Space-time permutation statistics application to detect cloud-to-ground lightning prone area (Case study: Pasuruan, Indonesia)

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Abstract. Lightning is a natural phenomenon caused by the release of positive and negative charges occur in cumulonimbus (CB) clouds. CG lightning is a lightning that strikes from the clouds to the ground. This lightning is dangerous for human activities which can cause burns, blindness, and even temporary deafness. This research will determine the areas prone to CG lightning strikes and lightning characteristics in the city of Pasuruan. Space-time permutation scan statistics is a method used to detect prone areas by considering spatial and temporal aspects. This method merely requires case data, such as location and time without using population data. The detected prone areas will be tested for significance using Monte Carlo. It is used to determine the distribution of the sample. In this study, the Monte Carlo test for scanning window is 0.048 (p-value < 0.05), Sekargadung. Thus, making Sekargadung a hotspot for lightning-prone areas. Furthermore, the value is taken based on the highest ratio test (LRT), 11.46, which is the most likely cluster. Based on space-time permutation statistics Sekargadung is the main hotspot prone area in this case study. It has 226 strikes with the intensity of their occurrence with characteristics area dominant paddy field.

Keywords. Space-time permutatio scan, likelihood ratio test, hazardous areas, cloud-to-fround, lightning prone area

1 Introduction

One of the unavoidable natural events is lightning. Lightning is caused by the discharge of positive (+) and negative (-) clouds that meet each other in cumulonimbus (CB) clouds. It can produce flashes followed by loud noises called thunder, then produce lightning and hurricanes. Cloud-to-ground (CG), one of the pretty dangerous lightning types, occurs in certain areas [1]. Indonesia is a tropical country which has two seasons. It is a dry season and a wet season [2]. The dry season falls between May and October then the wet season falls between November and April. The part of Indonesia is Pasuruan that located in East Java.

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Thus, the area is adjacent to the capital city of East Java and is one of the major industrial cities in East Java [3].

Pasuruan is one prone area to lightning strikes. It is located in the highlands between the mountains and the ocean. Therefore, it is easy to form cumulonimbus clouds. It is high vertical clouds, solid, and involves thunderstorms and other cold weather [4]. Population growth in this city is higher than in Madiun and Mojokerto, where population growth increased annually by 3.5 % from 2018 to 2020 [5]. The data shows that the higher the population growth, the higher the risk of being negatively affected by lightning strikes [6]. Some of the adverse effects of a lightning strike are that it will cause an electric shock that can cause a heart attack, burning sensation, and even temporary deafness. Lightning can strike tall objects such as trees, transmitter towers, high-rise buildings, skyscrapers, and others [7]. Pasuruan is one of city in Indonesia that famous with city of mountain. Since it has one famous tourism icon, Bromo mountain [7]. In addition, Pasuran has many areas that have the potential for lightning strikes such as Beji, Bangil, Gempol, half of Prigen area. It has category of extreme lightning-prone areas with a percentage of 68 % [6]

The previous research regarding lighting strikes is a study that discusses the DBSCAN (Density-Based Spatial Clustering of Application with Noise) method for analysis of lightning spread patterns in Pasuruan. The research resulted in two clusters, the first spreading in the Pasuruan area and the second around the Prigen sub-district [8]. Space-time permutation scan statistics used by Yakin for detecting earthquake hotspots in central Sulawesi. Furthemore, the result showed that the main hotspot of earthquake was Donggala district and Palu then the second hotspot was Banggai district, Banggai Laut district, Tojo Una-Una district, and North Morowali district.

The statistical method used to determine prone areas in an event is space-time permutation scan statistics. Scan statistics is a method to detect cluster based on space location or time. The procedure is carried out based on the dimensions of space and time to detect the occurrence of events in specific areas statistically. This method was discovered by J. I Naus in 1965 [9]. Martin Kulldorf developed the application of the statistical scan method by integrating spatial aspects, temporal aspects, and spatial and temporal characteristics [10]. Space-time permutation scan statistics are based on space and time dimensions to detect whether events occur in certain areas statistically. Hypothesis testing is based on the Likelihood Ratio Test (LRT) when detecting prone areas. The method observes lightning strike-prone areas using sample data without population data [11].

