

Search method for optimal interpolation of thermomechanical coefficients for conventional and low alloy steels

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Abstract. One of the most important factors influencing the behaviour of a metal when subjected to temperature and pressure is the material's ability to resist deformation. So-called thermomechanical coefficients are widely used for mathematical description of this property. These are the ones that determine the direct effect on deformation resistance. There are temperature coefficient (K_t) deformation degree coefficient (K_ϵ) and deformation rate coefficient (K_u) describing the influence of temperature, degree of deformation and deformation rate respectively. The situation becomes complicated because the thermomechanical coefficients are not constants. For example, as the temperature increases, the deformation resistance decreases. Also, the deformation resistance increases with increasing degree of deformation and the deformation rate increases with the deformation resistance increases too. The aim of this paper is to find the most accurate method of interpolating thermomechanical coefficients for various steels and alloys using the least squares method.

1 Introduction

The thermomechanical coefficient method is widely used for modelling the deformation of metals, but mainly for manual yield strength calculations. There are several research articles [1-5] describing the interpolation of a small number of steels to create various kinds of automated design systems. However, it is not clear from these papers why certain interpolation methods were chosen. And the total number of steels under consideration leaves a lot to be desired. As well as the whole research in the field of interpolation of thermomechanical coefficients. In addition, mathematical models are also applied the microstructure-based finite element modeling in the same way [6-9]. However, these mathematical models are difficult to implement with approximately the same accuracy.

This topic shows its relevance in the development of thermo-deformation mathematical models and subsequent application in automated control systems of thermo-deformation processes.

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2 Problem statement

In fact, all these coefficients such as temperature coefficient (K_t), deformation degree coefficient (K_ε) and deformation rate coefficient (K_u) are functions of temperature, deformation degree and deformation rate. Thermomechanical coefficients are widely used in mathematical calculations describing complex thermo-deformation states. For example, in the field of thermo-mechanical methods for forming non-breakable joints. However, they are presented in an inconvenient form for calculations in the form of graphs and tables. Therefore, they cannot be applied in automated control and optimization algorithms [10]. Obtaining a smooth function by interpolation from such tables and graphs will solve this problem. But another problem arises, which can be formulated as follows. Is which a method the most accurate and at the same time simple for interpolation of thermomechanical coefficients?

3 Research questions

Let us pose the first question of this study. What are methods acceptable for interpolating thermomechanical coefficients? The second and main question is which of the presented interpolation methods are more accurate for each of the considered materials?

4 Purpose of the study

The present work is devoted to the construction of a wide range of mathematical models related to thermo-deformation processes.

5 Research methods

The first thing to decide on is the representation of the initial data. Initial data are tabulated values of thermomechanical coefficients. A step of 100 degrees was taken for temperature. A step of 10% degrees was taken for the degree of deformation. A step of 0.01 was taken in the data period from 0.01 to 0.1 for the deformation time. A step of 0.1 was taken in the data period from 0.1 to 1 for the deformation time. A step of 1 was taken in the data period from 1 to 10 for the deformation time. And a step of 10 was taken in the data period from 10 to 80 for the deformation time. As a result, 6 to 10 tabulated values are obtained for most steels. In addition, there are limits on minimum and maximum temperature values. These constraints are simultaneously defined as interpolation bounds. These data are presented in Tables 1,2,3. Next, the list of methods used to interpolate tabular data is determined. The following interpolation methods were chosen: linear, quadratic, logarithmic, cubic, polynomial of degree four. Lastly, finding deviations from tabulated data using the least squares method and selection of the most accurate representation of the smooth function.

6 Results and discussion

The method resulted in optimal representations of thermomechanical coefficients in the form of smooth functions. There are temperature coefficient (K_t), deformation degree coefficient (K_ε) and deformation rate coefficient (K_u). Functions are bounded on partial intervals of temperature, deformation degree and deformation rate of the metal. The data are presented in the following tables.

Table 1. Optimal representation of the temperature thermomechanical coefficient (K_t) as a smooth function for conventional and low alloy steels.

