Risk quantification using skewed distributions: An application to the South African Financial Index (J580)

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Abstract. This study assesses the reproducibility of a recent publication on the risk quantification of the South African Financial Index (J580) using skewed distributions. That is, four skewed distributions (Burr, exponential, gamma and Weibull) are fitted to the returns (split into losses and gains) of the J580 dataset. In this paper, we redo the analysis in an effort to highlight some of the quantifiable differences in the values of the descriptives, goodness-of-fit and risk measures for all four distributions. In addition, other goodness-of-fit tests are computed for all four distributions to check consistency, and based on this extension, it is observed that the Weibull is a better model for gains due to a majority of the goodness-of-fit test inferring that and yields better risk measures. Finally, the Burr distribution is recommended for losses as it better captures the heavy tail of the loss returns.

1 Introduction

Investment portfolios and risk management strategies play a pivotal role in financial decision-making, particularly in assessing potential losses and gains [1]. In the financial sector, there are four main classes of losses: (i) high frequency and high severity, (ii) high frequency and low severity, (iii) low frequency and low severity, (iv) low frequency and high severity. It is important to note that [2] stated that high frequency claims that are high in severity is not feasible/implausible in the financial industry. Next, high frequency with low severity claims as well as low frequency with low severity claims are unimportant and can often be both prevented. However, low frequency with high severity claims tends to cause the most devastating losses, with the best example being the 1995 Barings Bank's collapse (also portrayed in the movie Rogue Trader). For other well-known examples of company's poor risk management, see Chapter 20 of [1] and Chapter 1 of [2]. Additional examples of poor risk management in the South African context are Steinhoff, Hullett, Venda Building Society (VBS) mutual bank, Eskom, South African Airways (SAA) and in 2022, the Capitec bank computer systems were down during a peak period of the month [3]. Another best example is the South African Post Office bank (called PostBank) where there have been numerous reported cases of internal staff defrauding millions of South African rands (currency) due to poor risk management framework therein. Thus, the latter has led to catastrophic debt that

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has put the Post Office and PostBank to a blink of collapse and currently in the year 2024 asking the South African government for a bailout to avert total collapse. Therefore, overall, banks must be extremely cautious of these different types of losses as they tend to cause bankruptcy in many financial institutions.

Low frequency with high severity losses can be extreme in size when they are compared to the rest of the data. If you construct a skewed to the right histogram of the data distribution, the low frequency with high severity losses events would be placed in the far-right end, which is often referred to as 'tail event'. Due to loss data exhibiting such tail events, it is said that the data is heavy tailed. Such heavy-tailed distributions have a higher probability of encountering extreme events which pose distinct challenges in the domain of risk analysis. Model uncertainty arises as traditional models may inadequately capture extreme events which affects risk predictions. A heavy-tailed data distribution is characterized by an increased probability of extreme events, deviating from the characteristics of a normal distribution. Heavy-tailed distributions exhibit a slower decay in tail probabilities (compared to normal distributions) which makes extreme events more probable and accounted for. Heavy tailed distributions can be used to assess several risks which are mentioned in [2]. These include credit risk, market risk, operational risk, and many others.

Upon reading [4], what stands out is that the authors considered four distributions (i.e., Burr, exponential, gamma and Weibull) to separately analyse the goodness-of-fit for losses and gains returns of the South African Financial Index (J580). [4] concluded that the best distributions to fit were the Burr (for losses returns) and exponential (for gains returns) distributions. Thereafter, they computed the corresponding risk metrics using the 'single best model' approach where the best-fitting model is selected and assumed to be correct based on the AIC and BIC criterion. In [4], the risk metrics were computed assuming that the Burr and exponential distributions are the best fitting models under J580's losses and gains observations, respectively. The main objective of this paper is to re-evaluate the four distributions fitted by [4] and highlight where we believe the authors may have made an error as well as fill in the gaps where the authors opted not to do computations – for the sake of completeness. For other publications that discuss heavy-tailed distributions in a financial context, see [5-12].

A brief theoretical background of the four distributions used by [4] and their sensitivity analysis is provided in Section 2. Data and the corresponding analysis are discussed in Section 3. Future research ideas pertaining to the given data are provided in Section 4. Section 5 provides the concluding remarks.

