Performance of Brownian-motion Process Generated Universal Portfolio in Times of COVID-19 Pandemic

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Abstract. The universal portfolio is a portfolio investment strategy which theoretically achieves good return. Brownian motion is a stochastic process which is heavily applied in various financial derivative pricing. The goal of this study aims to investigate the performance of finite order Brownian motion process generated universal portfolio (BMPGUP) using 3 independent stocks during COVID-19 pandemic. The research will include 3 daily traded stocks from respective countries, Malaysia KLCI, Singapore STI and Thailand SET 50 from January 2020 to June 2022. The performance is benchmark against Equally Weighted portfolio (EWP). According the investigation, BMPGUP performed better than the benchmark (compare with respective listed index within each country), especially in term of Sharpe ratio and Sortino ratio. This suggests that BMPGUP is a good alternative strategy for investors adopting the universal portfolio strategies.

1 Introduction

An investment portfolio is a combination of financial assets such as stocks, bonds, funds, cash, etc. Markowitz's Modern Portfolio Theory (MPT) [1] is a classic, aiming to construct a portfolio that maximizes mean return given a specific level of risk. MPT utilizes the mean return, the volatility, and the correlation of the financial assets as weight vectors to cater to different investor risk appetites and is rebalanced at the beginning of each trading period.

In contrast to MPT, Cover [2] introduce the concept of universal portfolio. The Universal Portfolio by Cover is an investment strategy that involves an algorithm that uses a computational procedure to predict the future of a stock market based on its past data. However, universal portfolio needs to be readjusted and rebalanced daily, as it is not same as the constant rebalance portfolio that maintain the same proportion of capital among a set of stocks daily. The universal portfolio that was pioneered by Cover keeps the highest return investments near the centre of gravity of the constant rebalances portfolios. Extensions of the study by Cover, Cover and Ordentlich [3] have employed both distribution and side information in the universal portfolio, demonstrating outperformances compared to buy-and-

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hold strategy. The strategy proposed by Bhatt et al. [4] achieved the result similar to best constant rebalanced portfolio after the study extended on work done by Cover [3].

To overcome the computational complexity of Cover's universal portfolio strategy, Tan [5] introduced the finite order probability distribution generated universal portfolio. In this paper, we introduced a finite order Brownian-motion Process Generated Universal Portfolio (BMPGUP) and using the most recent trading days. This strategy, comparable to the best constant rebalanced portfolio, simplifies the computational challenges of Cover's universal portfolio. Pang et.al [6] evaluated the performance of finite order BMPGUP in Malaysia stock market from 2000 to 2015 with different length of investment period and the results proved to be successful and obtained good returns.

Various portfolio performance measurements, including CAPM [7], Sharpe ratio [8] and Sortino Ratio [9], are proposed to assess portfolios relative to different risk dimensions and objectives. The Sharpe ratio assesses consistency and risk-reward relationship, while Sortino ratio considers downside risk of a portfolio performance.

According to World Health Organization (WHO), the COVID-19 pandemic was started on 11^{th} March, 2020. The first confirmed case of COVID-19 was originated from Wuhan City, China at December, 2019. Since then, the disease has spread across 210 countries and have infected over 560 million people as of today. After the first few breakouts in numerous cities, many financial analyst and reports predict a catastrophic impact of the COVID-19 crisis will have the same or worse than the Great Depression during WW1. Geopolitical instability, inflation, supply-chain disruption, and energy price are threats that were the most concerned by majority as most economic activities are forced to shut down.

Investment in stock market is very volatile these days due to uncertainty and ongoing economic recession, especially during COVID-19 pandemic period. Being able to survive in the market or being able to preserve the value of portfolios become important. Cheng et al. [10] suggest a significant connection of stock market volatility to the COVID-19 pandemic with varying levels of connectedness in Asian market, while the volatility of Europe, United State and Australia market show a consistent closely connection to the COVID 19 development. Liu el al. [11] demonstrate more negative abnormal experiences in Asian market compared to Western markets. A study of the impact of the domestic and global outbreak of the COVID-19 pandemic on the trading size of the Malaysian stock have been carried out by Gamal et al. [12]. The findings of his study showed that COVID-19 has no effect on the Malaysian stock market. Therefore, we would like to assess and compare the performance of the stock market that had immediate impact of the unexpected outbreak that cause havoc globally.

