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System approach to building a mathematical model of a complex organizational-technical system in transport

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Abstract. The current situation in the synthesis of complex organizational and technical systems in transport is characterized by a significant lag in the level of development of special mathematical and software, which significantly reduces the pace of development and efficiency of complex organizational and technical systems. This, in turn, does not allow to fully use the potential opportunities of scientific and technical progress to improve complex organizational and technical systems in transport. The fact of system and circuit synthesis of complex systems, a number of important fundamental tasks of system synthesis of multi-level hierarchical systems, to which complex organizational and technical systems belong. In the literature the objective prerequisites determining the necessity, expediency and possibility of their solution have not been sufficiently considered yet.

1 Introduction

Currently, the synthesis of complex organizational-technical systems in transport is usually carried out taking into account the following assumptions:

- Synthesis of complex organizational-technical systems as a creative procedure cannot be fully formalized and is characterized by the following features, such as, the presence of a decision maker in conditions of incomplete information and bearing full responsibility for the decision, the presence of the decision maker's propensity to risk, the possibility of the decision maker to change the assessment of

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the importance of various properties and various characteristics of complex organizational-technical systems in the process of synthesis and implementation.

- Complex organizational-technical systems - a large organizational-technical system as an object of synthesis is characterized by the following properties, such as, has a large parametric dimensionality, which fundamentally excludes the possibility of direct quantitative comparison of synthesized alternatives of the system by both the method of direct enumeration and the method of blind dialogue, alternatives of synthesized solutions at the stage of synthesis can not be fully ordered by properties and the situation when one alternative is inferior to another is not excluded.

Here structural uncertainty at the early stages of the life cycle is expressed by the incompleteness of the necessary information about the links between functional subsystems (elements) of different levels of the hierarchical structure of the system. Parametric uncertainty is expressed by incompleteness of information about parameters and characteristics of functional elements of the system.

First of all, it refers to the tasks of evaluating the effectiveness and quality of alternatives of synthesized solutions and the selection of optimal options for the structure of complex organizational and technical systems, which are, the central issues of synthesis and the solution of which is important. Solving the problems of evaluating the effectiveness and quality of alternatives of synthesized solutions and selecting the optimal options for the structure of complex organizational and technical systems is impossible without a mathematical model of the object of a complex organizational and technical system.

2 Model of functioning of complex organizational and technical system on transport

Under the conditions of the adopted assumptions from the position of ensuring optimal parameters of functioning of a complex organizational and technical system, we obtain a formalized description of the synthesis object, which logically links the general principles of abstract description of hierarchical systems and the formulated principles of adaptive situational synthesis of structures of complex organizational and technical systems [1]. Such a position corresponds to the general approach to the mathematical description of systems from the point of view of decision making [2]. One of the essential aspects of the implementation of the system approach in such a description is a comprehensive consideration of the formalization process of both the structural description of the system and the procedure for finding the optimal structure and optimal parameters of functioning of a complex organizational-technical system. The main task is to ensure consistency of the whole sequence of decision-making procedures for selecting alternatives of the structure of a complex organizational-technical system and subsystems (elements) of different levels with the evaluation criteria of the complex organizational-technical system as a whole. However, such a problem has been solved [1, 3] only for the case of parametric optimization and is not studied for the practically most interesting variant of structural optimization, although the necessity and practical importance of it is obvious [4-6].

The description is based on the formal concepts of the system approach in the construction of theoretical and multiple schemes of the general theory of systems [7, 8], methods of applied systems engineering [9, 10], and methods of the theory of operations research.

$$x_0^* = \mathrm{Arg}\max_{x \in X} F_0(x_0) \tag{1}$$

Based on the general definition of the mathematical model [11], we assume that the model of functioning of a complex organizational and technical system is determined by the dependence

$$y = F(x, z) \tag{2}$$

where y - vector of output parameters of the object $y \in Y$; x - vector of internal parameters $x \in X$; z - vector of external perturbing influences, which includes uncertain factors $z \in Z$; F - operator determining the relationship between the specified quantities.