2 Methodology

2.1 Properties of the distributions

While overall there are many basic statistical distributions, we intend to limit our discussion to the four non-Gaussian statistical distributions for skewed to the right data used in [4] that are summarized in Table 1. Their corresponding properties, i.e. parameter(s) domain, probability density function (pdf) and cumulative distribution function (cdf) are also given in Table 1.

Based on the three illustrations of sensitivity analysis in Figure 1 for the Burr distribution, firstly, it is observed that for a fixed shape parameter (γ) and a fixed scale parameter (θ), the smaller the shape parameter (α), the heavier is the tail. Secondly, for a fixed α and θ, the smaller γ, the heavier is the tail. Thirdly, for a fixed α and γ, the larger the θ, the heavier is the tail. Similarly, in Figures 2 to 4, the sensitivity analysis of the exponential, gamma and Weibull distributions are illustrated, respectively. Firstly, for the exponential distribution, the

 https://doi.org/10.1051/itmconf/20246701009

larger the scale parameter (θ), the heavier is the tail. Secondly, for the gamma distribution, with a fixed scale parameter (θ), the larger the shape parameter (α), the heavier is the tail; however, for a fixed α, the larger the θ, the heavier is the tail. Finally, for the Weibull distribution, with a fixed scale parameter (θ), the smaller the shape parameter (τ), the heavier is the tail; however, for a fixed τ, the larger the θ, the heavier is the tail. Figures 1 to 4 investigates how varying specific parameters of a distribution can affect the heaviness of the corresponding tail area.

Table 1. Distributions and their properties.

Distribution	Parameters	PDF	CDF
Burr	$\alpha > 0, \gamma > 0, \theta > 0$	$\dfrac{\alpha\gamma\left(\frac{x}{\theta}\right)^{\gamma}}{x[1+\left(\frac{x}{\theta}\right)^{\gamma}]^{\alpha+1}}$	$1-u^{\alpha},\quad u=\dfrac{1}{1+\left(\frac{x}{\theta}\right)^{\gamma}}$
Exponential	$\theta > 0$	$\dfrac{e^{-\frac{x}{\theta}}}{\theta}$	$1-e^{-\frac{x}{\theta}}$
Gamma	$\alpha > 0, \theta > 0$	$\dfrac{\left(\frac{x}{\theta}\right)^{\alpha} e^{-\frac{x}{\theta}}}{x\Gamma(\alpha)}$	$\Gamma\left(\alpha;\dfrac{x}{\theta}\right)$
Weibull	$\tau > 0, \theta > 0$	$\dfrac{\tau\left(\frac{x}{\theta}\right)^{\tau} e^{-\left(\frac{x}{\theta}\right)^{\tau}}}{x}$	$1-e^{-\left(\frac{x}{\theta}\right)^{\tau}}$

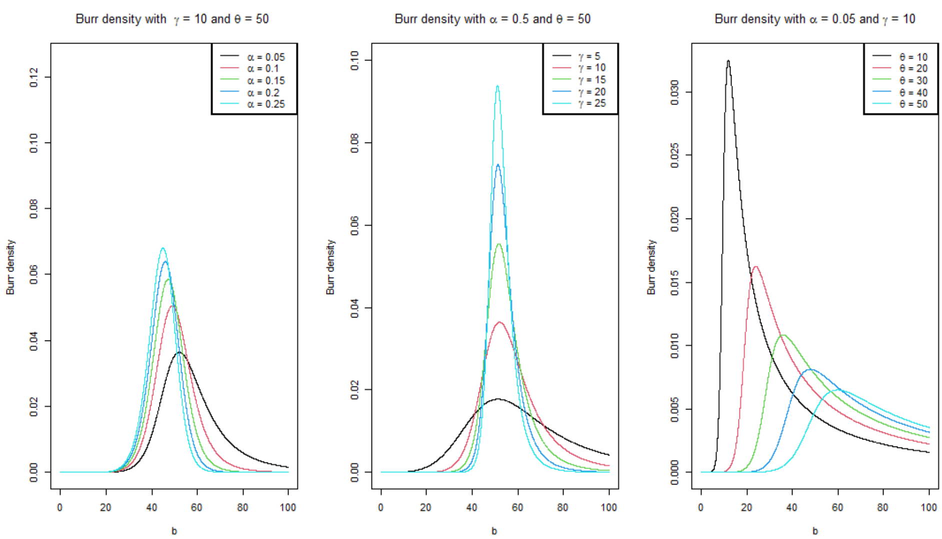

Fig. 1. Sensitivity analysis of the Burr distribution with varying parameters.

