A product recovery inventory model with a circular economy indicator

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Abstract. The circular economy concept has been proposed as a way to increase sustainability, where manufacturers reduce waste by keeping materials in circulation as much as possible through product recovery, and consumers support these manufacturers by buying from them. Hence, the aim of this paper is to demonstrate the advantage of investing in circular economy activities by proposing an Economic Production Quantity inventory model for a finished product in a circular economy, where the finished product can be manufactured from raw materials and remanufactured from used items. The variable level of circularity is indicated by an index between 0 and 1. Both the production quantity and the circularity level are taken as a decision variables. The proposed model also takes carbon emission costs into account. A solution procedure to find the optimal policy is presented and is illustrated with numerical examples. Our analysis showed that investing in circular economy activities is advantageous, even when it is more profitable to manufacture than it is to remanufacture.

1 Introduction

Harris [1] proposed the economic order quantity (EOQ) inventory model and gave a simple square root formula for the optimal replenishment size. This model assumes that the demand rate is constant and replenishment is instantaneous. Taft [2] proposed the economic production quantity (EPQ) model by extending the EOQ model to the case of finite replenishment rate. Today, these models are still useful because they are simple yet effective [3]. These models have sparked the development of deterministic inventory models in a variety of directions. One important direction is dropping the assumption of constant demand that is rarely applicable in the real world. Some early examples include Wagner and Whitin [4], who proposed a procedure to find the optimal lot sizes and optimal ordering points under the assumption of time-varying discrete demand, and Donaldson [5], who proposed a solution procedure under the assumption of a linear time-varying demand operating over a finite planning horizon. Other directions include the assumptions of decaying items, lost sales and backlogged shortages, unit price subject to inflation, defective items, and recovery of used items.

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In the recent past, the sustainability agenda has become the focus of attention. Hence, researchers have been sparing more effort to incorporate sustainability into their inventory models. Sustainability is a consumption paradigm in which the current generation satisfies its needs without compromising the ability of the next generation to satisfy its needs. Hence, a natural way to incorporate sustainability into an inventory model is to recover used items instead of simply disposing of them. Half a century ago, Schrady [6] produced pioneering work in this direction. He considered an inventory system that carries service items and used items awaiting repair. Most of the used items are repaired but some are scrapped due to poor condition and must therefore be replaced by new items. Schrady gave formulas for the optimal repair size and optimal procurement size. Next, Richter [7] proposed an EOQ model with multiple procurement setups and multiple repair setups per cycle. The following authors have immediately extended this model. In Dobos and Richter [8], the conditions under which the optimal setup numbers can be found were given. In Dobos and Richter [9], the assumption of infinite procurement rate was relaxed to finite rate. They found that a policy of either pure procurement or pure repair is optimal. In Dobos and Richter [10], the assumption that all used items are reparable is relaxed. All these works assumed a constant return rate of used items. However, in reality, the return rate is not fixed and is influenced by different factors. Several works have explored a varying return rate, of which, three are surveyed as follows. In Saadany and Jaber [11], the purchasing price and acceptance quality level of used items are considered as factors affecting the return rate. The authors derived a cost function with the return rate as one of the decision variables. In Saadany et al. [12], used items that meet reparability requirements can be repaired a finite number of times rather than an unlimited number of times. In Dobos et al. [13], the limited reparability of used items is taken further – a used item is returned in a variable condition that is determined by the number of times the item underwent repair.