Assuming

$$\begin{gathered} x = \left\{x_j \middle| j \in J,\ J = \overline{1,n},\ n < \infty\right\} \\ y = \{y_i | i \in I,\ I = \overline{1,m},\ m < \infty\} \\ z = \left\{z_z \middle| \tau \in J_z,\ J_z = \overline{1,k},\ k < \infty\right\} \end{gathered} \tag{3}$$

from (1) we have a certain form of the object model for deterministic conditions

$$F : X \rightarrow Y \tag{4}$$

Where *X, Y* are the domain of definition and the range of values of the operator *F*.

$$X = \left\{x \mid x \in D \subset R_n, D = \cup_{j=J} D_j, D_j = \left[d_j^-, d_j^+\right]\right\} \tag{5}$$

$$Y = \{y \mid y \in B \subset R_m, B = \cup_{i \in I} B_i, B_i = [b_i^-, b_i^+]\} \tag{6}$$

Similarly, we believe that perturbing factors are determined by a variety of

$$Z = \left\{z \mid z \in V \subset R_k, V = \cup_{z=J_k} V_z, V_z = [v_z^-, v_z^+]\right\} \tag{7}$$

On the basis of (4), we will find models that establish a relationship between the output and internal parameters for the *k*-th level S_k of the hierarchical structure

$$F_k: X_k \rightarrow Y_k, k \in N_k, \tag{8}$$

where is the highest level $k=0$ corresponds to the product as a whole, and the lower $k=N_s$ – the level of functional elements or structural links.

Multitude $N_k = \{k \mid k-1 < k, k = \overline{0, N_x} N_x < \infty\}$ indices k are strictly ordered in the sense that the S_{k-1} takes precedence over S_k in the degree of influence on the overall efficiency of the functioning of complex organizational and technical systems.

The recurrence relation (8) establishes the relation of the output parameters to the input parameters at any level of the hierarchy, provided that the F_k and $X_k \forall k \in N_k$. In this case, assuming the influence of each level on the output parameters of the sample to be independent, it is possible to introduce the assumption accepted in [11] that sets X_0 and Y_0 can be represented in the form of Cartesian works $X_0 = X_1 \cdot \ldots \cdot X_n, Y_0 = Y_1 \cdot \ldots \cdot Y_N$.

This assumption is the basis of the implementation of the decomposition principle adopted in [11].

However, in practice, the influence of the internal parameters of the hierarchical structure on the efficiency of the functioning of complex organizational and technical systems is much more complex than is accepted in (8). First of all, the efficiency of the functioning of complex organizational and technical systems depends not only on the values of internal parameters determined by sets X_K and the structure of the different levels of the hierarchy defined by the functionalities F_K, but also on the type of structure of the relationships between the different levels of the hierarchy. Model (8) does not fully take into account this feature, since it is valid only if the model of each level is directly decomposable. However, in general, for complex organizational and technical systems, the conditions of direct decomposition are not met, and at the early stages of development, due to the axioms of synthesis, they cannot be tested. Secondly, the output parameters of the k-th level of the hierarchy depend not only on the internal parameters of this level, but are generally functions of the internal parameters of all previous levels. Model (8) does not take this feature into account and is valid provided that at each level of the hierarchy the task is directly aggregated. Third, model (8) assumes that $X_{k-1} \leq Y_k$ although, in general, between Y_k and X_{k-1} There is a functional dependency.

Taking into account these factors, let us present the model of the object of synthesis in a more general form $\forall k \in N_k$

$$\begin{gathered} F^k: X_k \rightarrow Y_k; F^{k-1}: X_{k-1} \rightarrow Y_{k-1} \\ F_{xx}^{k-1}: X_k \rightarrow X_{k-1}; F_{yx}^{k-1}: Y_k \rightarrow X_{k-1} \\ F_{xy}^{k-1}: X_k \rightarrow Y_{k-1}; F_{yy}^{k-1}: Y_k \rightarrow Y_{k-1} \end{gathered} \tag{9}$$

