articles should have specifically mentioned or included the research design used. However, most articles on geometric thinking were in the form of experimental studies. The number of mixed methods and research and development articles was 14 (Figure 4). The research trends in the last four years tend to use quasi-experimental design (Figure 5).

### 3.3 Participant

Most selected participants are secondary school students (19 articles), followed by elementary and undergraduate students (Figure 7). This result indicates that the research trend in studying geometric thinking mainly focuses on school students. It aligns with the literature review by [19], which showed the same results. However, there is still a gap in the extent to which geometric thinking is owned or understood by educators (postgraduates, teachers, and lecturers). The quality of the students depends on how well the teachers-to-be understand and use geometric thinking in learning [26].

![Fig. 7](image.png)

**Fig. 7.** The distribution of participants in geometric thinking studies.

According to [12,27,28], the teacher's ability to comprehend geometric thinking and implement it in learning is one of the most significant elements in students' geometric performance. Therefore, researchers can concentrate their investigations on educators' level of geometrical thinking.

### 3.4 Mathematics Topics Selected

According to the analysis, some articles support only one topic, while others cover a wide range. For example, an article used more than three topics (coordinate system, straight line, circle, conic, point in space, plane, straight line in space, surfaces and curves) [11]. An article used three topics (properties of rectangle, square and triangle) [29]. Four articles used two topics (pyramid and prism, triangle and quadrilateral, triangular and quadrilateral, and quadrilateral and triangle) [30–33]. Furthermore, 27 articles used one mathematics
topic, such as 3D figures, quadrilateral, transformation, spatial, conics, Pythagorean theorem, and distance between two geometry elements in a solid.

The researchers selected many topics to use as pilots for their articles at various levels of participants. Geometry was the most popular research topic selected. A total of seven articles examined students’ geometric thinking on the square theme \cite{7,29,30,32–35}. Besides, other topics have also been used in studying geometric thinking, such as triangles \cite{29,30,32,33}, 3D \cite{36–38}, Pythagorean theorem \cite{39,40}, distance \cite{41}, pyramid and prism \cite{31}, spatial \cite{42}, transformation \cite{43}, conics \cite{44}, and coordinate system \cite{11}. In addition, it indicates that fundamental geometrical concepts such as quadrilaterals and triangles are still regarded as standard topics in studying geometric thinking.

3.5 Intervention

We can see that various forms of intervention have been used in the study of geometric thinking. Most interventions used by researchers in Indonesia are van Hiele-based learning. When classified, eight articles used the van Hiele model \cite{10,33,42,45–47} as a learning principle integrated with other learning tools or instructions such as GeoGebra \cite{11}, Metacognition \cite{47,48}, Video \cite{42,49}, Rotary Wheel Media \cite{33}, multimedia \cite{7,36,50}, computer manipulative \cite{29}, and IDEAL model of problem solving \cite{51}. Researchers widely used the van Hiele model to improve geometric thinking. In addition, van Hiele (1959) introduced and suggested using van Hiele-based learning as an intervention in the classroom. Many studies have shown that using van Hiele-based learning in the form of modules or incorporating it into technology applications effectively develops geometric thinking \cite{20,50,52,53}. Similar results were discovered by \cite{43} when integrating van Hiele-based learning with learning media aids (modules, multimedia, computerized origami programs, and GeoGebra) had a significant effect.

3.6 Data Collection Techniques or Instruments

The level of geometric thinking could be measured through various instruments developed by previous researchers. The test, which was a generic and self-made test apart from the van Hiele Geometry test, had become the most widely used instrument for collecting data about the level of geometric thinking and interview (Figure 8). Geometric thinking was a level of thinking that could be examined based on students' answers to questions arranged according to van Hiele. Data collection through tests is more objective than questionnaires and documentation \cite{22}. After the test, the most common test used was the van Hiele Geometry test developed by \cite{1}. Besides, interviews were one of the prevalent techniques used to explore students' geometric thinking in a qualitative approach. Teachers and researchers may use the combination of the van Hiele Geometry test, interview, and documentation, which refers to students' work notes, to examine the geometric thinking level of students in depth.
Unfortunately, only some researchers fail to identify the instruments used to collect data on geometric thinking in their investigations. Seventeen articles that relied on tests (generic and self-made tests) based on the geometric thinking level as their primary data collection technique are unaware that the test must pass a validity and reliability evaluation. They did not mention that the instruments passed the evaluation in their articles. Before the instrument is used to collect data, its validity and reliability should be thoroughly verified [54,55]. In other words, the target reader needed to know about the validity and reliability of the data to be persuaded.

3.7 Data Analysis Techniques

The selection accuracy of the data analysis techniques would determine the level of accuracy of a study. The test is used to examine geometric thinking (Figure 8). Two data analysis techniques are often used in nine articles, descriptive analysis and weighted van Hiele geometric thinking test scores, followed by a t-test (Figure 9). Most of the analysis techniques used for the test instrument are t-tests (eight articles). Several other studies used descriptive, z-test, and ANOVA. Weighted van Hiele geometric thinking test scores, which [1] introduced in analyzing the data from the van Hiele geometric thinking test he developed, were used in nine articles [9,11,25,26,30,33,47–49]. In addition, the articles tend to use parametric statistics compared to non-parametric statistics to see the extent to which students’ geometric thinking levels were.

For the interview instrument, most articles used data analysis of triangulation (seven articles) and Miles and Huberman’s [56] qualitative analysis, namely, data reduction, data display (data presentation), and conclusions (drawing/verification) (six articles). Therefore, it is beneficial for the researcher to explore further what is occurring in the student's mind regarding how geometric thinking can be advanced.