As part of the sustainability agenda, the concept of Circular Economy (CE) is being promoted as a replacement for the current largely linear economic model in which resources are consumed to make products that are eventually disposed as waste [14]. In contrast, CE is restorative and regenerative – it maximizes the reduction of waste and pollution, keeps materials circulating in the economy as long as possible to maximize value, and exploits renewable resources and energy as much as possible [14]. The economic benefit of implementing CE is estimated to be worth USD 700 billion globally for fast-moving consumer goods [14]. However, to implement CE effectively, effective methods to measure the degree of implementation are required. Linder et al. [15] proposed a transparent circularity indicator at product level. Saidani et al. [16] reviewed 55 sets of circularity metrics and proposed a method to find the appropriate indicators to meet specific requirements. In the same vein, Parchomenko et al. [17] reviewed 63 sets of CE metrics. CE has attracted the attention of policy makers in China [18]; Japan, the UK, France, Canada, the Netherlands, Sweden, and Finland [19]; and in the European Union [20]. In [18], the authors suggested that governments should lead the promotion of CE by imposing regulations, incentivizing cleaner technologies, and organizing appropriate education to improve awareness. The government plays an important role – government taxation, subsidies, and environmental and safety regulations can substantially affect the costs incurred by firms [21]. Besides governments and firms, CE has also attracted the attention of consumers. Studies have shown that information transparency affects consumer purchasing behavior in terms of being willing to pay steeper prices [22]. The use of appropriate labelling can shape consumer knowledge, purchasing behavior, and consumption patterns, herding the market to increase demand for quality in food products [23]. Additionally, studies on using ethical labels such as Fair Trade [24], organic food [25], charity-linked [26], and eco-friendly [27] have indicated that consumers mostly appreciate such labels and will respond positively to products that share such information.

Consumers are found to be less sensitive to price when considering products with ethical labels and are willing to pay higher prices to purchase such products compared to products without such labels [24, 28]. According to a survey [29], a third of the respondents stated that they would buy from sustainable brands if sustainability information were made transparent in the packaging and in marketing.

However, while there are opportunities to implement CE, industrial practitioners faced challenges that all but stopped them from orienting their businesses to become more circular [30]. A survey on 245 supply chains found that only a tenth of the respondents actively manage their carbon footprints and over a third are not aware of their carbon emission levels [31]. Firms are reluctant to transform because they perceive that it is risky and costly to do so, since it incurs additional costs, changes established practices, changes supply chain relationships, and adds additional uncertainties to demand. Therefore, it is desirable to demonstrate that the transformation is advantageous, and an effective way to do so is by demonstration using mathematical models. Hence, in Rabta [32], the classical EOQ model is modified by adding a circularity level index as a decision variable in addition to the classic order quantity. The circularity level index is measured as a number between 0 and 1, where 0 means minimal circularity adoption and 1 means maximal adoption. Rabta [32] assumed that the circularity index has a positive effect on the demand rate and a negative effect on the profit of selling the circular product. His results indicated that running an inventory policy using the optimal circularity level can increase profits significantly. Furthermore, if the firm fixes its desired level of circularity, optimal order quantities can be determined to maximize profits. Thomas and Mishra [33] implemented Rabta's circularity index approach on a two-echelon vendor-buyer supply chain in the plastic reforming industry. They included fixed carbon emission costs and fixed waste costs in their model. Khan et al. [34] also proposed an EPQ model that considers circularity index as a decision variable. Their work included fixed carbon emission costs for setup activities, which was absent in [33]. Hence, in this paper, we will apply Rabta's approach to the EPQ model with remanufacturing. Motivated by the above literature, we will consider a variable return fraction instead of a constant one. For simplicity, we assume that the return fraction is linearly increasing with circularity level. In other words, the circularity level of the inventory policy is dependent on the amount of used items that are remanufactured. Additionally, we will consider variable costs for managing carbon emissions from manufacturing, remanufacturing, and items storage, such that the carbon costs increase with the circularity level. The remainder of this paper is organized as follows: we provide the mathematical formulation of our proposed model in Section 2; we propose a solution procedure to find the optimal cycle length and optimal circularity index that maximizes the average profit in Section 3, and we provide four numerical examples to illustrate our model and draw some managerial insights in Section 4. Finally, the conclusion of the paper is given in Section 5.