In addition, Abuoliem, Kalyebara and Ibrahim [13] stated shariah-compliant portfolios tend to have a better performance than the conventional rivals in the Malaysian stock market during the COVID-19 pandemic period. This is also supported by Haseeb, Mahdzan and Ahmad [14], in which they revealed that the firm's Shariah compliance is less prone to the stock price crash risk as well as less likely to assemble bad news. Other than that, Subekti et al. [15] proposed that their modified Black-Litterman (BL) model was able to mitigate the implications of the COVID-19 pandemic to the Shariah-complaint stock market in Indonesia by comparing the Sharpe ratio of their proposed portfolio model with the two reference models which are the Mean-Variance (MV) Method and the Black-Litterman Capital Asset Pricing Model (BL-CAPM).

The universal portfolio is particular class of portfolio investment strategies which theoretically achieves good return based on combinations of non-correlated financial assets. Given the satisfactory result on the earlier work for normal economic cycles, we investigate the performance of the BMPGUP (Brownian motion process generated universal portfolio) with three-stock combinations during COVID-19 pandemic using Malaysia KLCI, Singapore

STI and Thailand SET stock markets. Brownian motion process is a stochastic process which is heavily applied in various financial derivative pricing.

2. Methodology

This study consists of stocks selected from Malaysia and its two neighbouring countries, i.e. Singapore and Thailand from January 2020 to June 2022 with daily opening and daily closing price are collected from Yahoo Finance using open-source software library. Stocks that with complete data from the beginning of 2020 to the end of June 2022 during the pandemic period are considered in this study. Therefore, only twenty-nine Malaysia KLCI component stocks, twenty-eight Singapore Strait Time Index stocks and twenty-five stocks from Thailand SET 50 Index are chosen. Three stocks are randomly form by generated from above collected stocks. Total combination formed is 3654 for Malaysia stocks, 3276 and for Singapore stocks and 2300 for Thailand stocks. By varying parameters of BMPGUP, every 3 stocks formed are then generated by BMPGUP with m = 3, wealth returns are collected and compare to Sharpe ratio, Sortino ratio and Maximum Drawdown and the Equally Weighted portfolio (EWP).

2.1 Brownian-motion process generated universal portfolio (BMPGUP)

Refer to the generation method for universal portfolio in [6] , the initial wealth $\hat{S}_0$ is assumed to be one unit and the wealth at the end of n^th^ trading day $\hat{S}_n\left(\boldsymbol{x}_1^n\right)$ is given by

$$\hat{S}_n\left(\boldsymbol{x}_1^n\right)=\prod_{j=1}^{n} \hat{\boldsymbol{b}}_j^{\,t}\, \boldsymbol{x}_j\,, \tag{1}$$

given the universal portfolio $\left\{\hat{\boldsymbol{b}}_j\right\}$.

Let $\left\{Y_{n1}\right\}_{n=1}^{\infty}$, $\left\{Y_{n2}\right\}_{n=1}^{\infty}$, ……., $\left\{Y_{nm}\right\}_{n=1}^{\infty}$ be m given independent stochastic processes and assume the price-relative sequence $\boldsymbol{x}_1^n$ is known. For a fixed positive integer ν , the ν -order universal portfolio $\left\{\hat{\boldsymbol{b}}_{n+1}\right\}$ generated by the m given stochastic processes is defined as :

$$\hat{b}_{n+1,k}=\frac{E\left[Y_{nk}\left(\boldsymbol{Y}_n^{\,t}\boldsymbol{x}_n\right)\left(\boldsymbol{Y}_{n-1}^{\,t}\boldsymbol{x}_{n-1}\right)\ldots..\left(\boldsymbol{Y}_{n-(\nu-1)}^{\,t}\boldsymbol{x}_{n-(\nu-1)}\right)\right]}{\sum_{j=1}^{m}E\left[Y_{nj}\left(\boldsymbol{Y}_n^{\,t}\boldsymbol{x}_n\right)\left(\boldsymbol{Y}_{n-1}^{\,t}\boldsymbol{x}_{n-1}\right)\ldots.\left(\boldsymbol{Y}_{n-(\nu-1)}^{\,t}\boldsymbol{x}_{n-(\nu-1)}\right)\right]} \tag{2}$$

for $k=1,2,...,m$ where the notation $\boldsymbol{Y}_l=\left(Y_{l1},\ldots.,Y_{lm}\right)$ for $l=1,2,\ldots\ldots$ To simplify (2), note that the numerator of (2) can be written as $E\left[Y_{nk}\left(\sum_{i_1=1}^{m}Y_{ni_1}x_{ni_1}\right)\left(\sum_{i_2=1}^{m}Y_{n-1,i_2}x_{n-1,i_2}\right)\ldots..\left(\sum_{i_\nu=1}^{m}Y_{n-\nu+1}x_{n-\nu+1,i_\nu}\right)\right]=$

$$\sum_{i_1=1}^{m}\sum_{i_2=1}^{m}\ldots\ldots\sum_{i_\nu=1}^{m}\left(x_{ni_1}x_{n-1,i_2}\ldots\ldots x_{n-\nu+1,i_\nu}\right)E\left[Y_{nk}Y_{ni_1}Y_{n-1,i_2}\ldots..Y_{n-\nu+1,i_\nu}\right]. \tag{3}$$