Definition domains and operator value domains are characterized by sets

$$\begin{gathered}
R_{xx}^{k-1} = R(F_{xx}^{k-1}) = \{x^{k-1} \in X_{k-1}; x^{k-1} = F_{xx}^{k}(x^{k}); x^{k} \in D_{xx}^{k}\} \\
R_{xy}^{k-1} = R\left(F_{xy}^{k-1}\right) = \left\{y^{k-1} \in Y_{k-1}; y^{k-1} = F_{xy}^{k}(x^{k}); x^{k} \in D_{xy}^{k}\right\} \\
R_{yx}^{k-1} = R\left(F_{yx}^{k-1}\right) = \left\{x^{k-1} \in X_{k-1}; x^{k-1} = F_{yx}^{k}(y^{k}); y^{k} \in D_{yx}^{k}\right\} \\
R_{yy}^{k-1} = R\left(F_{yy}^{k-1}\right) = \left\{y^{k-1} \in Y_{k-1}; y^{k-1} = F_{yy}^{k}(y^{k}); y^{k} \in D_{yy}^{k}\right\} \qquad (10) \\
R^{k} = R(F^{k}) = \{y^{k} \in Y_{k}; y^{k} = F^{k}(x^{k}); x^{k} \in D^{k}\} \\
D^{k}(F^{k}) = D_{xx}^{k} \cup D_{xy}^{k}; D^{k}(F^{k}) \subseteq X^{k}; D_{xx}^{k} \cap D_{xy}^{k} = \emptyset \\
R^{k}(F^{k}) = D_{xy}^{k} \cup D_{yy}^{k}; D_{yx}^{k} \cap D_{yy}^{k} = \emptyset
\end{gathered}$$

The mathematical model of the object is the starting point in the decision-making procedure in the synthesis of a complex organizational and technical system. The purpose of the procedure is to select alternatives to functional elements of different levels of the hierarchy that determine the structure and appearance of complex organizational and technical systems, based on a hypothetical model of the future system, taking into account the factors of the external environment in which the system should operate. The selection criterion for complex organizational and technical systems is to maximize the efficiency of the functioning of complex organizational and technical systems. Consequently, the criterion for the selection of synthesized alternatives at any level of the hierarchical structure of complex organizational and technical systems in accordance with the principles of the system approach is the optimization of the integral indicator (efficiency) of the highest level of the hierarchy – complex organizational and technical systems as a whole.

Let us proceed to the formal description of the process of synthesis of the hierarchical structure of complex organizational and technical systems in order to give, on the one hand, more constructive procedures for finding the optimal structure of functional elements of different levels of the hierarchy than is provided by heuristic algorithms [8, 13], and, on the other hand, more general procedures than formal methods of parametric optimization [13].

Let us describe the synthesis procedure in accordance with the system principle of the first leader [4], who began with *k=0*. It should be taken into account that at the initial stage of the synthesis, the requirements are determined for complex organizational and technical systems as a whole and the indicators of comparison and criteria for preference of alternatives at each level of the hierarchy are determined. Mathematically, the problem of synthesis in the most general formulation is reduced to structural optimization and consists in finding such a set of functions $\bar{F}_0^0 = \{\bar{F}_0^k \mid k = 1, N_s\}$ and such parameter values $\bar{x}_0^0 = \left\{x_k^0 \mid k = \overline{1, N_s}\right\}$ so that for deterministic conditions

$$\bar{x}_0 = \text{Arg} \underset{\bar{F}_1^0 \in M(\bar{F})}{\text{absmax}}[\text{absmax } \alpha_{\text{Э}i}(\bar{F}^0, \bar{x})] \qquad (11)$$

and for uncertainty conditions

$$\bar{x}_0 = \text{Arg} \underset{\bar{F}_l^0 \in M(F)}{\text{absmin}} \mid \text{absmin } [\text{abs } max[1 - \alpha_{\text{Э}i}^{-}, \alpha_{\text{Э}i}^{+}] \qquad (12)$$

In here $\alpha_{\text{Э}i}$ is determined by the ratio

$$\alpha_{\text{Э}j} = \frac{\text{Э}_{Hj} - \text{Э}_H^{+}}{\text{Э}_H^{+} - \text{Э}_H^{-}} \qquad (13)$$

Where is $\text{Э}_H^{-}, \text{Э}_H^{+}$ determine the minimum and maximum efficiency of a product of a given class per unit of cost, and Э_{Hj} – standardized efficiency *j*-th alternatives (13), and the *l*-th variant of the alternative corresponds to the *l*-th vector functional F_l^0 as an element of a set $M(\bar{F})$. Sets of $M(\bar{F})$ and $\overline{X^0}$ are defined by developers on the basis of specified requirements, technological, structural, and other constraints.