2 Mathematical formulation

We consider a single-product single-location inventory system in which one manufacturing run and one remanufacturing run occur per cycle. The demand for the finished product is satisfied as much as possible by remanufacturing used items and by manufacturing new items to satisfy the remaining demand. The finished product is associated with a circularity index, denoted by ω ($0 \leq \omega \leq 1$), where 0 indicates minimal circularity adoption by the inventory operator and 1 indicates maximal adoption. We assume that the demand rate, the unit gross profit for manufacturing, the unit gross profit for remanufacturing, and the carbon emission management costs are functions of ω. To develop the model, we make the following assumptions:

- The inventory system is considered over an infinite planning horizon.
- The demand rate, $D(\omega)$, is a known function of ω.
- The manufacturing rate of new items is constant at P and the remanufacturing rate of used items is constant at R, where $P, R > D(\omega)$.
- The return rate of used items, $U(\omega)$, is a known function of ω, where $U(\omega) < D(\omega)$ for $0 \leq \omega \leq 1$.
- There is only one manufacturing run and one remanufacturing run per cycle.
- Shortages and excess inventory are prohibited.
- All used items are remanufactured as good as new.
- The following fixed costs are considered:
  - K, the total setup costs per cycle.
  - h_p, the unit holding cost of the finished product per unit time.
  - h_r, the unit holding cost of used items per unit time.
- The unit carbon emission management cost per unit time for inventory storage, $c(\omega)$, is a function of ω. This cost applies to both finished items and used items.
- The unit gross profit functions for remanufacturing and for manufacturing are denoted by $p(\omega)$ and $q(\omega)$, respectively.

We also use the following notations:

- T, the length of time of a cycle.
- T_1, the elapsed time in a cycle until the end of the remanufacturing run.
- T_2, the elapsed time in a cycle until the start of the manufacturing run.
- T_3, the elapsed time in a cycle until the end of the manufacturing run.
- Q_p, the manufacturing batch size.
- Q_r, the remanufacturing batch size.

The unit costs for making the finished product, by manufacturing and by remanufacturing, increase with the circularity index. This means that the higher the circularity index, the more expensive it is to manufacture and to remanufacture. We note that these unit costs also include the unit carbon emission management costs for manufacturing and for remanufacturing. Hence, assuming a fixed unit selling price, the unit gross profit for manufacturing and the unit gross profit for remanufacturing decrease with the circularity index. We propose to express this phenomenon by using the following exponential functions for the unit gross profits:

$$p(\omega) = p_0 - ae^{\alpha(\omega-1)} \tag{1}$$

$$q(\omega) = p_0 - be^{\beta(\omega-1)} \tag{2}$$

where p_0, a, b, α, and β are positive constants (see Fig. 1).

On the other hand, the demand rate for the finished product increases with the circularity index. As mentioned in the literature review, there is evidence to support such a demand pattern. It is reasonable to assume that the demand rate increases rapidly at first but the increase gradually slows down. We propose to express this phenomenon by using a logarithmic function for the demand rate:

$$D(\omega) = D_0 + c\ln(1 + \gamma\omega) \tag{3}$$

where D_0, c and γ are positive constants (see Fig. 2).

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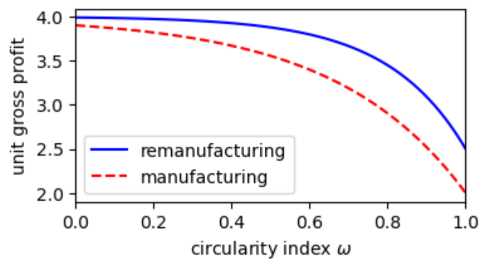

Fig. 1. Examples of the proposed decreasing unit gross profit functions. The unit gross profit for remanufacturing is always greater than that of manufacturing.

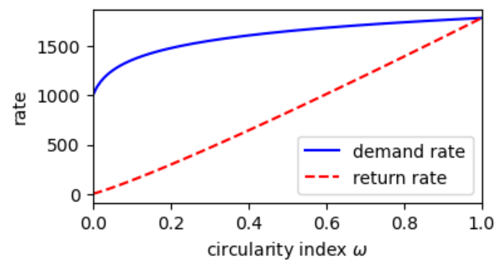

Fig. 2. Examples of the proposed increasing demand rate and the increasing return rate. The demand rate is always greater than the return rate.

