Comparison and selection of criteria and features in morphological analysis

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Abstract. The article deals with the issues of using the method of pairwise comparisons in the selection of attributes and criteria in morphological analysis. The paper specifies the possibilities of solving the problems of collecting, analysing, evaluating and selecting rational options. The fundamental impossibility of building accurate models leads to the necessity of decision making under conditions of uncertainty. The analytic hierarchy process is used to solve the problems. The formulation and method of solving the problem are given. Using an example, the application of the proposed approach is shown. In conclusion and conclusions are drawn and the advantages of the stated algorithm are considered. The use of the proposed approach allows increasing the validity of the decisions made in morphological analysis and thus increasing the efficiency of research.

1 Introduction

The proposed methodology is intended for analyzing and selecting engineering solutions at the pre-design stage. This stage is the main stage in the creation of innovative engineering systems [1,2]. Structuring information and using decision-making methods requires solving the problem of collecting, analyzing, evaluating and selecting the best options. The fundamental impossibility of building accurate mathematical models leads to the need to make decisions under conditions of uncertainty. Significant factors of uncertainty include:

- incomplete information about the object of study;
- variability of parameters and situation on which the decision should be based;
- uncertainty about the validity of assumptions of environmental changes;
- inability to recognize complex data, situations with long-term consequences, to judge without bias.

To solve these problems, multi-criteria decision-making methods (MCDM) have been developed. The approaches were developed to help decision makers select the most preferable solutions.

Heuristic methods are an effective tool for solving problems and finding solutions in the natural and technical sciences. The approaches consider the search space by sampling solutions, evaluate them, and adjust the search towards promising solutions. However, in many cases, this fitness function involves performing expensive calculations, which

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dramatically reduces the reasonable number of evaluations [3]. These methods are widely used in applied engineering tasks. The paper presents a modification of the Analytic Hierarchy Process (AHP) in order to improve its efficiency in multi-criteria comparison of generating station combinations. Energy sources are represented by thermal, solar, wind and diesel power plants. This paper [4] considers the AHP parameterized generator of scalable and customizable benchmark problems for multi-criteria problems. An iterative method is proposed, with the solution consisting of synthesizing a small subset of solutions to obtain pairwise judgments.

2 Morphological method

The developed method is based on the morphological box method of F. Zwicky [5,6]. Thanks to this method, he was able to obtain a significant number of original technical solutions in astrophysics in a short time. The researcher defined the purpose of morphological analysis as follows: "The purpose of morphological research is to see the perspective of the whole space of knowledge about the object of research".

Morphological box is a method of problem solving based on selection of possible solutions for separate parts of the problem (so-called morphological features characterizing the device) and subsequent systematized obtaining of their combinations (combining). It refers to heuristic methods [7].

3 Statement of the choosing solutions problem

Thus, when choosing the vector of elements of the design object, it is necessary to find a compromise between the desire for modeling accuracy and the simplicity of the model. The external conditions affecting the functioning of the vehicle are known and fixed. In accordance with this, the criterion for the functioning of the vehicle is a function $F(x)$ of only the elements $X^* \in X_M$. And accordingly, the design task is to find [8,9]

$$X^* = \underset{X^* \in X_M}{argmax\, F(x)}, \tag{1}$$

X* - the desired technical solution
X_M – morphological set of solutions

The problem can be formulated as finding the preferred engineering solution (ES) belonging to the morphological set of solutions. Let the initial X^0 and the desired TR X^* belong to the set of ES X (Figure1).

$$X^0 \in X \tag{2}$$

$$X^* \in X \tag{3}$$

As a rule, the process of ES searching X^* is iterative and consists in iterative selection of ES

$$X^0 \rightarrow X^1 \rightarrow X^2 \rightarrow \ldots \rightarrow X^* \tag{4}$$

In the object under study, there are many signs of P. Many features of P can be identified, for example, from the formulas of ES and descriptions of inventions.

$$P = \{P_i,\ i = 1, k\}, \tag{5}$$

where k is the number of features

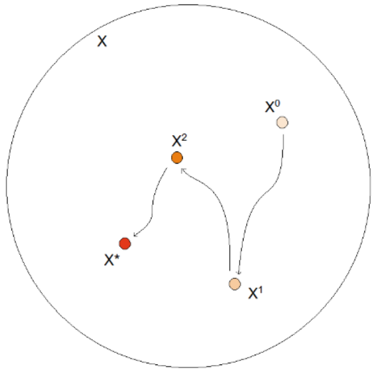

Fig. 1. Iterative process of ES search.