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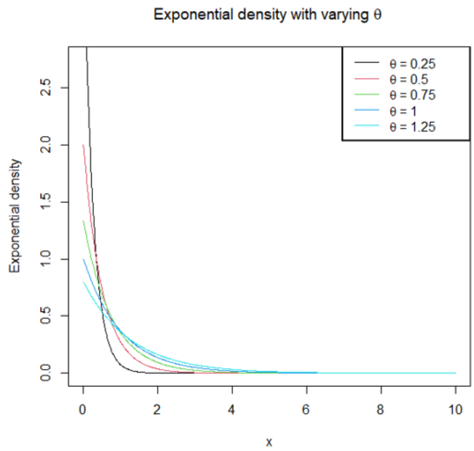

Fig. 2. Sensitivity analysis of the exponential distribution with varying parameter.

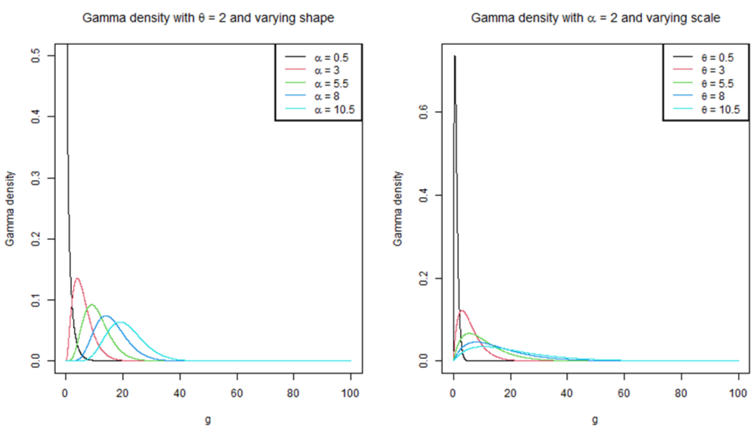

Fig. 3. Sensitivity analysis of the gamma distribution with varying parameters.

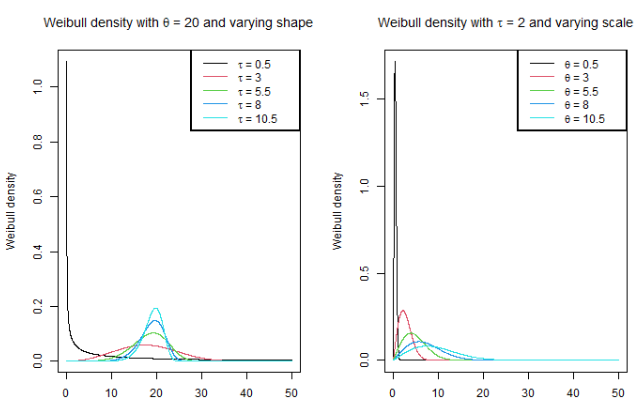

Fig. 4. Sensitivity analysis of the Weibull distribution with varying parameters.

2.2 Parameter estimation and goodness-of-fit

The parameters are estimated by maximum likelihood method. If $\boldsymbol{x} = \langle x_1, x_2, \dots, x_n \rangle$ is a random sample of n observations, then the likelihood function is $L(\theta) = \prod_{i=1}^{n} f(x_i)$ and the log-likelihood function is defined as $\ell(\theta) = \ln L(\theta) = \ln \prod_{i=1}^{n} f(x_i) = \sum_{i=1}^{n} \ln f(x_i)$.