We propose to use linearly increasing functions for the return fraction k $(0 \leq k \leq 1)$ and the inventory storage unit carbon emission management cost per unit time $c(\omega)$. For the former, the rationale is that when the circularity index is higher, the return rate should be higher as well to force a higher level of remanufacturing. For the latter, to achieve a higher level of circularity index would require more costs being incurred to manage the carbon emissions. Hence, we may write:

$$U(\omega) = kD(\omega) = (u_0 + x\omega)D(\omega) \tag{4}$$

$$c(\omega) = c_0 + y\omega \tag{5}$$

where u_0, x, c_0, and y are positive constants.

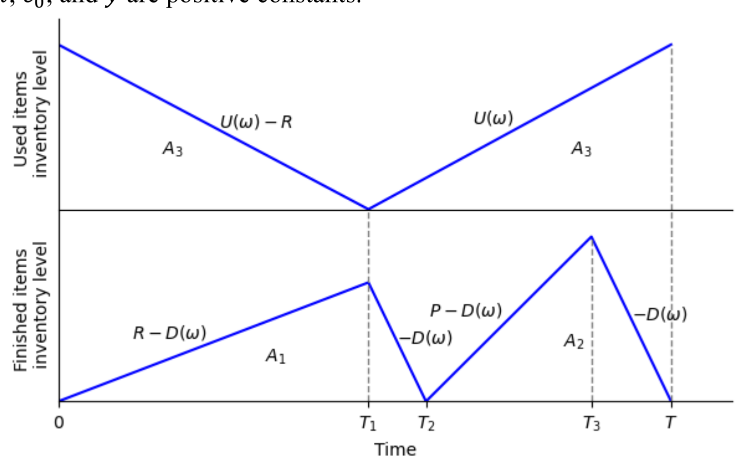

Fig. 3. An example of the movement of the inventory levels with respect to time.

Fig. 3 illustrates the inventory movements of the used items and the finished items over time in a cycle of duration T. Suppose that the circularity index has been fixed. First, consider the finished items inventory. At the start of the cycle (at time 0), the remanufacturing run commences and converts used items to finished items. Since $R > D(\omega)$, the remanufacturing run accumulates finished items over the interval $[0,\, T_1]$. Once the remanufacturing run ends at time T_1, the accumulated finished items are gradually consumed by demand over the interval $[T_1, T_2]$. Just when the finished items inventory is zero, the manufacturing run commences at time T_2 to supply the items to satisfy the remaining demand. Since $P > D(\omega)$, the manufacturing run also accumulates finished items over the interval $[T_2,\, T_3]$. Once the manufacturing run ends at time T_3, the accumulated finished items are gradually consumed by demand as well over the interval $[T_3, T_4]$.

Next, consider the used items inventory. Over the period $[0, T_1]$, the used items are gradually consumed by the remanufacturing run. Although used items are also collected over this period, $R > U(\omega)$ leads to a decrease in the inventory level. On the other hand, over the period $[T_1,\ T]$, used items are gradually collected at a rate of $U(\omega)$ units per unit time.

The remanufacturing run must satisfy all demand over the interval $[0, T_2]$ and all used items are remanufactured as good as new. Hence, we have

$$RT_1 = D(\omega)T_2 = U(\omega)T \tag{6}$$

which yields

$$T_2 = \frac{U(\omega)T}{D(\omega)} \quad \text{and} \quad T_1 = \frac{U(\omega)T}{R}. \tag{7}$$

The manufacturing run must satisfy all remaining demand over the period $[T_2, T]$. Hence, we have

$$P(T_3 - T_2) = D(\omega)(T - T_2) \tag{8}$$

which yields

$$T_3 = \frac{U(\omega)[P - D(\omega)] + [D(\omega)]^2}{D(\omega)P}T. \tag{9}$$

The total inventory cost per unit time, $TCUT$ is expressed as the sum of the unit time setup costs, unit time holding costs, and unit time carbon emission management costs for inventory storage. From Fig. 3, the areas of triangles A_1, A_2, and A_3 gives the amount of items held in storage over time. Hence, we can write

$$TCUT = \frac{K + \left[h_p + c(\omega)\right](A_1 + A_2) + [h_r + c(\omega)]A_3}{T}. \tag{10}$$