Lightning is a deadly natural disaster that can be destructive on a short time scale. Moreover, accompanied by bad weather, hail and floods, which can result in significant economic losses [12]. Lightning strikes can cause cardiac arrest at the time of injury. However, some victims of strikes appear to experience delayed death several days later when the victim is revived. However, victims can suffer irreversible brain damage [13].

2 Space-time permutation statistics method

A space-time permutation scan statistics method developed by Martin Kulldorf was used to detect the occurrence of prone areas by using a scanning window in the form of a cylindrical window with spatial and temporal aspects [9]. Scan Statistics is a technic that Joseph Naus developed in 1965. A space-time permutation scan statistics is an extension of scan statistics. Space-time permutation scan statistics have the advantage of being the location of a specific size and the location of the clusters formed [14]. Data that only has case data without using population data, then with information about the spatial location and time of day for each case.

The study use a retrospective approach, which is a temporal approach that deals with the past, and a prospective approach that deals with the present. The detection of prone areas in the statistical scan method can be seen from the likelihood ratio test. The first step is determining a scanning window. All scanning window in the data point is possibility area centered in each point. Since it is carried out up to a maximum cluster size of 50 % of the total population and is carried out for all location.

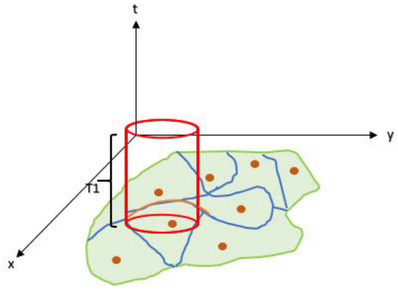

Fig. 1. Cylindric window

The scanning window is an established cluster, then thus a set region in a specific time interval, and it has the most likely cluster potential. In detecting prone areas, a specific opportunity model will be used and test the hypothesis with Monte Carlo and determine the Likelihood Ratio Test value. Hence, the data has spatial and temporal aspects [11] then use a cylindric window (Fig. 1) to determine most likely cluster. Figure 1 show that t is time period. Then, scanning window formed by cylinder. The base is a red circle in the shape of a cylinder on the x-axis, y-axis and the height of the cylinder on the T1-axis.

The cylindric window determined by [9]:

1. Choose any center point from the subregion represented by the center coordinates. Where, center point, S_i, write as (x_i, y_i). Then, x_i is latitude and y_i is longitude.
2. Calculate Euclid's distance from the coordinate center point to the other sub-region coordination center point (Equation (1)). For instance, , $S_j\left(x_j, y_j\right)$.

$$S_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (1)$$

3. Sort from closest to farthest.

After the steps to determine the cylindric window, then build a cylindric window.

1. Select any sub-region represented by the coordinate centre point,
2. Create a cylinder centered at the center of coordinates with a height of one unit of time with a radius along the shortest distance of the center point to the nearest sub-region. For each point, the centre coordinates enter the cylinder,
3. Calculate n_A, where is the number of events in A, and μ_A is the population in the cylinder. Then $RR = \frac{n_A}{\mu_A}$, omit when $RR < 1$,
4. Calculate the likelihood ratio of each pair $(n_A\ ;\ \mu_A)$. This likelihood ratio is used to test the similarity of the distribution inside and outside the scanning window. Repeat steps 2 and 3,
5. Enlarge the radius of the cylinder bed continuously up to the limit specified by the user, which has reached a maximum of 50 % of the total population in the study area. Repeat steps 3 and 4.

$\sum_m C_{mt}$ is lightning strikes at spatial locations at all times. Let $\sum_t C_{mt}$ be the number of lightning strikes for all spatial areas. Therefore, the expected cases in the scanning window formed are $\mu_A = \sum_{(m,t)\epsilon A} \mu_{mt}$. The number of lightning strike cases in *scanning window,* for instance, C_A.