The first three commonly used model selection criteria or goodness-of-fit statistics (to assess how closely the observed data mirrors the fitted distribution) are: Kolmogorov-Smirnov (KS), Cramer-von Mises (CvM), and the Anderson-Darling (AD). The KS, CvM and AD test statistics are computed by the following expressions: KS $= \max_x |F_n(x) - F(x)|$, CvM $= n \int \left(F_n(x) - F(x)\right)^2 f(x)dx$ and AD $= n \int \frac{\left(F_n(x)-F(x)\right)^2}{F(x)\left(1-F(x)\right)} f(x)dx$, where n is the number of observations, $F_n(x)$ is the empirical cdf, $F(x)$ is the theoretical (fitted) cdf and $f(x)$ is the corresponding pdf. The KS statistic computes the maximum absolute vertical differences between the empirical cdf and the theoretical cdf. KS is described as a statistic which captures differences between the middle of the data and the proposed model [2]. The CvM statistic considers the integral of the squared differences between the empirical cdf and the theoretical cdf rather than just considering differences between points; while the AD statistic places emphasis on the tails of the distribution, i.e., where $F(x)$ or $1 - F(x)$ are small [2].

Let $\ell(\theta)$ denote the maximised log-likelihood function of a model, then the negative log-likelihood (NLL) is defined as NLL $= -\ell(\theta)$. The AIC is defined as AIC = 2NLL + $2p$, and the BIC is defined as BIC = 2NLL + $p \log(n)$, where p is the number of parameters or degrees of freedom and n is the number of observations.

2.3 Risk measure

A risk measure, which can also be referred to as a key risk indicator [13], is defined as a mathematical function of the probability of an event and the consequences of that event. It is important to realize that decision-making regarding risks is very complex and risk measures are essential for actuaries, investors, and financial institutions to make informed decisions about investments and risk management strategies. Two main risk measures are considered, i.e., value-at-risk (VaR) and tail VaR (TVaR). VaR can be interpreted as the lower bound for the capital required to avoid insolvency, whereas TVaR can be interpreted as the expected value of total loss, given that the loss exceeds VaR and thus, TVaR tends to be more attractive for an organisation with many business lines [13]. Note that TVaR is also known as expected shortfall (ES), a term used in [4].

Let $F(\cdot)$ and $F^{-1}(\cdot)$ denote the cdf and inverse cdf of a continuous random variable X, respectively. Then, the VaR of X at a $100p\%$ security level denoted by $\mathrm{VaR_p}(X)$, is the $100p\%$ quantile of F such that

$$P\left(X < \mathrm{VaR_p}(X)\right) = p, \qquad F^{-1}(p) = \mathrm{VaR_p}(X).$$

The ES of X at a $100p\%$ security level represents the average of all VaR values exceeding security level, p, such that

$$ES = \frac{1}{1-p} \int_p^1 \mathrm{VaR_u}(X)du = \mathbb{E}\left[X | X > \mathrm{VaR_p}(X)\right].$$

Finite values of ES can only be obtained if the first moment of the distribution of X exists.

3 Data analysis

3.1 Data and software

We will be using the monthly data based on the South African Financial Index (denoted by J580). The dataset has been used in other publications, for example, [4], [14-16]. The portion of the J580 dataset we are interested in is for the period June 1995 to January 2018 (this is the period used by [4]), and it consists of 272 observations. The J580 index is one of the three main sub-indices of the South African All Share Index (ALSI) and it consist of the banking, insurance and securities industries [4]. The total economic growth relies heavily on the financial sector as it has a significant majority of the tertiary/college educated work force in South Africa and sponsors financially many community-based projects as well as sporting activities. Note that the daily J580 index is also available publicly online on Yahoo Finance.

For the purpose of the data analysis, the open-source R software is used, i.e. R version 4.2.2 and R Studio version 2023.6.0.421 (codes can be requested from the authors). The R add-on package **moments** built by [17] is used, for instance, to compute the skewness and the kurtosis of a sample data. For graphics like histograms and box plot, the package **graphics** is used. Other very important packages to be used are **actuar** (contains the Burr distribution) and **stats** (contains the exponential, gamma and Weibull distributions). The **actuar** add-on package was developed by [18] but the R package **stats** is part of R base. The *fitdist*() and *gofstat*() functions from the add-on package **fitdistrplus** by [19] are used to obtain maximum likelihood estimates of the parameters and to compute the goodness-of-fit measures, respectively. The integral in ES is evaluated using the *integrate*() function in R package **stats**.