Note that by independence assumption,

$E\left[Y_{s_1 i_1} Y_{s_2 i_2} \ldots. Y_{s_u i_u}\right] = E\left[Y_{s_1 i_1}\right] E\left[Y_{s_2 i_2}\right] \ldots. E\left[Y_{s_u i_u}\right]$ if the u integers $i_1, i_2, \ldots\ldots, i_u$ are distinct. Otherwise, we can use the moment-generating function of $Y_{s_1 j}, Y_{s_2 j}, \ldots\ldots, Y_{s_u j}$ to determine the expression for $E\left[Y_{s_1 j} Y_{s_2 j} \ldots. Y_{s_u j}\right]$.
Also , by independence,

$E\left[\left(Y_{s_1 j} Y_{s_2 j} \ldots Y_{s_u j}\right)\left(Y_{r_1 k} Y_{r_2 k} \ldots. Y_{r_p k}\right)\right] = E\left[Y_{s_1 j} Y_{s_2 j} \ldots. Y_{s_u j}\right] \times E\left[Y_{r_1 k} Y_{r_2 k} \ldots Y_{r_p k}\right]$
for $j \neq k$. In general,

$$E\left[\prod_{i=1}^{q}\left(Y_{s_{i_1} j_i} Y_{s_{i_2} j_i} \ldots. Y_{s_{i_{u_i}} j_i}\right)\right] = \prod_{i=1}^{q} E\left(Y_{s_{i_1} j_i} Y_{s_{i_2} j_i} \ldots Y_{s_{i_{u_i}} j_i}\right) \tag{4}$$

for any set of distinct integers $j_1, j_2, \ldots, j_q$.

Let $\{Y_{n1}\}_{n=1}^{\infty}, \{Y_{n2}\}_{n=1}^{\infty}, \ldots\ldots, \{Y_{nm}\}_{n=1}^{\infty}$ be m given independent Brownian motions with positive drift coefficients $\mu_1, \mu_2, \ldots\ldots, \mu_m$ and variance parameters $\sigma_1^2, \sigma_2^2, \ldots., \sigma_m^2$ respectively. Then it is well-known [16] that the process $\{Y_{nl}\}$ has stationary and independent increments, where Y_{nl} has a Gaussian distribution with a mean $n\mu_l$ and a variance $n\sigma_l^2$ for $l = 1, 2, \ldots., m$ and $n = 1, 2, \ldots\ldots$. The covariance of $Y_{n_1 l}$ and $Y_{n_2 l}$ is given by :

$$COV\left(Y_{n_1 l}, Y_{n_2 l}\right) = n_1 \sigma_l^2 \quad \text{for } 0 < n_1 \leq n_2. \tag{5}$$

Furthermore, the ν random variables $Y_{n-\nu+1,l}, Y_{n-\nu+2,l}, \ldots., Y_{nl}$ have a joint multivariate Gaussian distribution with a positive mean vector $\boldsymbol{\mu}_l = \left(\mu_{l1}, \mu_{l2}, \ldots., \mu_{l\nu}\right) > \mathbf{0}$ and a $\nu \times \nu$ covariance matrix

$$K_l = \sigma_l^2 L = \sigma_l^2\left(\lambda_{ij}\right) \qquad \text{for} \quad l = 1, 2, \ldots., m \tag{6}$$

where

$$\mu_{lk} = (n - \nu + k)\mu_l \qquad \text{for} \qquad k = 1, \mathrm{K}\ 2, \ldots., \nu \tag{7}$$

and

$$\lambda_{ij} = \begin{cases} n - \nu + i & if \quad i \leq j \\ n - \nu + j & if \quad i > j, \end{cases} \tag{8}$$

for $i, j = 1, 2, \ldots., \nu.$
The matrix $L = \left(\lambda_{ij}\right)$ in (6) is independent of *l*. The elements λ_{ij} in *L* are the covariance components of $Y_{n-\nu+1,l}, \ldots., Y_{n,l}$ that depend on the time *n*. The means μ_{lk} are nonnegative

for $n \geq \nu - 1$, $k = 1, 2, \ldots, \nu$ and $l = 1, 2, \ldots, m$. Similarly, $\lambda_{ij} \geq 0$ for all $i, j = 1, 2, \ldots, \nu$ if $n \geq \nu - 1$. The portfolio starts at $n = \nu - 1$ given $\boldsymbol{x}_1^{\nu-2}$ for $\nu \geq 3$.