Problem (11) is unsolvable in a direct formulation and must be reduced to a sequence of problems of much smaller dimensions, each of which is solvable. One of the directions is the representation of the problem (11) in the form of a logically determined and mathematically interrelated sequence of problems

$$x_0^k = \text{Arg} \underset{\bar{F}_k \in M^k(F)}{\text{absextr}} [\underset{x^k \in X^k}{\text{absextr}} f^k(\bar{F}^k, x^k)] \tag{14}$$

Where is $f^k(\bar{F}^k, x^k)$ –functionality of the quality of the project alternative of the k-th level of the hierarchical structure, complex organizational and technical systems. When forming this functionality, take into account that for any hierarchical level, changing only the parameters of one structure cannot get another structure. For example, by changing the value of parameters and the type of connections between elements of only elements, it is impossible to obtain complex organizational and technical systems, or by changing the value of parameters and links of elements of complex organizational and technical systems, it is impossible to obtain a parametric amplifier. It follows that the two different structures of the k-th level of the hierarchy S_k^i and S_k^j as elements of space $M^k(S)$ form non-intersecting neighborhoods $U_{S^k}^j$ and $U_{S^k}^i$ such that

$$U_{S^k}^j \cap U_{S^k}^i = \emptyset \tag{15}$$

Therefore, $M^k(S)$ satisfies the definition [14] of Hausdorff space. This circumstance makes it possible to f^k take the Hausdorff distance between the corresponding sets $U_{S^k}^0$ and $U_{S^k}^i$, formed by the structure S_k^0, providing a solution to the problem (11), and an arbitrarily chosen structure S_k^i when you change their parameters x^k within X^k. Let's consider the following circumstances: Firstly, the quality of each functional element of the k-th level of the hierarchical structure of the system, in accordance with the accepted approaches of system engineering and circuit design [7], is characterized by a set of output parameter values$y^k \in Y_k$, which are determined by the structure of the functional element and the values of the internal parameters, $\overline{F^k}$ и x^k. Secondly, that the Hausdorff distance between arbitrary sets *A* and *B* is defined as

$$\rho(A, B) = max[\Delta(A, B); \Delta(B, A)] \tag{16}$$

Where is

$$\Delta(A, B) = \underset{x \in A}{sup} \underset{y \in B}{inf} |x - y| \Delta(B, A) = \underset{y \in B}{sup} \underset{x \in A}{inf} |x - y| \tag{17}$$

Then for $f^k(.)$ we get

$$f^k(.) = \max [\Delta(U_{S^k}^0, U_{S^k}^i); \Delta(U_{S^k}^i, U_{S^k}^0)] \tag{18}$$

Where is,

$$\Delta(U_{S^k}^i, U_{S^k}^0) = \underset{x_i^k \in X_i^k}{sup} \underset{x_0^k \in X_0^k}{inf} \left|\overline{F_0^k(x_0^k) - F_\iota^k(x_\iota^k)}\right| \tag{19}$$

From this it follows in accordance with (11) that in order to maximize the magnitude of the $\alpha_{\ni i}$ it is necessary to ensure that there is a minimum difference in the selected project alternative S_k^i from the alternative S_k^0 which by definition is the solution to the problem (11). Therefore, the distance $f^k(.)$ should be minimal and therefore the ratio (2.88-12) is converted to the form

$$x_0^k = Arg \underset{\overline{F^k} \in M^k(F)}{abs\ min} [\underset{\overline{x^k} \in X^k}{abs\ min} f^k(\overline{F^k}, x^k)] \tag{20}$$

The substantive formulation of this task consists in finding such a structure and such values of the parameters of functional subsystems (elements) of the structure of complex organizational and technical systems that ensure the minimum deviation of the obtained external parameters from the required ones. Provided that the required parameters are specified, possible general algorithms for solving the problem (11) and specific ways of implementing algorithms for a number of typical functional subsystems (elements) of complex organizational and technical systems have been developed in a number of works [3, 9]. Here, this problem is studied systematically, in the sense that there are models that provide both the determination of the requirements for the output parameters of functional elements for each level of the hierarchy, and the determination of the structure of the values of internal parameters from the condition for ensuring the implementation of the specified requirements.