3.2 Descriptives

Let Y_t be the monthly index for the period June 1995 to January 2018; hence, the corresponding monthly returns (r_t) are computed by

$$r_t = \ln(Y_t/Y_{t-1}).$$

Note that some of the returns are positive (termed gains) and others are negative (termed losses) - these are separately analysed in this study and also in the study by [4], where each of the losses (negative returns) are redefined as positive values by introducing absolute values thereof: $l_t = |r_t|$ and thus are positive.

Table 2. Descriptives of the J580 returns by [4] (2nd column) and this paper (last 3 columns).

Descriptive	Combined by [4]	Combined	Losses	Gains
Mean	**-0.835**	0.00835	0.04123	0.04223
Median	**-0.0101**	0.01015	0.02867	0.03188
Maximum	**0.51195**	0.21652	**0.**51195	0.21652
Minimum	**-0.21652**	-0.51195	0.00104	0.00028
Variance	**0.36512**	0.00365	0.00299	0.00128
Std deviation	0.06042	0.06042	0.05468	0.03577
Skewness	**2.194**	-2.2065	6.05210	1.50399
Kurtosis	**19.440**	22.6061	51.28085	6.44453
Count	271	271	110	161

Table 2 shows a comparison of the descriptives of the J580 data from our computations and that of [4]. It is observed that while [4] conducted the data analysis based on separate gains and losses, however, descriptives were computed for the combined data (second column on Table 2) – which is a bit confusing. Also, some of the values do not make sense, for example, the (standard deviation)^2 ≠ variance since $(0.06042)^2 = 0.003651 \neq 0.3651$ as given in [4]. However, here we compute the descriptives for combined returns (on the 3rd column, for comparative purpose), losses and gains (shown on the last 2 columns) on Table 2. The values in boldfaced from 2nd column are not the same as those we obtained in the 3rd column for combined negative and positive returns, except the standard deviation which is the same. Next, the descriptives of the combined (with losses negative) and separate (losses and gains) histograms are depicted in Figures 5 and 6, respectively. Overall, from Figure 5, the data is skewed to the left (negatively skewed) but separated with absolute losses and gains, it is skewed to the right (positively skewed), see Figure 6. Consequently, from Table 2, it is clear that most of the descriptives by [4] are incorrect.

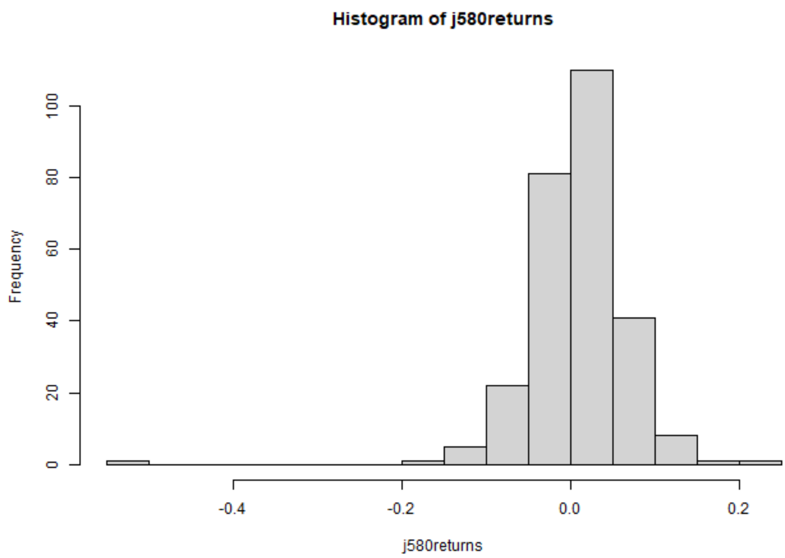

Fig. 5. Histograms of the combined J580 returns.