However, if there are no events or interactions of space and time, then C_A is assumed that have Hypergeometric distribution with the probability density function as follows [9].

$$P(C_A) = \frac{\binom{\sum_{m\in A} C_{mt}}{C_A}\binom{C - \sum_{m\in A} C_{mt}}{\sum_{t\in A} C_{mt} - C_A}}{\binom{C}{\sum_{t\in A} C_{mt}}} \tag{2}$$

Suppose, C_{mt} is number of monthly cases of lightning strike at a spatial location that occurs at t=1,2,...,T. Then, C is number of lightning strike cases that occurred.

$$C = \sum_m \sum_t C_{mt} \tag{3}$$

For μ_{mt} is a number of expected cases, the estimated number of cases at the time. Accordingly, t is multiplied by the proportion of all cases at a location equal to the number of cases at spatial locations. Hence,

$$\mu_{mt} = \frac{1}{C}\left(\sum_m C_{mt}\right)\left(\sum_t C_{mt}\right)$$

When there is no event or space and time interaction of an event (in this case, it is the number of lightning strikes contained in the scanning window), C_A has a hypergeometric distribution. Because in Equation (2), $\sum_{t\in A} C_{mt}$ and $\sum_{m\in A} C_{mt}$ are relatively smaller than the total population. It is C then it can be approximated by a binomial distribution with a value of $p = \frac{\sum_{m\in A} C_{mt}}{C}$. Therefore, $\sum_{m\in A} C_{mt}$ is large and the probability $p \to 0$, the binomial distribution approximated by the Poisson distribution [9],

$$f(C_{mt}) = \begin{cases} \dfrac{e^{-p\mu_{mt}}(p\mu_{mt}(x_{mt}))^{C_{mt}}}{C_{mt}!}, & x_{mt} \in A \\ \dfrac{e^{-q\mu_{mt}}(q\mu_{mt}(x_{mt}))^{C_{mt}}}{C_{mt}!}, & x_{mt} \notin A \end{cases}$$

Let p be the lightning strike probability inside the formed scanning window. Then, q s the lightning strike probability outside the created scanning window. In the Poisson distribution, the probability value depends on the average of the experimental results that occur in a given time interval or area. Therefore, $p\mu_{mt}$ is the number of test results inside the scanning window during a specific time interval. At the same time $q\mu_{mt}$ in Equation (3) is the number of test results outside the scanning window during a certain time interval.

Subsequently, the natural logarithm of $\mathrm{L}(\Omega)$ or $\ln \mathrm{L}(\Omega)$ determined to get for likelihood function based on $L(\Omega)$ is true if $p = q$. Where, p is number of lightning strikes inside the scanning window and q is opportunities outside the scanning window. Therefore, parameter space $\Omega = \{(p, q), 0 \leq p \leq 1, 0 \leq q \leq 1\}$ [9],

$$L(\Omega) = \frac{e^{-p\mu_A - q(C-\mu_A)}}{C_{mt}!} p^{C_A} q^{C-C_A} \prod_{i=1} (x_i) \tag{4}$$

Then, natural logarithm value from $\mathrm{L}(\Omega_1)$ or $\ln \mathrm{L}(\Omega_1)$ determined by likelihood function based on H_1 or $L(\Omega_1)$ true if $p > q$.

$$\begin{aligned} L(\Omega_1) &= \frac{e^{-p\mu_A - q(C-\mu_A)}}{C_{mt}!} p^{C_A} q^{C-C_A} \prod_{i=1} (x_i) \\ \ln\ L(\Omega_1) &= ln\left(\frac{e^{-p\mu_A - q(C-\mu_A)}}{C_{mt}!} p^{C_A} q^{C-C_A} \prod_{i=1} (x_i)\right) \\ &= n(e^{-p\mu_A - q(C-\mu_A)}) + \ln(p^{C_A}) + \ln(q^{C-C_A}) + ln\left(\prod_{i=1} (x_i)\right) - \ln\,(C_{mt}!) \\ &= -p\mu_A - q(C-\mu_A) + C_A \ln(p) + C - C_A \ln(q) + ln\left(\prod_{i=1} (x_i)\right) - \ln\,(C_{mt}!) \end{aligned}$$