For the practical implementation of the method of converting a task (11) into a sequence of tasks (20), it is necessary to:

- -to develop models that make it possible to determine the requirements for external parameters of the k-th level of the hierarchy based on the known properties of functional subsystems (elements) of a higher *(k-1)* level;
- develop a procedure for determining a set of acceptable design alternatives for each level of the hierarchical structure Complex organizational and technical systems;
- to propose methods for finding the optimal structure of functional subsystems (elements) of different levels of hierarchy;
- -to propose methods for solving the main optimization problems of the synthesis of functional subsystems (elements) of various levels of hierarchical structure, complex organizational and technical systems;
- consider the principles of implementation of computational optimization processes.

Formally, the solution of the first problem gives a sequence of recurrence relations obtained from (2.84-7) on the basis of the properties of the operators

$$\begin{gathered} F_{K-1}^{-1}: Y_{k-1} \to X_k;\ F_{K-1}^{-1}: Y_{k-1} \to X_k \\ {}^{(k-1)}F_{xx}^{-1}: X_{k-1} \to X_k;\ {}^{(k-1)}F_{yx}^{-1}: X_{k-1} \to Y_k \\ {}^{(k-1)}F_{xy}^{-1}: Y_{k-1} \to X_k;\ {}^{(k-1)}F_{yy}^{-1}: Y_{k-1} \to Y_k \end{gathered} \tag{21}$$

Hence, according to the well-known requirements for complex organizational and technical systems, given in the form of specific quantitative limitations, permissible limits for changes in external factors and other parameters and characteristics, it is possible to determine the requirements for indicators of complex organizational and technical systems.

At the same time, the practical use of model (21) is fraught with a number of fundamental difficulties. First, the inverse operators F^{-1} are multivalued, which excludes the unambiguous definition of requirements for functional subsystems (elements) of the k-th level according to known requirements and elements of a higher level of the hierarchy. Secondly, with the transition from the highest level of the hierarchy to the next levels, both the dimensionality of the output parameter space and, especially, the internal parameter space increases dramatically. Therefore, the equations linkingY_k with Y_{k-1}, turns out to be less than the output parameters of the k-th level of the hierarchical structure. Thirdly, the output parameters of the k-th level of the hierarchy are determined not only by the properties of the higher *(k-1)* level, but also by the structure of the interconnection of the functional elements of this level. Fourth, a number of the most important indicators of complex organizational and technical systems are determined by the chosen structure and the values of internal parameters of the entire hierarchical structure of the system at the same time. An example is the amount of costs, which significantly affects the indicator of resource intensity and efficiency of complex organizational and technical systems and is determined both by the cost of functional subsystems (elements) of all levels of the hierarchical structure of complex organizational and technical systems, and by the amount of associated capital investments and operating costs. Fifth, the presence of uncertainty in the early stages of synthesis determines the structural and parametric uncertainty of the synthesized system $\mu = \mu(S)$ object S or products S not in nature. These factors determine the fundamental difficulties in the development of direct methods of decomposition in the synthesis of hierarchical structures of complex organizational and technical systems, and lead to the need to introduce such strong assumptions that significantly narrow the scope of practical application of these methods. At the same time, as the analysis [2] shows, due to the specifics of the iterative process, these factors are less significant in the development of iterative multilevel decomposition procedures, which makes this direction more promising. Therefore, let us analyze the possible mathematical schemes of the iterative decomposition algorithm.

The iterative algorithm, as well as the entire decision-making procedure in system synthesis, is based on practical purposefulness in the interests of ensuring a closer relationship between the general theory of complex systems and the practice of synthesizing specific classes of systems. Therefore, in addition to the principles of system synthesis formulated above, when developing the algorithm, we will take into account the availability of a priori information about the real requirements for complex organizational and technical systems and for individual functional subsystems (elements) of all levels of the hierarchical structure.

In meaningful terms of systems engineering, the idea of an algorithm is as follows.

On the basis of inaccurate and incomplete data of the initial stage of synthesis, a roughly approximate model of the system is built, according to which the requirements for functional subsystems (elements) are determined in the form of a certain range of values for each external parameter.