4 Discussions

There has been a rise in research focusing on students' geometric thinking and learning geometry. Over the last four years, significant studies in this area have increased. This
study looked at research trends in Indonesia related to geometric thinking over the past ten years, examining the methods, topics, learning interventions, instruments, and data analyses used. The study's findings are summarized and analyzed, with a focus on how they can be applied to advance mathematics education, specifically geometry, for Indonesian students. This research is important as it addresses the ongoing problem of Indonesian students' weak geometric thinking skills [47] and helps identify effective instructional interventions and media to improve them.

![Fig. 9. The distribution of data analysis used in geometric thinking studies.](image)

Increased research on geometric thinking has a positive impact on education development, particularly in geometry education in Indonesia. The results of this study can influence educational practices by providing reliable information for teachers, serving as a basis for educational decision-making at various levels, and shaping the way teachers think and act. The study also reveals that geometric thinking research has been gaining popularity in recent years to address the problem of low ability in student's geometric thinking [5,32,57]. This problem can be addressed by implementing interventions that are suitable for their level of geometric thinking. Research in geometry education is essential as the geometry domain is among the most difficult for students in various countries, according to TIMSS [2].

Qualitative research is the dominant method for investigating geometric thinking in Indonesia, although there has been a significant increase in the use of quantitative research from 2017-2020. These two approaches have been integrated through mixed methods research to better understand how to improve geometric thinking [31,43]. Quasi-experiments are most frequently used to examine geometric thinking, as they can address the issue of non-randomly selected students. Experimental research design is also common, particularly in developing learning instruction [7,10,31,33,51]. It is consistent with previous findings that experimental research design is frequently used in investigating geometric thinking. Through these research methods, the effectiveness of geometric thinking can be studied on a non-random sample.

The majority of the studies analyzed in the review focused on secondary school students. For example, [33] explored the geometric thinking processes of secondary school students using the Van Hiele learning model and rotary wheel media, while [58] conducted
a quasi-experimental study using the van Hiele model. However, many secondary school students still struggle with visualization and analysis levels of geometric thinking, which may affect their academic performance in college [31,32,39]. According to [59,60], students often face difficulties in geometry learning at the college level due to a lack of geometric thinking skills at the secondary school level. Therefore, it is crucial for educators and researchers to enhance the geometric thinking skills of secondary school students to ensure they are well-prepared for college.

Moreover, the studies included a variety of topics at different levels of participation, with the most popular research topic being geometry, particularly on quadrilaterals and triangles. This suggests that fundamental geometric concepts such as quadrilaterals and triangles remain important in the study of geometric thinking [7,33,35]. These findings have implications for educators and researchers in selecting appropriate topics and adjusting the level of education to facilitate the enhancement of students' geometric thinking skills.

The majority of studies reviewed utilized van Hiele-based learning to enhance students' level of geometric thinking, which involves five steps according to [3]. The studies suggest that this approach can be integrated with various learning approaches, tools, or media. For instance, [33] combined van Hiele-based learning with rotary wheel media to enhance students' thinking ability. Other studies [36] examined the effectiveness of different learning interventions, such as Augmented Reality learning media, in enhancing students' geometric thinking abilities. However, more research is needed to compare the effectiveness of different interventions. Most studies in Indonesia used tests and interviews to evaluate students' level of geometric thinking, but the tests and interview questions were often general and self-made [10,44,61]. Therefore, it is essential to have a rubric with validation criteria to ensure that the results produced are valid.

The studies reviewed primarily used weighted van Hiele's geometric thinking test scores and descriptive analysis for data analysis. Weighted van Hiele geometric thinking test scores were utilized when the data was generated using the van Hiele geometry test, which is quantitative. While many articles used the t-test to compare pre- and post-intervention effects, ANOVA is recommended as it can evaluate more than two data sets and draw richer conclusions. ANOVA enables researchers to control external variables that impact the relationship between independent and dependent variables and can also uncover group differences based on pre-test data averages [54,62]. On the other hand, several qualitative articles analyzing data referred to Miles and Huberman’s theory, including data reduction, data display, and conclusion [56]. Qualitative research also uses triangulation to test validity by combining data from multiple sources. However, the analysis of mixed-method research is still limited, and it is essential to examine whether the integration of the two databases will be merged, connected, or embedded. Therefore, it is recommended that future studies use mixed methods.

5 Conclusion

This systematic review study reviewed articles highlighting geometric thinking published worldwide in educational and social science journals in an Indonesian context from 2011 to 2021. The trend is that the number of publications with geometric thinking as the primary concern in the last four years has increased. Due to students' low level of geometric thinking, the researchers focused on studying geometric thinking, which has implications for increasing student geometry achievement. Among hundreds of articles, the qualitative approach is the most used. The most common type of geometric thinking experimental research is quasi-experimental design. Secondary school students are mainly selected as participants, while the most preferred topic is quadrilateral. Van Hiele-based learning is the
intervention that is most widely employed. Tests and interviews are the most popular instruments used to collect data. Meanwhile, descriptive analysis and weighted van Hiele geometric thinking test scores followed by a t-test are data analysis techniques often used in analyzing data according to the instrument used, the test, and the van Hiele Geometry Test.

6 Recommendations

Educators must increase the quantity of qualitative and quantitative studies from various nations about geometric thinking. In addition, it is also essential to examine the correlation between student performance in geometry and the teacher's level of geometric thinking. Finally, combining the van Hiele Geometry test with interviews is strongly advised to provide comprehensive and in-depth data for examining geometric thinking.

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