Subsequently, maximize the value of $\ln\ L(\Omega_1)$ by finding p and q from $\frac{\partial \ln \mathrm{L}(\Omega_1)}{\partial p} = 0$ and $\frac{\partial \ln \mathrm{L}(\Omega_1)}{\partial q} = 0$. Then, it get $\hat{p} = \frac{C_A}{\mu_A}$ dan $\hat{q} = \frac{C-C_A}{C-\mu_A}$. Hence, both are substituted for Equation (4) as follows.

$$\begin{aligned} \mathrm{L}(\Omega_1) &= \frac{e^{-\left(\frac{C_A}{\mu_A}\right)\mu_A - \left(\frac{C-C_A}{C-\mu_A}\right)(C-\mu_A)}}{C_{mt}!} \left(\frac{C_A}{\mu_A}\right)^{C_A} \left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A} \prod_{i=1} (x_i) \\ &= \frac{e^{-C_A - C + C_A}}{C_{mt}!} \left(\frac{C_A}{\mu_A}\right)^{C_A} \left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A} \prod_{i=1} (x_i) \\ &= \frac{e^{-C}}{C_{mt}!} \left(\frac{C_A}{\mu_A}\right)^{C_A} \left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A} \prod_{i=1} (x_i) \end{aligned} \tag{5}$$

Determination of lightning strike-prone areas must have a higher probability in the scanning window than outside the scanning window then the alternative hypothesis is $\mathrm{H}_1 : p > q$ using the test statistic of the likelihood function as follows.

$$\lambda(x) = \frac{L(\Omega_1)}{L(\Omega)} \tag{6}$$

$$= \frac{\frac{e^{-C}}{C_{mt}!}\left(\frac{C_A}{\mu_A}\right)^{C_A}\left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A}\prod_{i=1}(x_i)}{\frac{e^{-C}}{C_{mt}!}\prod_{i=1}(x_i)}$$

$$= \left(\frac{C_A}{\mu_A}\right)^{C_A}\left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A}$$

Based on hypothesis $p > q$ or $\left(\frac{C_A}{\mu_A}\right) > \left(\frac{C-C_A}{C-\mu_A}\right)$ then likelihood ratio test [9]:

$$\lambda(x) = \left(\frac{C_A}{\mu_A}\right)^{C_A}\left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A}, \text{when } \left(\frac{C_A}{\mu_A}\right)^{C_A} > \left(\frac{C-C_A}{C-\mu_A}\right)^{C-C_A}$$

The critical area used likelihood ratio test is to reject H_0 if $\lambda(x) \leq k$. While k is constant then it it has chosen to provide a test of size α. The value of k is at $0 < k \leq 1$ [15]. The test criteria used were obtained based on the Monte Carlo simulation because the distribution of the calculated likelihood ratio is unknown.

2.1 Kolmogorov Smirnov test

One of the commonly used tests is the Kolmogorov Smirnov test. The Kolmogorov Smirnov test is used to determine whether a data has a certain distribution or not. Before testing the distribution of data will be grouped according to class. The Kolmogorov Smirnov test determines whether data has a specific distribution. However, the distribution of data should be grouped according to class. The class divided used the Sturgess rule [16]:

$$k = 1 + 3{,}3\log(n)$$

where, k is the number of classes. Then, n is the number of data studied. Since determining the class, each class's interval length should be determined by Equation (5) [16],

$$c = \frac{max - min}{k}$$

where, c is the length of the interval. The max is the maximum number of data while min is the minimum number of data. Moreover, k is the number of classes. Hypothesis test in this study is,

H_0: Poisson distribution data
H_1: Data is not distributed Poisson

Statistics test is,

$$D = \max\ |(S(x) - F_0(x))|$$

where, $S(x)$ is the cumulative number of case data. While, $F_0(x)$) is the cumulative number of the Poisson distribution. The criteria test to be used to reject H_0 when $D > D_\alpha$. It is obtained from the Kolmogorov-Smirnov table or can use the p-value $\leq \alpha$ [17].

2.2 Monte Carlo hypothesis testing

Monte Carlo is used to determine the distribution of the sample. The complex space-time permutation scan statistics method uses another way to obtain critical points. Since the probability can be through Monte Carlo hypothesis testing, then evaluated with test statistics using the likelihood ratio test [9]. The hypothesis used is as follows.