According to these requirements, the permissible set of structures of functional subsystems (elements) is determined, their approximate models are built, and the requirements for functional subsystems (elements) are found. This information similarly serves as the basis for building models of subsystems (elements) and forming requirements for elements and selecting a permissible set of classes of corresponding elements. By taking a step for all levels of the hierarchy, and obtaining certain information about the hierarchical structure of complex organizational and technical systems, it is possible to build more accurate models, clarify requirements and permissible sets, as well as solve problems of optimization of functional subsystems (elements) of various levels of the hierarchy.

Mathematical formalization is described as follows. We assume that at step k of the iterative process of synthesizing a hierarchical structure, approximate models are used for functional subsystems (elements) of all levels in the form of a tuple

$$\widetilde{\mu_q^k} = \langle \widetilde{F_q^k}, \widetilde{X_q^k}, \widetilde{Y_q^k} \rangle \tag{22}$$

Then, to determine the requirements for the functional elements of the hierarchical structure, we obtain the following recurrence equation

$$\widetilde{Y_q^{k-1}} = \widetilde{F_q^{k-1}}(\widetilde{Y_q^k}) \tag{23}$$

where the set is known $\widetilde{Y_q^{k-1}}$, and those who are sought are multitudes $\widetilde{Y_q^k}$.

Definition $\widetilde{Y_q^k}$ for vector functionality $\widetilde{F_q^{k-1}}$ leads in general to an incompatible system of nonlinear equations.

Multitude $\widetilde{Y_q^k} \forall k \in N_k$ is described as

$$\widetilde{Y_q^k} = \{y | y \in \widetilde{B_q^k}, \widetilde{B_q^k} = \bigcup_{i \in I} \widetilde{B_{iq}^k}, \widetilde{B_{iq}^k} = [\widetilde{b_{iq}^-}, \widetilde{b_{iq}^+}]\} \tag{24}$$

Equation (17) makes it possible to determine the requirements for the external parameters of functional subsystems (elements) of all levels of the hierarchical structure according to known requirements$\widetilde{Y_q^0}$ to the system as a whole. Initial approximation $\widetilde{Y_q^0}$ is determined by the requirements of the decision-maker, and subsequent approximations $\widetilde{Y_q^0}$ – both the requirements of the decision-maker and the information obtained in previous iterations.

According to the well-known $\widetilde{Y_q^k} \forall k \in N_k$ a procedure for finding a valid set is built S_{kq}^+ design alternatives for each level of the hierarchical structure. The following classes of synthesized alternatives are permissible S_{kiq}^+, which fundamentally ensure the fulfillment of the specified requirements. Since only approximate models are known for the q-th iteration $\mu(S_{klq})$ l-го класса функциональных подсистем (элементов) k-го уровня иерархической структуры, то допустимое множество of the l-th class of functional subsystems (elements) of the k-th level of the hierarchical structure, the permissible set S_{kq}^+ define from the condition

$$S^{+}_{kq} = \{S^{+}_{klq} \mid \mu(S^{+}_{klq}) = \langle \tilde{F}^{k}_{lq}, \tilde{X}^{k}_{lq}, \tilde{Y}^{k}_{lq} \rangle, \exists l \in N^{+}_{k}, B^{+}_{kl} \cap \tilde{B}^{k}_{q} \neq 0\}, \quad (25)$$

Where

$$B^{+}_{kl} = \bigcup_{i \in I} B^{+}_{ilq};\ B^{+}_{ilq} = [b^{+}_{ilq}, b^{-}_{ilq}],$$

$$b^{+}_{ilq} = \sup_{F^{1}_{iq} \in \tilde{F}^{k}_{lq}} \sup_{x \in \tilde{X}^{k}_{lq}} F^{1}_{iq}(x) \quad (26)$$

$$b^{-}_{ilq} = \inf_{F^{1}_{iq} \in \tilde{F}^{k}_{lq}} \sup_{x \in \tilde{X}^{k}_{lq}} F^{1}_{iq}(x)$$

Hence the detection algorithm S^{+}_{kq} is reduced to the procedure of finding B^{-}_{kl}, and the subsequent exclusion from the selected set of decision makers S_{kq} possible design alternatives of the k-th level at the q-th iteration of the set S^{-}_{kq}, determined by the ratio (25).

The solution to the latter problem can be found on the basis of known methods, in particular, on the basis of the sequential exclusion method [1].