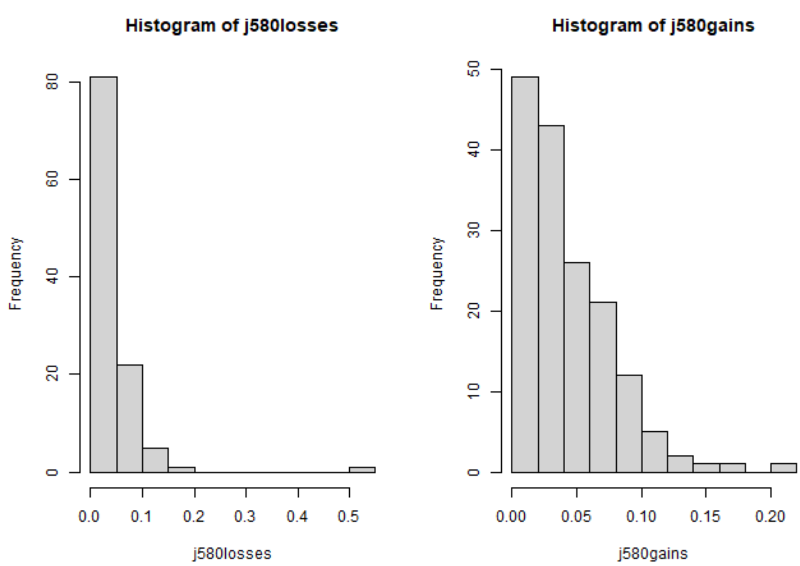

Fig. 6. Histogram of the J580 absolute losses (on the left) and the gains (on the right).

3.3 Analysis

The parameter estimates and standard errors are the same as those obtained in [4], for the sake of brevity, they are not shown here. While [4] computed AIC and BIC only for the goodness-of-fit evaluation, in Tables 3 and 4 we present these as well as the KS, AD, CvM and NLL for the losses and gains, respectively. Note that the AIC and BIC we obtained are equal to those of [4]. The boldfaced values in Tables 3 and 4 correspond to the distribution that has the lowest goodness-of-fit measure. As can be seen in Table 3, it makes logical sense to pick Burr distribution as a better fit for losses as done by [4]. However, in Table 4, it seems as though the Weibull would be a better competitor over the chosen exponential distribution in [4]. It is understandable why [4] utilised the exponential as the best fit due to principle of parsimony, note though, in the modern day of technology, accuracy needs to be prioritized over parsimony (thus, in this study we prefer the Weibull over the exponential as the latter is not as flexible as the Weibull, fewer parameters).

Table 3. Goodness-of-fit test for losses of J580 returns.

Distribution	KS	AD	CvM	NLL	AIC	BIC
Burr	**0.0377**	**0.1943**	**0.0241**	247.2442	**-488.4884**	**-480.3870**
Exponential	0.0720	1.0285	0.1252	**240.7529**	-479.5058	-476.8054
Gamma	0.0616	0.5299	0.0697	241.7943	-479.5886	-474.1877
Weibull	0.0638	0.8362	0.0993	240.8620	-477.7239	-472.3229

Table 4. Goodness-of-fit test for gains of J580 returns.

Distribution	KS	AD	CvM	NLL	AIC	BIC
Burr	0.0610	1.1541	0.1581	**341.8976**	-677.7952	-668.5697
Exponential	0.0810	1.3320	0.2442	348.5136	-695.0273	**-691.9459**
Gamma	0.0498	0.6122	0.0860	349.8089	-695.6179	-689.4550
Weibull	**0.0375**	**0.4368**	**0.0499**	350.5195	**-697.0390**	-690.8762

Unlike in [4], in this research work, the fit of the theoretical model is also assessed by comparing the empirical risk estimates to the theoretical risk estimates. The CDF of the empirical distribution is computed by $F_n(x) = \frac{1}{n}\#\{i: x_i \leq x\}$, where # denotes the number of observations $\leq x$, and n is the total number observations in the sample. This is done by computing the percentage deviation as follows: %deviation VaR = $\frac{(\mathrm{VaR_{Theoretical}} - \mathrm{VaR_{Empirical}})}{\mathrm{VaR_{Empirical}}}$ and %deviation ES = $\frac{(\mathrm{ES_{Theoretical}} - \mathrm{ES_{Empirical}})}{\mathrm{ES_{Empirical}}}$, where theoretical implies either Burr, exponential, gamma or Weibull. This is important because underestimating the risk measures may result in under-reserving - which may lead to insolvency, i.e., not enough capital to cover future claims. Overestimating the risk measures may result in over-reserving - which may affect the profitability of the insurer due to fewer funds available for investment purpose.