Steel	K_t	Constraints K_t
12XH3A	$18.8765 - 2.58912 \times \ln(x);$	$850 < x < 1200$
08KП	$-1.68704 \times 10^{-7} \times x^3 + 0.000257389 \times x^2 - 0.177323 \times x + 47.9671$	$600 < x < 1200$
St25	$7.25 \times 10^{-6} \times x^2 - 0.017495 \times x + 11.2235$	$900 < x < 1200$
St40	$-1.46309 \times 10^{-7} \times x^3 + 0.000222782 \times x^2 - 0.155662 \times x + 43.6076$	$725 < x < 1200$
St45	$3.46057 \times 10^{-6} \times x^2 - 0.00962476 \times x + 7.13877$	$900 < x < 1200$
45X	$-2.39014 \times 10^{-7} \times x^3 + 0.000383032 \times x^2 - 0.276019 \times x + 76.9766$	$900 < x < 1200$
45XH	$9.14399 \times 10^{-9} \times x^3 - 0.0000214517 \times x^2 + 0.0117855 \times x + 1.50174$	$900 < x < 1200$
18XГТ	$5.375 \times 10^{-6} \times x^2 - 0.0136225 \times x + 9.20675$	$900 < x < 1200$
20XГHP	$4.75 \times 10^{-6} \times x^2 - 0.012165 \times x + 8.3845$	$900 < x < 1200$
30XГCA	$3.25 \times 10^{-6} \times x^2 - 0.009275 \times x + 7.0175$	$900 < x < 1200$
4X13	$3.25 \times 10^{-6} \times x^2 - 0.009875 \times x + 7.6525$	$900 < x < 1200$
X17H2	$2.75 \times 10^{-6} \times x^2 - 0.008785 \times x + 7.0305$	$900 < x < 1200$
Chromium-molybdenum steel	$5.75 \times 10^{-6} \times x^2 - 0.014545 \times x + 9.8085$	$900 < x < 1200$
18XHBA	$4 \times 10^{-6} \times x^2 - 0.01078 \times x + 7.744$	$900 < x < 1200$
XБГ	$7.75 \times 10^{-6} \times x^2 - 0.018985 \times x + 12.2255$	$900 < x < 1200$
Manganese silicon steel	$4 \times 10^{-6} \times x^2 - 0.01078 \times x + 7.784$	$900 < x < 1200$
Chromium-nickel-molybdenum steel	$4 \times 10^{-6} \times x^2 - 0.011 \times x + 7.98$	$900 < x < 1200$

Table 2. Optimal representation of the deformation degree thermomechanical coefficient (K_ϵ) as a smooth function for conventional and low alloy steels.

Steel	k_ϵ	Constraints k_ϵ
12XH3A	$-0.000535686 \times x^2 + 0.0389465 \times x + 0.645214$	$5 < x < 40$
08KП	$-3.06562 \times 10^{-7} \times x^4 + 0.0000372636 \times x^3 - 0.00181941 \times x^2 + 0.0514509 \times x + 0.633886$	$5 < x < 50$
St25	$-1.0832 \times 10^{-6} \times x^4 + 0.000138518 \times x^3 - 0.00643073 \times x^2 + 0.137308 \times x + 0.094732$	$5 < x < 50$
St40	$-2.64845 \times 10^{-7} \times x^4 + 0.0000370788 \times x^3 - 0.00194861 \times x^2 + 0.0508209 \times x + 0.650743$	$5 < x < 50$
St45	$-0.000535686 \times x^2 + 0.0379465 \times x + 0.675214$	$5 < x < 40$
45X	$-1.03534 \times 10^{-6} \times x^4 + 0.00013071 \times x^3 - 0.0059338 \times x^2 + 0.121687 \times x + 0.230558$	$5 < x < 50$
45XH	$-1.32546 \times 10^{-6} \times x^4 + 0.000161756 \times x^3 - 0.00707205 \times x^2 + 0.139817 \times x + 0.120522$	$5 < x < 50$
18XГТ	$-1.22348 \times 10^{-6} \times x^4 + 0.000155404 \times x^3 - 0.00712727 \times x^2 + 0.147504 \times x + 0.0476328$	$5 < x < 50$
20XГHP	$-1.59399 \times 10^{-6} \times x^4 + 0.000204642 \times x^3 - 0.00945156 \times x^2 + 0.193078 \times x - 0.195202$	$5 < x < 50$
30XГCA	$8.29235 \times 10^{-6} \times x^3 - 0.00104223 \times x^2 + 0.0484588 \times x + 0.60886$	$5 < x < 40$
4X13	$0.0000123224 \times x^3 - 0.00143011 \times x^2 + 0.0570638 \times x + 0.566119$	$5 < x < 40$
X17H2	$0.255454 \times \ln(x) + 0.410839$	$5 < x < 40$