If $\forall k \in N_k$, $S_{kq} = \emptyset$ then it is possible to solve the following problem of the procedure for forming the hierarchical structure of the product – finding the optimal classes of functional subsystems (elements) of each level of the hierarchical structure from the condition of maximizing performance indicators (13) or optimizing the criteria when the conditions are met. The challenge is to find such a $\overline{S}^{0}_{q} \in S^{+}_{q}$ for every $k \in N_k$, that

$$\alpha_{\ni}(\overline{S^{0}_{q}}) = \underset{S^{+}_{lq} \in \overline{S^{+}_{q}}}{abs \max} \alpha_{\ni}(\overline{S^{+}_{lq}}) \quad (27)$$

Where is $\overline{S^{+}_{lq}}$ –the l-th variant of the hierarchical structure is complex organizational and technical systems in the q-th iteration, presented as a set of structures of all levels in the form of

$$\bar{S}^{+}_{lq} = \{S_{klq} | k = 1, \overline{N_S}, l \in N_k\} \quad (28)$$

S^{+}_{q} – set of valid product structures in the q-th iteration

$$S^{+}_{q} = \{S^{+}_{kq} | k = 1, \overline{N_S}\} \quad (29)$$

S^{0}_{q} – optimal hierarchical structure for the q-th iteration

$$S^{0}_{q} = \{S^{0}_{lq} | l^{0} \in N_k\} = \underset{S^{+}_{lq} \in S^{+}_{q}}{Arg\ abs \max} \alpha_{\ni}(S^{+}_{lq}) \quad (30)$$

Thus, the algorithm for determining the optimal structure of complex organizational and technical systems includes the solution of extreme and inverse problems and the procedure for sequential exclusion of a subset of options interrelated by a single goal - ensuring maximum efficiency of complex organizational and technical systems as a whole. At the same time, the decision-making procedure for the choice of alternatives for each level of the hierarchical structure is carried out on the basis of sequential disaggregation of variables S_{klq} and sets S^{+}_{lq} and is closely related to the requirements procedure so that at each iteration for each level of the hierarchy, there is freedom to choose options. As uncertainty decreases $(B^{k}_{q} \to B^{k}_{0})$ freedom is diminished and the number of permissible alternatives is reduced.

As a result, the procedure for coordinating the interrelations of functional subsystems (elements) of different levels of the hierarchical structure is transferred from the stage of prototyping to the stage of forming the appearance of complex organizational and technical systems.

This algorithm is relatively easy to implement interactively, with the possible variants of project alternatives at each level of the hierarchy set by the decision-maker, and then the permissible set of alternatives is determinedS_{kq} and background information for the selection of the optimal structure. As a condition for the end of an iterative process, you can take a given degree of difference between the arguments or the efficiency indicator by q and *(q+1)* step.

It should be noted that when any variant of structures is chosen, the synthesis problem turns into the form of parametric optimization, the properties and features of which have been studied in sufficient detail.

From now, for any structure option, $S_{lq}^{+} \in S_{q}^{+}$ the theorems and properties of the parametric optimization procedure are valid. At the same time, the described procedure also has a number of specific properties, the study of which is an independent task of the mathematical theory of complex systems. In particular, of practical interest is the study of the regularization of the equation (27), the coordination of the accuracy of models and initial data.

3 Conclusion

For the practical implementation of this algorithm, it is also necessary to solve a number of issues of information support, in particular, obtaining data on the limit values of external parameters of various classes of functional elements of different levels of the hierarchy $b_{\bar{i}lq}^{+}$, $b_{\bar{i}lq}^{-}$. For a number of classes of functional elements, the corresponding data are presented in reference books, for example [17]. However, in the future, in order to function, it is necessary to concentrate information about $b_{\bar{i}lq}^{+}$, $b_{\bar{i}lq}^{-}$ for the whole possible set of synthesized alternatives of several higher levels of hierarchical structure, complex organizational and technical systems. This can be done based on experience and information processing. Information about the parameter limits for the lowest level of the hierarchy can be easily found on the basis of the solution of the parametric optimization problem. These questions reflect the specifics of the procedure and may have different aspects depending on the specific applications of the described algorithm. In this paper, the general properties of the synthesis procedure are studied. Therefore, the purpose of this section is to select the most rational methods for solving optimization problems of the system synthesis of the hierarchical structure, complex organizational and technical systems.

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