Depending on the task, essential P_n features are selected from this set of features, i.e., those that can greatly influence the solution of the problem.

$$P_n \leq P \tag{6}$$

$$P_n = (P_1 , P_2 , P_3 , ..., P_k) \tag{7}$$

The choice is made by experts evaluating each feature and ranking them, for example, by pairwise comparison (Table 1).

Table 1. Selection of essential features.

		Coefficients of relative importance of criteria	**Ranking of features**
Set of features *P*	P_1	w_1	R_1
	P_2	w_1	R_2
	P_3	w_1	R_3
	…		
	P_i	w_i	R_i
	…		
	P_k	w_k	R_k

Pairwise comparison methods are paved research tools and are widely used in research. Researchers solve the problem of approximating the pairwise comparison matrix for each criterion simultaneously by a general consistent matrix of unit rank that defines a vector of scores [10]. A new method, " “Objective-Subjective Weighted method for Minimizing Inconsistency (OSWMI)", which considers both pairwise comparisons of criteria and alternatives along with their corresponding performance evaluations, is investigated [11]. The best-worst method (BWM) has been proposed to reduce inconsistency using a concept that requires significantly fewer pairwise comparisons. BWM includes a nonlinear model solution to obtain weights based on comparisons [12]. The work is devoted to the development of an approach to modeling information systems of multi-criteria design based on expert ranking of criteria. The theory of fuzzy sets and probability theory are used, which allow to carry out a comprehensive analysis of criteria [13]. This paper describes sets of weights obtained by "reasonable" methods: those that are efficient for the (vector-)optimization problem of simultaneous minimization of discrepancies [14]. The aim of the study [15] is to construct a non-reciprocal matrix of pairwise comparisons and obtain perfectly matched weights.

For each feature of P_i elements P^j are selected, i.e., possible variants of its execution or implementation (Table 2), a morphological table is filled in. The morphological ES set X_M is a subset of all ES X.

$$X_M \subset X \tag{8}$$

The table contains the original X^0 and the desired TR X^* (Figure 2)

$$X^0 = \{ P_1^0,\ P_2^0, P_3^0, \dots, P_n^0 \} \tag{9}$$

$$X^* = \{ P_1^*,\ P_2^*, P_3^*, \dots, P_n^* \} \tag{10}$$

The power of the morphological set of solutions is equal to

$$|\, X_M \,| = \prod_{i=1}^{n} m_i \tag{11}$$

where m is the number of elements in the i-th row

Morphological set of solutions:

$$X_M = \{X_{mi},\, i = 1, p\}, \tag{12}$$

$$p = |X_M| \tag{13}$$

Table 2. Morphological table.

									Number of elements in the column
	Elements (alternatives) P^k								
		P^1	P^2	P^3	…	P^j	…	P^{mmax}	
P_n	P_1	P_1^1	P_1^2	P_1^3	…	P_1^j	…	P_1^{m1}	m_1
	P_2	P_2^1	P_2^2	P_2^3	…	P_2^j	…	P_2^{m2}	m_2
	P_3	P_3^1	P_3^2	P_3^3	…	P_3^j	…	P_3^{m3}	m_3
	…	…	…	…	…	…	…	…	
	P_i	P_{i1}^1	P_{i2}^2	P_{i3}^3	…	P_{ij}^j	…	P_i^{mi}	m_i
	…	…	…	…	…	…	…	…	
	P_n	P_n^1	P_n^2	P_n^3	…	P_n^j	…	P_n^{mn}	m_n

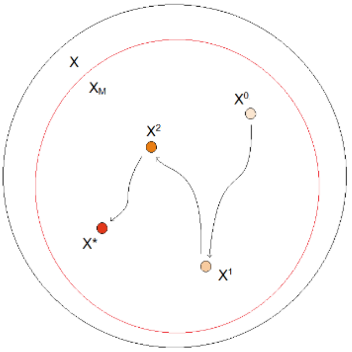

Fig. 2. Morphological set of solutions X_M and the desired variant X^*.