Chromium-molybdenum steel	$0.0000174044 \times x^3 - 0.00170911 \times x^2 + 0.0599219 \times x + 0.545129$	$5 < x < 40$
18XHBA	$0.0000187978 \times x^3 - 0.00187718 \times x^2 + 0.0632602 \times x + 0.530801$	$5 < x < 40$
XBF	$-0.000506106 \times x^2 + 0.0375408 \times x + 0.675837$	$5 < x < 40$
Manganese silicon steel	$0.0000121311 \times x^3 - 0.00117718 \times x^2 + 0.0419269 \times x + 0.670801$	$5 < x < 40$
Chromium-nickel-molybdenum steel	$0.0000231421 \times x^3 - 0.00209707 \times x^2 + 0.0660297 \times x + 0.514682$	$5 < x < 40$

Table 3. Optimal representation of the deformation rate thermomechanical coefficient (k_u) as a smooth function for conventional and low alloy steels.

Steel	k_u	Constraints k_u
12XH3A	$0.0869207 \times \ln(x) + 0.715447$	$0.1 < x < 1$
	$0.000233447 \times x^3 - 0.00637573 \times x^2 + 0.0751723 \times x + 0.651376$	$1 < x < 10$
08KII	$-0.0000253654 \times x^2 + 0.00724987 \times x + 0.930519$	$10 < x < 100$
	$0.0550712 \times \ln(x) + 0.61794$	$0.004 < x < 0.1$
St25	$0.117514 \times x^3 - 0.399129 \times x^2 + 0.508502 \times x + 0.45312$	$0.1 < x < 1$
	$0.000540439 \times x^3 - 0.011359 \times x^2 + 0.100936 \times x + 0.586161$	$1 < x < 10$
St40	$4.45806 \times 10^{-8} \times x^3 - 0.000033381 \times x^2 + 0.00886502 \times x + 0.943481$	$10 < x < 100$
	$0.000123583 \times x^3 - 0.00405847 \times x^2 + 0.0503711 \times x + 0.797141$	$1 < x < 10$
St45	$-3.76377 \times 10^{-7} \times x^3 + 0.0000417064 \times x^2 + 0.00692043 \times x + 0.947316$	$10 < x < 100$
	$-7.85268 \times 10^{-8} \times x^3 + 0.0000473232 \times x^2 - 0.00438994 \times x + 1.72283$	$100 < x < 285$
45X	$0.0786554 \times \ln(x) + 0.719091$	$0.004 < x < 1$
	$0.114975 \times \ln(x) + 0.737073$	$1 < x < 10$
45XH	$5.97136 \times 10^{-8} \times x^3 - 0.0000354485 \times x^2 + 0.00797992 \times x + 0.955289$	$100 < x < 300$
	$-0.401639 \times x^4 + 0.988265 \times x^3 - 0.968176 \times x^2 + 0.613683 \times x + 0.487759$	$0.1 < x < 1$
18XГТ	$0.121374 \times \ln(x) + 0.712371$	$1 < x < 10$
	$-1.37451 \times 10^{-8} \times x^4 + 3.17434 \times 10^{-6} \times x^3 - 0.000273981 \times x^2 + 0.0151299 \times x + 0.876139$	$10 < x < 100$
20XГHP	$0.000681818 \times x^2 + 0.00742308 \times x + 0.853965$	$1 < x < 10$
	$6.35646 \times 10^{-8} \times x^3 - 0.0000371304 \times x^2 + 0.00923748 \times x + 0.921222$	$10 < x < 250$
18XГТ	$-0.000210139 \times x^3 + 0.00364914 \times x^2 + 0.00211162 \times x + 0.824736$	$1 < x < 10$
	$-8.99134 \times 10^{-9} \times x^4 + 1.88412 \times 10^{-6} \times x^3 - 0.000170547 \times x^2 + 0.0137159 \times x + 0.878727$	$10 < x < 100$
20XГHP	$0.891942 \times \ln(x) - 2.56359$	$100 < x < 300$
	$-0.0000488388 \times x^4 + 0.000425223 \times x^3 + 0.00146066 \times x^2 + 0.0243669 \times x + 0.673509$	$1 < x < 10$
20XГHP	$1.35947 \times 10^{-7} \times x^3 - 0.0000749876 \times x^2 + 0.0166038 \times x + 0.864896$	$10 < x < 260$
	$0.0000443762 \times x^4 - 0.00114875 \times x^3 + 0.00885248 \times x^2 - 0.000325939 \times x + 0.822908$	$1 < x < 10$