After some algebraic manipulation, we obtain

$$TCUT = \frac{K}{T} + \left\{\left[h_p + c(\omega)\right]\frac{P[R - D(\omega)][U(\omega)]^2 + R[P - D(\omega)][D(\omega) - U(\omega)]^2}{2RPD(\omega)} + [h_r + c(\omega)]\frac{U(\omega)[R - U(\omega)]}{2R}\right\}T. \tag{11}$$

For a fixed ω, it is clear that the value of T that minimizes $TCUT$ is given by the following formula:

$$T^* = \sqrt{\frac{2KRPD(\omega)}{h_1(\omega)\{Pf(\omega)[U(\omega)]^2 + Rg(\omega)[D(\omega) - U(\omega)]^2\} + h_2(\omega)Ph(\omega)U(\omega)D(\omega)}} \tag{12}$$

where $f(\omega) = R - D(\omega)$, $g(\omega) = P - D(\omega)$, $h(\omega) = R - U(\omega)$, $h_1(\omega) = h_p + c(\omega)$, and $h_2(\omega) = h_r + c(\omega)$. It is clear that $T^* > 0$.

We express the average profit as

$$\Pi(T,\omega) = \frac{p(\omega)Q_r + q(\omega)Q_p}{T} - TCUT(T,\omega),$$

$$\Pi(T,\omega) = p(\omega)U(\omega) + q(\omega)[D(\omega) - U(\omega)] - TCUT(T,\omega). \tag{13}$$

where the remanufacturing batch size $Q_r = U(\omega)T$ and the manufacturing batch size $Q_p = [D(\omega) - U(\omega)]T$. Our objective is to find the optimal values of the decision variables T and ω that maximizes average profit subject to the constraints $\omega \leq 1$ and $\omega \geq 0$. We note that the following cases are trivial:

- if the demand function and the unit gross profit functions are increasing functions of ω, then $\omega^* = 1$ for maximum average profit; and
- if the demand function and the unit gross profit functions are decreasing functions of ω, then $\omega^* = 0$ for maximum average profit.

Since ω^* is fixed at either 0 or 1, the T^* that maximizes the average profit can be calculated from (12). Hence, we will focus on the situation where the demand function is increasing with ω but the unit gross profit functions are decreasing with ω, which will result in $\omega^* \in (0,1)$.

3 Solution procedure

The Lagrangian of the constrained maximization problem

$$\max \Pi(T,\omega) \text{ subject to } \omega \leq 1, \omega \geq 0$$

is expressed as

$$L(T,\omega,\mu_1,\mu_2) = \Pi(T,\omega) - \mu_1(\omega - 1) + \mu_2\omega. \tag{14}$$

The optimal solution (T^*, ω^*) must satisfy the following Karush-Kuhn-Tucker (KKT) necessary conditions:

$$\frac{\partial L(T,\omega,\mu_1,\mu_2)}{\partial T} = -\frac{\partial TCUT(T,\omega)}{\partial T} = 0, \tag{15}$$

$$\frac{\partial L(T,\omega,\mu_1,\mu_2)}{\partial \omega} = F(\omega) - \frac{\partial TCUT(T,\omega)}{\partial \omega} - \mu_1 + \mu_2 = 0, \tag{16}$$

$$\mu_1(\omega - 1) = 0, \tag{17}$$

$$\mu_2\omega = 0, \tag{18}$$

$$\mu_1, \mu_2 \geq 0. \tag{19}$$

where

$$F(\omega) = p'(\omega)U(\omega) + p(\omega)U'(\omega) + q'(\omega)[D(\omega) - U(\omega)] + q(\omega)[D'(\omega) - U'(\omega)].$$

Complementary slackness (Conditions 17 to 18) leads to three possible cases, from which candidate solutions can be determined:

Case 1: $\omega^* = 0$. Then, (17) implies that $\mu_1 = 0$ and (12) gives the optimal T^*. This is the case of minimal remanufacturing. Then, (16) gives

$$\mu_2 = \left.\frac{\partial TCUT(T,\omega)}{\partial \omega}\right|_{\omega=0, T=T^*} - F(0)$$

This solution is feasible and optimal if $\mu_2 \geq 0$.