As a rule, in practical tasks it is not always possible to clearly define the goal and, accordingly, quantify the degree of its achievement. Therefore, the goal is decomposed into a number of criteria. As a consequence, such a description of the goal using a variety of criteria is a prerequisite for solving the task. The criteria should cover all the features of the task. The number of criteria is considered complete and sufficient if the addition of a new criterion does not change the result of the decision. To evaluate TR, a set of criteria is introduced K.

$$K = \{K_i, i = 1, n\}, \tag{14}$$

where n is the number of selected criteria

To evaluate the criteria, a kind of ordinal scale is used, namely the verbal-numerical scale, which allows you to measure the degree of intensity of the criterion property. To compare the criteria according to their degree of importance, a point comparison based on a survey of experts or a pair comparison method is used. As a result of this process, two ranking orders can be obtained: linear or partially linear. With a partially linear ranking, there are criteria with equal indicators of importance. Such criteria are called related. In the future, a square matrix of paired comparison of criteria with dimension n is constructed and this matrix is filled with coefficients k_{ij}, where $i = 1,...n$, $j = 1...n$, showing the ratio of the importance of the criterion K_i with to K_j [16,17].

$$K = \begin{matrix} & K_1 & K_2 & \cdots & K_n \\ K_1 & k_{11} & k_{12} & \cdots & k_{1n} \\ K_2 & k_{21} & k_{22} & \cdots & k_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ K_n & k_{n1} & k_{n1} & \cdots & k_{nn} \end{matrix} \tag{15}$$

In the future, indicators of the importance of each of the k_i criteria are calculated:

$$k_i = \sum_{j=1}^{n} k_{ij}\ , \tag{16}$$

where i=1,…n

and calculate the total indicator of the importance of all criteria k_s:

$$k_s = \sum_{i=1}^{n} k_i\ , \tag{17}$$

$$a_i = \frac{k_i}{k_s}, \tag{18}$$

where i=1,...n

For paired comparison, the criteria method uses the hierarchy analysis method. The matrix is filled with coefficients according to the condition:

$$k_{ij}\ \ k_{ji} = 1 \tag{19}$$

If in the matrix of paired comparison of criteria, the criterion of row (i) has a certain superiority over the criterion of column (j), then the corresponding number k_{ij} is assigned to this element of the matrix (Table 3).

Table 3. AHP table.

	K							
K		K_1	K_2	K_3	…	K_n	**Estimates of the components of the eigenvector**	**Normalized estimates of the priority vector**
	K_1	k_{11}	k_{12}	k_{13}	…	k_{1n}	V_1	S_1
	K_2	k_{21}	k_{22}	k_{23}	…	k_{2n}	V_2	S_2
	K_3	k_{31}	k_{32}	k_{33}	…	k_{3n}	V_3	S_3
	…	…	…	…	…	…	…	…
	K_n	k_{n1}	k_{n2}	k_{n3}	…	k_{nn}	V_n	S_n

After selection and evaluation of criteria and option evaluation, operations are performed according to the algorithms of the advanced morphological approach [18].

4 Conclusion

The article is devoted to the use of the method of pairwise comparisons and AHP for morphological analysis of engineering solutions. The brief state of affairs in the mentioned field is given and the characterization of the morphological box method is given. The main part of the paper is devoted to the research, problem formulation and solution methods. Pairwise evaluation methods are introduced in morphological analysis to evaluate the criteria and attributes of the morphological table. Conclusions are drawn about the prospectivity of the considered approach.

The use of the proposed approach allows increasing the validity of the decisions made in morphological analysis and thus increasing the efficiency of research.

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