3. Result and discussion

This study provides a comprehensive analysis of portfolio investment strategies, focusing on the impact of the COVID-19 pandemic on different financial markets utilizing the finite order Brownian motion process generated universal portfolio. Results obtained by using a first order Brownian motion generated universal portfolio on three selected stocks compared to benchmark (the KLCI, STI and SET 50) and traditional equally weighted portfolios in Malaysia, Singapore and Thailand are shown in Table 1, Table 2 and Table 3.

Table 1. Return of BMPGUP, EWP of Malaysia stocks.

	BMPGUP	**EWP**	**Benchmark: KLCI**
Count	3654	3654	1
Mean return	0.3313	0.1392	- 0.0942
Minimum return	- 0.2544	-0.2578	
Maximum return	1.3172	1.3013	

Table 2. Return of BMPGUP, EWP of Singapore stocks.

	BMPGUP	**EWP**	**Benchmark: STI**
Count	3276	3276	1
Mean return	0.9012	0.4173	- 0.0360
Minimum return	- 0.2137	-0.3461	
Maximum return	5.5737	2.6637	

Table 3. Return of BMUP, EWP of Thailand stocks.

	BMPGUP	**EWP**	**Benchmark: SET 50**
Count	2300	2300	1
Mean return	1.3423	0.6718	- 0.0060
Minimum return	- 0.1929	-0.2245	
Maximum return	5.3151	4.7016	

From the descriptive statistic Tables 1, 2 and 3, the study reveals that BMPGUP portfolios exhibit a higher mean return 0.3313, 0.9012 and 1.3423 compared to the benchmark (KLCI, STI, SET) and traditional equally weighted portfolios with mean return 0.1392, 0.4173 and 0.6718. While for the maximum return, Malaysia stocks is 1.3172, Thailand stocks is 5.3251 and Singapore stocks is 5.5737 by using BMPGUP strategy. The results indicated that all the three maximum returns outperform EWP.

Further investigation within each economy exposes the vulnerability of some portfolios, performing exceptionally well or poorly due to a lack of diversification. The combination of three consistently under performing stocks can lead to suboptimal results. To minimize over-concentration risk, this study further investigates into the risk profiles of the portfolios, employing metrics such as Sharpe ratio, Sortino ratio, and Maximum Drawdown. Results obtained shown in Tables 4, 5 and 6.

Table 4. Risk profile of BMPGUP, EWP of Malaysia stocks.

	BMPGUP	**EWP**	**Benchmark: KLCI**
Sharpe	0.0196	0.0106	-0.0170
Sortino	0.0279	0.0149	- 0.0234
Maximum Drawdown	-0.3176	-0.3186	- 0.2431

Table 5. Risk profile of BMPGUP, EWP and Singapore stocks.

	BMUP	**EWP**	**Benchmark: STI**
Sharpe	0.0331	0.0235	-0.0052
Sortino	0.0529	0.0339	- 0.0061
Maximum Drawdown	-0.3635	-0.3650	- 0.3193

Table 6. Risk profile of BMPGUP, EWP and Thailand stocks.

	BMUP	**EWP**	**Benchmark: SET50**
Sharpe	0.0396	0.0301	-0.0008
Sortino	0.0526	0.0381	- 0.0008
Maximum Drawdown	-0.4413	-0.4285	- 0.3599

Based on the risk profiles in Table 4, Table 5 and Table 6, despite lacking significant proof, the BMPGUP strategy closely aligns with practical portfolio investment. Besides outperforming benchmarks in mean return from Tables 1, 2 and 3. the research achieves better statistical outcomes by BMPGUP compare to EWP with higher positive Sharpe and Sortino ratios, indicating its practicality and promise. Both BMPGUP and EWP have showed the similar Maximum Drawdown results. Besides, BMPGUP also have achieved better results in Sharpe and Sortino ration when compared to all three above countries benchmarks. In addition, BMPGUP have obtained better results compare to EWP in Maximum Drawdown in both Singapore and Thailand stocks.

3.1 Recommendations for future work

In conclusion, this research shows the potential of the BMPGUP strategy to outperform the existing benchmarks in mean return and enhance risk profiles. Stock market perform differently on a recession period, and there are a lot of unthinkable factors to consider. Hence, future study of performances of universal portfolio on different period with more stocks is needed. Since we focus on the Malaysian, Singapore and Thailand stock markets in this study, it is also recommended to investigate the performance of BMPGUP in the developed economies such as United States and Europe.

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