Case 2: $\mu_2 = 0$ and $\omega - 1 = 0$. Then, $\omega = 1$ and (12) gives the optimal T^*. This is the case of maximal remanufacturing. Then, (16) gives

$$\mu_1 = F(1) - \left.\frac{\partial TCUT(T,\omega)}{\partial \omega}\right|_{\omega=1, T=T^*}$$

This solution is feasible and optimal if $\mu_1 \geq 0$.
Case 3: $\mu_1 = 0$ and $\mu_2 = 0$. Then, (15) gives

$$\frac{\partial TCUT(T,\omega)}{\partial T} = 0, \tag{20}$$

and (16) gives

$$F(\omega) - \frac{\partial TCUT(T,\omega)}{\partial \omega} = 0. \tag{21}$$

Solving (20) and (21) simultaneously allow us to find the optimal T^* and ω^*. The value of ω^* is feasible if $0 < \omega^* < 1$. The value of T^* is always feasible since it is always positive.

Case 1 and Case 2 result in trivial candidate solutions. However, solving (20) and (21) simultaneously gives us a non-trivial candidate solution. Any candidate solution must first meet all the KKT conditions. Finally, the global maximal average profit is found by comparing the average profit of all the feasible candidate solutions. Fig. 4 shows an example of a global maximal average profit from Case 3.

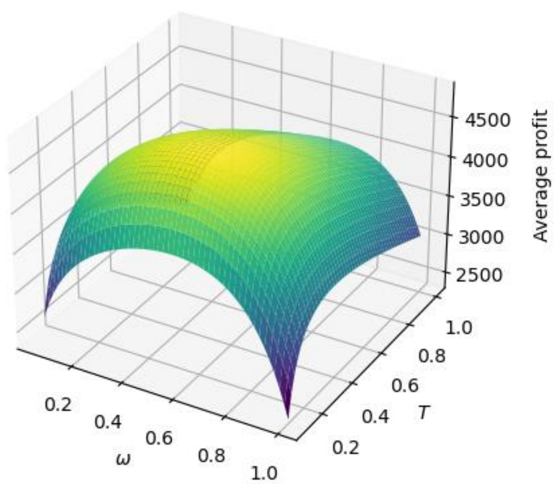

Fig. 4. Plot of the average profit function against T and ω.

4 Numerical examples

4.1 Example 1

In this example, the fixed costs are set as follows:

$$K = 200, \quad h_p = 6, \quad h_r = 4.$$

The unit time cost for managing carbon emissions of a single item in storage is set as follows:

$$c(\omega) = 1 + 0.5\omega$$

Additionally, the return rate, remanufacturing rate, and manufacturing rate are set as follows:

$$U(\omega) = \omega D(\omega), \quad P = 2500, \quad R = 2000.$$

The demand function is set as $D(\omega) = 1000 + 200\ln(1 + 50\omega)$. The unit gross profit function is set as $p(\omega) = 4 - 1.5e^{5(\omega-1)}$ for remanufacturing and $q(\omega) = 4 - 2e^{3(\omega-1)}$ for manufacturing. Both $p(\omega)$ and $q(\omega)$ are plotted in Fig. 1 with respect to ω. We assume that $q(\omega) < p(\omega)$ for all $\omega \in [0, 1]$, which means that it is always **less** profitable to manufacture than it is to remanufacture. Both are decreasing functions of ω, that is, the profitability from running these processes decreases as the level of circularity adoption increases. The optimal solution is $T^* = 0.3155$ and $\omega^* = 0.5737$, yielding the optimal average profit $\Pi^* = 4875.15$. The optimal remanufacturing quantity is $Q_r^* = 303.73$ and the optimal manufacturing quantity is $Q_p^* = 225.74$. For comparison, the two trivial feasible non-optimal solutions are:

- $\omega = 0$ and $T^* = 0.3086$, yielding the optimal average profit $\Pi^* = 2604.28$.
- $\omega = 1$ and $T^* = 0.4016$, yielding the optimal average profit $\Pi^* = 3469.81$.