$H_0 : p = q$ (clusters are unformed because the opportunities inside the scanning window are the same as the opportunities outside the scanning window).

$H_1 : p > q$, (clusters formed, because the opportunities inside the scanning window are greater than the opportunities outside the scanning window).

The statistical test that modified by [18] is,

$$p = \frac{1 + \sum_v^m (T_v > T)}{m + 1}$$

whereas, T_v is LRT summation of random data constructed under condition H_0. Then, T *is* number of original data LRT sums. Moreover, *m* is number of simulations. The critical area is rejecting H_0 if the p-value < α, where the p-value is equivalent to the p-value.

2.3 Data source

The study used lightning strike point data. The data source that will be used in this study is secondary data on Cloud to Ground (CG+) and CG- lightning strike points obtained from the Meteorology, Climatology and Geophysics Agency (BMKG) Geophysics Station Class II Pasuruan, with 34 villages in Pasuruan on December January 1st, 2021, to December 31st, 2021.

3 Results and discussion

Before determining the characteristics of lightning, the class division and frequency in each class will be determined. The division of classification classes used the formula in Equation (3) and obtained six classes. The interval calculation for each class of CG- lightning strikes is calculated using the formula in Equation (4) and obtained an interval of 44 and CG+ lightning with an interval of 24. The following are the results of the classification of CG- and CG+ lightning strikes in the city of Pasuruan. Table 1 shows the length of the CG+ and CG- lightning intervals and the frequency of CG- and CG+ lightning strikes in each class. The location of the lightning strike will be mapped in Pasuruan.

The legend in the Fig. 2 shows number of lightning strikes. Figure 2a knows that the Kepel and Blandongan villages have high strokes (light green and red). The highest strike in the Blandongan village occurred in December 2021, with 74 strikes out of a total of 271 strikes. CG-type lightning is the most common type of lightning (Fig. 2b), where this lightning usually falls along with convective rainfall. In the villages of Blandongan, Kepel, Pohjentrek, Krapyakrejo, and Bakalan, almost part of the area is paddy fields. However, strikes that occur very high in paddy fields allow negative impacts from lightning strikes such as damaging paddy fields, striking trees, and others.

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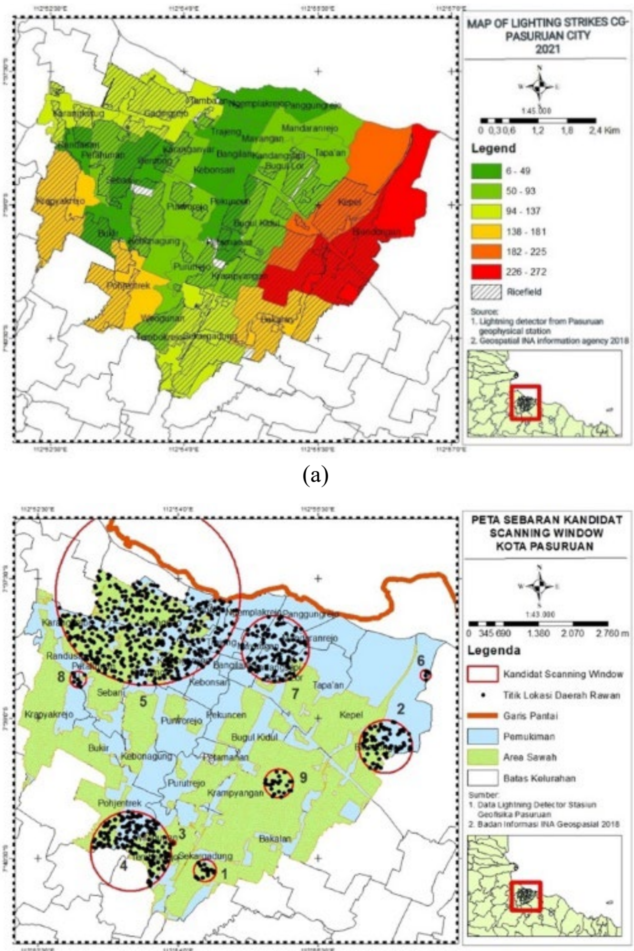
Table 1. Classification results of cg- and cg+ lightning strikes.