	$1.26878 \times 10^{-7} \times x^3 - 0.0000598464 \times x^2 + 0.0135558 \times x + 0.868555$	$10 < x < 250$
30XГСА	$0.140483 \times x^4 - 0.0173931 \times x^3 - 0.512198 \times x^2 + 0.648704 \times x + 0.460372$	$0.1 < x < 1$
	$-0.00255159 \times x^2 + 0.0580864 \times x + 0.672204$	$1 < x < 10$
	$-0.0000293565 \times x^2 + 0.00715631 \times x + 0.930054$	$10 < x < 100$
4X13	$0.0987298 \times x^3 - 0.360693 \times x^2 + 0.455561 \times x + 0.65618$	$0.1 < x < 1$
	$0.0000804979 \times x^3 - 0.00333345 \times x^2 + 0.0453756 \times x + 0.809179$	$1 < x < 10$
	$-0.0000136357 \times x^2 + 0.00506381 \times x + 0.966928$	$10 < x < 100$
X17H2	$-0.175265 \times x^2 + 0.359774 \times x + 0.69534$	$0.1 < x < 1$
	$0.0000771858 \times x^4 - 0.00165933 \times x^3 + 0.0100615 \times x^2 + 0.00112398 \times x + 0.870151$	$1 < x < 10$
	$-1.52391 \times 10^{-8} \times x^4 + 3.42142 \times 10^{-6} \times x^3 - 0.000276707 \times x^2 + 0.0127065 \times x + 0.898574$	$10 < x < 100$
Chromium-molybdenum steel	$-0.0000698315 \times x^4 + 0.00159832 \times x^3 - 0.0128388 \times x^2 + 0.0568911 \times x + 0.804848$	$1 < x < 10$
	$-0.0000305747 \times x^2 + 0.00682254 \times x + 0.925119$	$10 < x < 100$
18XHBA	$-0.055186 \times x^3 + 0.0040389 \times x^2 + 0.178249 \times x + 0.682982$	$0.1 < x < 1$
	$-0.0000401639 \times x^4 + 0.000988265 \times x^3 - 0.00968176 \times x^2 + 0.0613683 \times x + 0.757759$	$1 < x < 10$
	$-9.01128 \times 10^{-8} \times x^3 - 5.38072 \times 10^{-6} \times x^2 + 0.0041443 \times x + 0.949412$	$10 < x < 100$
XБГ	$-0.303279 \times x^4 + 0.71469 \times x^3 - 0.772446 \times x^2 + 0.59363 \times x + 0.577464$	$0.1 < x < 1$
	$-0.0000660291 \times x^4 + 0.00159315 \times x^3 - 0.0152194 \times x^2 + 0.0850237 \times x + 0.738805$	$1 < x < 10$
	$3.38196 \times 10^{-7} \times x^3 - 0.0000826192 \times x^2 + 0.00810703 \times x + 0.927269$	$10 < x < 100$
Manganese silicon steel	$-0.000306263 \times x^3 + 0.0047908 \times x^2 - 0.00323536 \times x + 0.878852$	$1 < x < 10$
	$8.25252 \times 10^{-8} \times x^3 - 0.0000367882 \times x^2 + 0.00709881 \times x + 0.955627$	$10 < x < 100$
Chromium-nickel-molybdenum steel	$-0.00187446 \times x^2 + 0.0426397 \times x + 0.760404$	$1 < x < 10$
	$-7.87568 \times 10^{-9} \times x^4 + 1.81793 \times 10^{-6} \times x^3 - 0.000172125 \times x^2 + 0.0108306 \times x + 0.907683$	$10 < x < 100$

7 Conclusion

The existence of smooth functions of thermomechanical coefficients makes it possible to develop mathematical models related to thermal and deformation processes describing the main factors that affect thermo-deformation processes. Mathematical models in turn enable us to develop methods and algorithms for optimal control of thermo-deformation processes. As well as use the data obtained in the process of this study in the development of computer-aided design systems, computer-aided control systems, research and education purposes.

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