In this example, setting $U(\omega) = \omega D(\omega)$ means that implementing $\omega = 0$ results in a pure manufacturing policy and implementing $\omega = 1$ results in a pure remanufacturing policy. Furthermore,

- the optimal circularity index is greater than 0 but less than 1 (we shall call this the hybrid policy),
- when it is less profitable to manufacture than it is to remanufacture, the optimal average profit from implementing circularity is higher than from implementing pure manufacturing or from implementing pure remanufacturing, and
- the implementation of 100% circularity (pure remanufacturing) returns lower optimal average profit than the hybrid policy, but it returns higher optimal average profit than implementing zero circularity (pure manufacturing).

From this example, the economic benefits of implementing circularity can be observed.

4.2 Example 2

In this example, all settings from Example 1 are retained except for the unit gross profit function for remanufacturing, which is set as $p(\omega) = 3 - 1.5e^{5(\omega-1)}$. Both $p(\omega)$ and $q(\omega)$ are plotted in Fig. 5 for reference. We assume that $p(\omega) < q(\omega)$ for all $\omega \in [0, 1]$, which means that it is always **more** profitable to manufacture than it is to remanufacture The optimal solution is $T^* = 0.3114$ and $\omega^* = 0.3055$ with average profit $\Pi^* = 4179.82$. For comparison, the two trivial feasible non-optimal solutions are:

- $\omega = 0$ and $T^* = 0.3086$ with average profit $\Pi^* = 2604.28$.
- $\omega = 1$ and $T^* = 0.4016$ with average profit $\Pi^* = 1683.44$.

In this example, under the assumption that it is more profitable to manufacture than it is to remanufacture, it can be observed that

- the optimal average profit from the hybrid policy is still higher than from implementing pure manufacturing, but
- compared to Example 1, the optimal circularity index is lower.

A crucial observation is that the economic benefits of implementing circularity can still be obtained despite manufacturing being more profitable. However, this conclusion should be interpreted cautiously because the higher demand from implementing circularity contributed to the higher profits and this phenomenon is a consequence of the assumption that demand is sensitive to the level of circularity implementation.

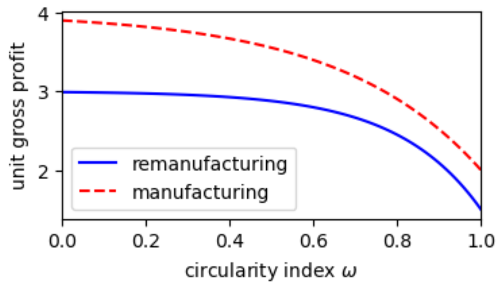

Fig 5. The case where the unit gross profit for remanufacturing is always less than that of manufacturing.

4.3 Example 3

In this example, all settings from Example 1 are retained except for the inventory storage unit carbon emission management cost per unit time, which is set as a range of linear functions (see the first column of Table 1). It can be observed that the both the y-intercept and slope of the linear function $c(\omega)$ increase by up to 300% from Example 1's $c(\omega)$. The first row of Table 1 gives the solution of Example 1 and we will refer to this solution as the base case. The percentage change in T^* from the base case's T^* is given by

$$PCT = \frac{T^* - T^*_{base}}{T^*_{base}} \times 100.$$

Similar formulas give the percentage change in ω^*, PCC, and the percentage change in Π^*, PCP.

Table 1. Change in the optimal solution when the carbon emission management cost function for inventory storage changes.