Class	CG- Interval Length	Frequency	CG+ Interval Length	Frequency
Class 1	6 – 49	12	2 – 25	13
Class 2	50 – 93	12	26 – 50	11
Class 3	94 – 137	5	51 – 74	8
Class 4	138 – 181	3	75 – 98	1
Class 5	182 – 225	1	99 – 122	0
Class 6	226 - 272	1	123 - 146	1



(a)

(b)

Fig. 2. Map of the number of cg- (a) and cg+ (b) lightning strikes.

The first use the space-time permutation scan statistical method to determine prone areas using the Kolmogorov Smirnov test to test whether the data used is Poisson distributed or not. Table 2 shows that the value is 0.164, where or p-*value* (0.999) is accepted. Hence,

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the data for lightning strikes in Pasuruan is distributed, Poisson. The next step is to determine the areas prone to lightning strikes using the ArcGis 10.0.2 software.

Figure 3 shows nine clusters formed with 863 cluster locations from a total of 3726 cases of lightning strikes that spread around the urban village of Pasuruan. In clusters, 1 to 9 lightning strikes occurred in some paddy fields and residential areas. Furthermore, the second cluster, the fifth cluster, seventh cluster, and eighth cluster are clusters that occur in areas close to the coast. The following are the results of candidates for lightning-prone areas in the city of Pasuruan.

Table 2. Kolmogorov Smirnov result test.

D	D_α	***P-value***
0.164	0.51926	0.999

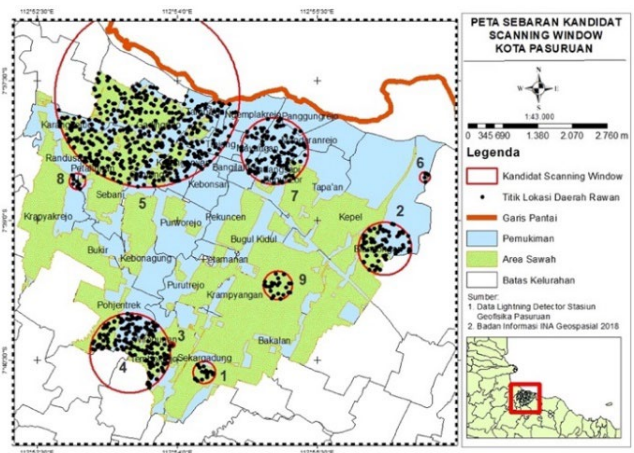

Fig. 3. Candidate prone areas with lightning strike point.

Table 3 shows nine clusters which are a collection of areas prone to lightning strikes. The cluster radius indicates the distance of the prone area from the central point where the closest distance from the center point will potentially become a prone area by considering the location and time of the incident; the p-value is used in determining the prone area in the Monte Carlo hypothesis testing while the LRT is used to determine the order of the prone areas.

Classification is divided based on the characteristics of the area formed in the scanning window, which can be seen in Fig. 3. The prone areas with the classification of paddy fields in the scanning windows 1, 3, and 9 are due to the high evaporation process and wind gusts moving water vapor to the plain higher or farther from the coast. Prone areas are classified as near the coast. High strikes are caused near the coast, experiencing a high evaporation process. In contrast, cumulonimbus clouds are easy to form, which causes lightning to occur. While the many tall buildings cause the areas prone to lightning strikes that occur in some settlements, and there are BTS towers that can trigger lightning strikes. The results of the scanning window classification Table 3 show that the most lightning strikes near the coast. However, suppose there is a thunderstorm in the paddy fields and areas near the beach. In that case, one of the efforts, such as moving away from the paddy fields or beach areas, then find shelter and not take shelter under trees.

Table 3. Candidates in prone areas detected.