Firstly, from the last row of Tables 5 and 6, the values from [4] are different from the ones we obtained from our computations of the corresponding distributions (i.e. only the Burr was computed for losses and only the exponential for gains). Secondly, from Table 5, it is observed that the VaR for losses at 95% and 99% confidence intervals have higher values than the empirical distribution; however, the ones for 99.5% are lower than those estimated by the empirical distribution. Thirdly, the ES from the theoretical distributions at all considered intervals are lower than the ones estimated with the empirical distribution. Fourthly, to interpret the VaR and ES for Burr distribution under losses is as follows: the loss returns are not expected to go beyond 11.51% (VaR=0.1151) at a 95% confidence level which is 7.0% higher than the empirical distribution one. However, if it goes beyond 11.51%, it will average 18.12% (ES=0.1812) at a 95% confidence level which is 4.4% lower than the empirical distribution one. The other values in Tables 5 and 6 are interpreted in a similar manner. Unlike losses, the gains in Table 6 have VaR and ES values for the theoretical distributions that are higher than the empirical one. Finally, from Table 5, in the different confidence levels, the Burr distribution seems to provide most results that are not too far from the empirical results, thus would be recommended as the best theoretical distribution. However, from Table 6, the exponential distribution (suggested by [4]) does not appear to have the lowest differences from the empirical results, in fact it is the third best. The Weibull distribution that we identified as the most ideal in terms of the goodness-of-fit seems to be the most appropriate here.

Table 5. Risk measures for losses of J580 returns and the % deviation from empirical distribution.

Distribution	$VaR_{0.95}$	$VaR_{0.99}$	$VaR_{0.995}$	$ES_{0.95}$	$ES_{0.99}$	$ES_{0.995}$
Empirical	0.1076	0.1545	0.3181	0.1895	0.3341	0.5119
Exponential	0.1235 (14.8%)	0.1899 (22.9%)	0.2184 (-31.3%)	0.1647 (-13.1%)	0.2311 (-30.8%)	0.2596 (-49.3%)
Gamma	0.1160 (7.8%)	0.1738 (12.5%)	0.1985 (-37.6%)	0.1519 (-19.8%)	0.2093 (-37.4%)	0.2339 (-54.3%)
Weibull	0.1212 (12.6%)	0.1838 (19.0%)	0.2106 (-33.8%)	0.1600 (-15.6%)	0.2223 (-33.5%)	0.2489 (-51.4%)
Burr	0.1151 (7.0%)	0.2113 (36.8%)	0.2680 (-15.7%)	0.1812 (-4.4%)	0.3158 (-5.5%)	0.3961 (-22.6%)
Burr from [4]	**0.1265**	**0.1945**	**0.2237**	**0.1621**	**0.2348**	**0.2648**

Table 6. Risk measures for gains of J580 returns and the % deviation from empirical distribution.

Distribution	$VaR_{0.95}$	$VaR_{0.99}$	$VaR_{0.995}$	$ES_{0.95}$	$ES_{0.99}$	$ES_{0.995}$
Empirical	0.1020	0.1572	0.1827	0.1385	0.1954	0.2165
Exponential	0.1265 (24.0%)	0.1945 (23.7%)	0.2237 (22.4%)	0.1687 (21.8%)	0.2367 (21.1%)	0.2659 (22.8%)
Gamma	0.1219 (19.5%)	0.1830 (16.4%)	0.2091 (14.4%)	0.1598 (15.4%)	0.2205 (12.8%)	0.2465 (13.9%)
Weibull	0.1157 (13.4%)	0.1688 (7.4%)	0.1910 (4.5%)	0.1486 (7.3%)	0.2003 (2.5%)	0.2220 (2.5%)
Burr	0.1157 (13.4%)	0.1689 (7.4%)	0.1910 (4.5%)	0.1486 (7.3%)	0.2004 (2.6%)	0.2221 (2.6%)
Exponential [4]	**0.1121**	**0.1628**	**0.1928**	**0.1463**	**0.2010**	**0.2317**

4 Future research ideas

Given the discussion above, there is a need for the J580 dataset to be properly analysed fully. In this section, we provide some suggestions that we believe are appropriate to follow. Firstly, given the combined losses and gains histogram in Figure 5, we suggest instead of splitting the data into (positive) losses and gains as in this study and in [4] to instead fit approximately symmetric distributions with flexible tails and compare their performance with the normal distribution. The competing distributions would include Laplace, scaled t, t, generalized error, skew-normal, skew t, generalized hyperbolic, etc.