$c(\omega)$	T^*	PCT	ω^*	PCC	Π^*	PCP
$1 + 0.5\omega$	0.315515	0.00	0.573652	0.00	4875.149770	0.00
$1.4 + 0.7\omega$	0.302377	-4.16	0.571501	-0.37	4820.440858	-1.12
$1.8 + 0.9\omega$	0.290765	-7.84	0.569478	-0.73	4767.933209	-2.20
$2.2 + 1.1\omega$	0.280406	-11.13	0.567561	-1.06	4717.380772	-3.24
$2.6 + 1.3\omega$	0.271087	-14.08	0.565730	-1.38	4668.580325	-4.24
$3 + 1.5\omega$	0.262647	-16.76	0.563973	-1.69	4621.361699	-5.21

Table 1 shows that the optimal average profit is insensitive to the carbon emission cost. It decreases by a maximum of 5.21% for a 300% increase in the parameters of $c(\omega)$. The results also show that the optimal circularity index ω^* is insensitive since it decreases by a maximum of 1.69%. The optimal cycle time T^* is the most sensitive since it decreases by a maximum of 14.08%. An important insight that can be drawn here is that the optimal policy compensates for the increasing circularity dependent carbon emission cost by reducing the optimal cycle time while keeping the optimal circularity index relatively stable. This results in a relatively low decrease in the optimal average profit. Therefore, the economic benefits of implementing circularity can still be obtained despite having to incur costlier carbon emission costs.

4.4 Example 4

In this example, all settings from Example 1 are retained except for the demand function $D(\omega)$, which is set as a range of logarithmic functions (see the first column of Table 2). It can be observed that the both the values of the parameters c and γ decrease until 10% of the values in Example 1's $D(\omega)$. The first row of Table 2 gives the solution of Example 1 and we will refer to this solution as the base case.

Table 2 shows that the optimal average profit Π^* and optimal circularity index ω^* is sensitive to the demand function. They decrease by up to 45.27% and 63.31%, respectively, for an 90% decrease in the parameter values of $D(\omega)$. However, the optimal cycle time T^* is insensitive since it changes up to a maximum of 2.82%. An important insight that can be drawn here is that the optimal average profit and optimal circularity index are highly dependent on the slope of the demand curve. That is, the steeper is the slope, the higher is the optimal average profit and optimal circularity index. Therefore, the economic benefits of implementing circularity are highly dependent on a positive response of demand to circularity.

Table 2. Change in the optimal solution when the demand function changes.

$D(\omega)$	T^*	PCT	ω^*	PCC	Π^*	PCP
$D(\omega) = 1000 + 200\ln(1+50\omega)$	0.315515	0.00	0.573652	0.00	4875.149770	0.00
$D(\omega) = 1000 + 160\ln(1+40\omega)$	0.307047	-2.68	0.512210	-10.71	4223.909740	-13.36
$D(\omega) = 1000 + 120\ln(1+30\omega)$	0.308562	-2.20	0.450583	-21.45	3651.321335	-25.10
$D(\omega) = 1000 + 80\ln(1+20\omega)$	0.315376	-0.04	0.380348	-33.70	3163.928823	-35.10
$D(\omega) = 1000 + 40\ln(1+10\omega)$	0.323261	2.46	0.284177	-50.46	2789.238807	-42.79
$D(\omega) = 1000 + 20\ln(1+5\omega)$	0.324411	2.82	0.210490	-63.31	2668.384447	-45.27

Conclusion

In this paper, an Economic Production Quantity inventory model that incorporates remanufacturing in a Circular Economy is proposed. The cycle length and a numerical indicator for the level of implementation of circularity are considered as decision variables. The rationale for considering the circularity level indicator (circularity index) as a decision variable is the assumption that the implementation of circularity by the manufacturer at a variable level (measured as a number between 0 and 1) affects the demand for the finished product and the profit from selling the product. We assume that as circularity increases, demand increases as well but unit profit decreases. We propose a solution procedure to find the optimal policy and give four numerical examples to derive managerial insights.

Our results provide preliminary evidence that implementing circularity in a manufacturing-remanufacturing business is economically justified. In the numerical examples that were given, where (1) the unit profit for remanufacturing is assumed to be always less than the unit profit for manufacturing, and (2) the unit cost for carbon emissions from inventory storage is increasing, we found that it is still optimal to implement circularity rather than not. However, getting economic benefits from implementing circularity is highly dependent on the demand being an increasing function of circularity.

One further research direction is to investigate other factors that cause the optimal circularity level to change, such as government incentive. Additionally, it would be desirable to provide the second order conditions under which the optimal solution of the proposed model exists and is unique.

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