Scanning Window	Village Name	Radius Cluster	P-value	LRT	Classification
1	Sekargadung*	0.22 km	0.048	11.46	Paddy Field
2	Blandongan* Kepel	0.53 km	0.085	10.66	Near the beach
3	Tembokrejo*	0.049 km	0.164	10.06	Paddy Field
4	Pohjentrek* Wigunan Tembokrejo	0.79 km	0.737	8.22	Part of Paddy Field
5	Gadingrejo* Karangketug Tamba'an Trajan Karanganyar Gentong Petahunan Randusari Ngemplakrejo	1.82 km	0.767	8.12	Near the beach
6	Blandongan*	0.10 km	0.972	6.88	Near the beach
7	Mandaranrejo* Panggungrejo Bugul Lor Mayangan Ngemplakrejo Kandangsapi Tapa'an	0.66 km	0.997	7.30	Near the beach
8	Petahunan* Randusari	0.16 km	0.997	6.86	Near the beach
9	Kepel* Blandongan	0.28 km	0.998	6.75	Paddy Field

Table 4. LRT lightning strike prone area.

Scanning Window	Village Name	LRT	Time Frame	Number of Cases
1	Sekargadung	11.46	1/5/2021 to 31/7/2021	5

The Monte Carlo test based on Table 1 shows the formed scanning window. Hence, it is a prone area when it has a significant p-value < 0.05. Accordingly, Table 1 shows that only the first scanning window has a p-value of 0.048 < 0.05. The second scanning window to the ninth scanning window has a p-value > 0.05, rejecting H_0. Therefore, the risk of lightning strikes inside and outside the scanning window is equal. Hence, the prone area to lightning strikes is the Sekargadung village, with the highest LRT (Likelihood Ratio Test) value of 11.46 (Table 4). The highest LRT value from the hotspot is the most likely cluster, and all other clusters are secondary clusters. Hence, the main cluster is Sekargadung. Figure 4 is prone area lightning strikes in Sekargadung area.

Based on lightning characteristics, Blandongan had the highest number of strikes in December 2021. However, the results of an analysis using space-time permutation statistics do not make this prone area. Therefore, the area has a far radius from the center point as well as the location and time of the incident. In Blandongan, there are 417 strikes which tend to be scattered in every area of the Blandongan sub-district. However, Sekargadung has 226 strikes with the intensity of their occurrence, which tends to be concentrated in one area and time. Sekargadung is ancarea with a dominant paddy field area than its residential area. It has resulted in Sekargadung being a prone area because the location of the strike point is concentrated in one paddy field area. In addition, the Sekargadung village is called the central cluster, which is a lightning strike-prone area in the city of Pasuruan.

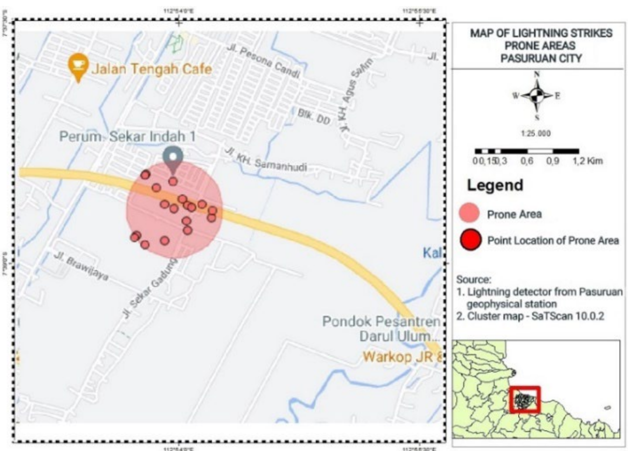

Fig. 4. Mapping results of space-time permutation statistics for lightning prone areas in Pasuruan.

4 Conclusion

Based on the results and discussion space-time permutation statistics main hotspot is Sekargadung village. It has LRT value of 11,462. Sekargadung has 5 cases of lightning with a period of 1 May 2021 to 31 July 2021. The characteristics area in Sekargadung is paddy field. Lightning strikes high in paddy fields allow negative impacts from lightning strikes, such as falling trees, damaging paddy fields, and can strike humans around the paddy fields. Hence, the local government should warn residents to be aware of the threat of lightning strikes, especially in November and April or during the wet season.

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