Secondly, examining the mean excess plot (a powerful tool for narrowing down the possible model/distribution to be fitted) in Figure 7, it is obvious that both losses and gains show mainly two different behaviours and thus, as per discussion by [20] can be best modelled by a combination of at least two distributions, for example, the composite or mixture distribution. More specifically, using page 27 of [21], it is easy to see from Figure 7 that roughly the losses may have a mixture of the exponential and (Pareto / lognormal), while the gains may have a mixture of (uniform / Weibull [with $\tau > 1$]) and (Pareto / lognormal / Weibull [with $\tau < 1$]).

Thirdly, given that [4] only considered four distributions while [22] considered nineteen distributions, it would be recommended to consider all nineteen distributions which can be classified into the following *three* categories:

(i) *The transformed beta family* (9 in total)

- 4-parameter: Transformed Beta distribution.
- 3-parameter: Generalized Pareto, Burr and Inverse Burr distributions.
- 2-parameter: Pareto, Inverse Pareto, Loglogistic, Paralogistic, & Inverse Paralogistic distributions.

(ii) *The transformed gamma family* (8 in total)

- 3-parameter: Transformed Gamma and Inverse Transformed Gamma distributions
- 2-parameter: Gamma, Inverse Gamma, Weibull and Inverse Weibull distributions.
- 1-parameter: Exponential and Inverse Exponential distribution.

(iii) *Other distributions* (2 in total)

- 2-parameter: Lognormal distribution.
- 2-parameter: Inverse Gaussian distribution.

Finally, for more research ideas on how to further or better analyse the J580 dataset, the reader is referred to [20]'s concluding remarks section where a number of research ideas are listed.

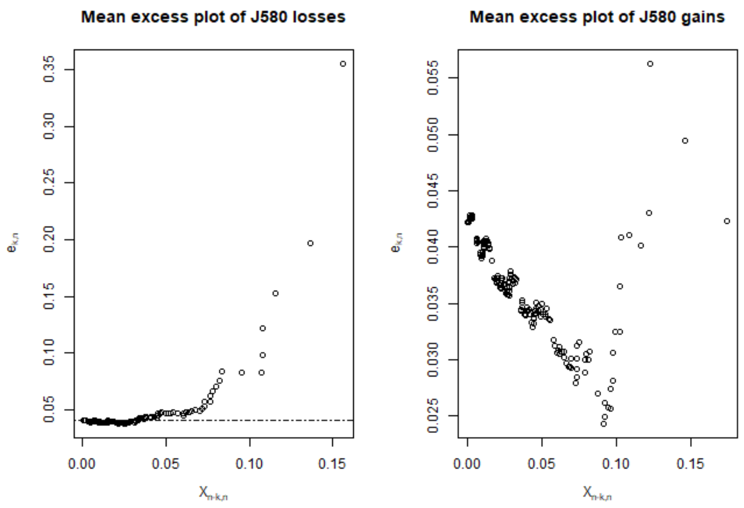

Fig. 7. Mean excess plots of the J580 absolute losses (on the left) and the gains (on the right).

5 Concluding remarks

The main purpose of this research is to emphasise the importance of reproducibility of novel research. Given that we obtained a large portion of different values for descriptives as compared to those reported in [4], we reevaluated the goodness-of-fit as well as risk measures. In short, we extended some of the analysis to provide correct and more in-depth analysis for the South African Financial Index (J580). The conclusion overall is similar with respect to the Burr distribution being the best out of the four fitted distributions when we consider loss returns; however, we showed a strong case for the Weibull to be more appropriate for the gain returns instead of the exponential distribution as suggested by [4]. Overall, the J580's expected losses have a higher likelihood of extreme events as they are recommended to be modelled by the much heavier-tailed Burr distribution; however, the J580's expected gains have a relatively lesser heavy-tailed pattern as it is recommended to be modelled by the Weibull distribution. Finally, we presented a number of future research ideas that can be persuaded for the J580 dataset.

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