An Economic Production Quantity inventory model with a circular economy indicator operating in two markets

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Abstract. A sustainable inventory system seeks to enhance production profits and minimize environmental impact. This study introduces an Economic Production Quantity (EPQ) model incorporating dual-market demand, recoverable items, external procurement, and variable item return rates, along with a circular economy indicator. Two scenarios are examined during the concurrent production and repair processes, and a model is devised to maximize the total profit per unit time (TPUT). In the first scenario, there is a sufficient or surplus quantity of moderate-quality return items available for repair. In the second scenario, there is an insufficient quantity of moderate-quality return items for repair, necessitating additional procurement from an external supplier. Items repaired from both scenarios will be marketed in the secondary market. Additionally, both scenarios involve the sale of repaired high-quality return items in the primary market. In the absence of a sufficient quantity of high-quality return items, new production items are anticipated to meet the remaining demand of the primary market. The proposed models are tested through numerical examples, and a numerical sensitivity analysis is conducted to explore how dual-market operations affect the total profit per unit time.

1 Introduction

The circular economy (CE) is predicated on the guiding principles of waste and pollution elimination through design, optimising product and material utilisation through extended lifecycles, and fostering the regeneration of natural systems. The circular economy approach emphasizes closed-loop resource utilization, aiming to extend the lifespan of products and materials through practices such as reuse, repair, and refurbishment. Ultimately, end-of-life products are envisioned to be recycled as raw materials for new products, completing the circular flow and minimising environmental impact [1].

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In circular economies, the significance of returned items lies in their crucial role in the production repair process. Some repaired items are anticipated to be less expensive than newly produced ones. While individual firms in Malaysia are pioneering circular economy initiatives, the current landscape remains fragmented due to the lack of robust legal and regulatory support. [2]

The circularity indicators offer an incomplete perspective of a system's environmental performance. Concurrently, it is observed that these indicators lend themselves to more accessible communication. An increased level of circularity is suggested to play a role in fostering positive customer relationships, enhancing reputation among stakeholders, and facilitating improved access to funding. [3] Our study is using the circularity indicator as index for the rate of return for high-quality and moderate-quality items, and the demand rates for both the primary and secondary markets. Opting to repair returned items is a more sustainable approach, allowing for the complete utilisation of their salvage value. However, it is anticipated that the quality of repaired items will be inferior to that of newly produced items. Therefore, one alternative is to explore a secondary market for selling the repaired items. Thus, the objective of this paper is to present an inventory model for a production system wherein the manufacturing process creates new items, and the high-quality repaired items are sold in the primary market, while simultaneously, a repairing process for moderate-quality returned items is established for sale in the secondary market. The rest of this paper is arranged as follows: we review the related literature in Section 2. In Section 3, we introduce the notations and assumptions of the model, illustrate the dynamics of inventory levels, and provide the mathematical formulation. In Section 4, we will demonstrate the solution procedures and examine a numerical example related to the two cases introduced in Section 3. In Section 5, we delve into the outcomes of a numerical sensitivity analysis conducted on the model. In the concluding section, Section 6, we present the summary and put forth prospects for future investigations.

2 Literature review

Table 1 provides a comparative analysis of gaps in research pertaining to inventory models functioning under circular economy indicators across two markets, in relation to the outcomes derived from the present study.

Table 1. Assessment of research gaps in the literature of inventory models under circular economy context with indicator operating in the two markets.

Paper	Type of model	Circular economy indicator	Two markets	Demand			Per unit gross profit	
				constant	linear in	nonlinear in	constant	dependent on
[1]	EOQ	√			circularity index	circularity index (logarithmic and logistic forms)		circularity index (linear, exponential and logistic forms)
[2]	supply chain		√	√			√	
[3]	supply chain				price and green level			price
[4]	EPQ		√	√			√	
[5]	supply chain	√			circularity index			circularity index (linear, exponential and logistic forms)

[6]	EPQ	√			circularity index			circularity index (linear, exponential and logistic forms)
[7]	Supply chain	√		√				
[8]	Supply chain			√				
This paper	EPQ	√	√			circularity index (logarithmic form)		circularity index (exponential form)

Rabta (2020) [1] mentioned about the critical obstacle to realizing the full potential of circular economy (CE) principles lies in the development and implementation of robust evaluation metrics by governmental entities. He argued that product circularity impacts demand, cost, and price. He proposed exploring diverse relationships, both linear and nonlinear, to investigate this influence. Under the assumptions of a linear model, his investigation demonstrated that increased costs associated with heightened product circularity can be counteracted by a simultaneous increase in demand, potentially leading to a state of profit equilibrium. In contrast, variations in nonlinear models revealed a more complex dynamic, showcasing significantly broader fluctuations in the profit function across different levels of circularity.

Khara et al. (2020) [2] investigated manufacturing using both virgin and reprocessed materials, acknowledging their inherent differences in infrastructure requirements and quality outcomes. Furthermore, the demand for reprocessed products was directly proportional to the return rate of used materials. In a recent study, Heydari et al. (2020) [3] demonstrated a linear relationship between the green level of products and demand within the context of supply chain coordination.

Yoo & Cheong (2021) [4] went beyond simply modelling price-demand relationships for refurbished items. They aimed to identify optimal integrated inventory and refurbishing strategies, encompassing decisions like production quantity, prices for defective and refurbished items, and refurbishing volume. This optimization process for pricing refurbished goods shares similarities with determining effective patient incentives in healthcare.

Thomas & Mishra (2022) [5] employed a linear decreasing demand model concerning the circularity level in a two-tier supply chain for plastic industries. However, the linear pattern may not be suitable for many real-world business environments. As a result, there is a need to formulate sustainable models that consider both linear and nonlinear demand with an awareness of carbon emissions arising from production and inventory activities.

In a recent study, Khan et al. (2023) [6] explored how a carbon tax affects the most profitable way for a manufacturer to manage product lifecycles under carbon tax regulations. Hegedűs & Longauer (2023) [7] examined a modification of Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) models that incorporates the reusability of raw materials and components across various product generations. The performance indicator was employed to quantify the quality characteristic, specifically as the reusability rate of the particular component. A closed-loop supply chain addresses challenges such as fulfilling the requirements of various entities within the network and ensuring the appropriate recycling or reuse of returned products Suhandi & Chen (2023) [8].

The main challenge in achieving positive results from Circular Economy (CE) principles lies in establishing and implementing a robust measurement framework by regulatory bodies. Several researchers have explored diverse parameters that impact the feasibility of incorporating CE into their inventory models, aiming to provide guidance to regulatory bodies. Recent research on incorporating circular economy principles into inventory models, such as EOQ, EPQ, or supply chain models, has attempted to include the circular economy indicator, two markets, demand rates (whether constant or modelled as a linear function), circularity index, price, or green level. Some researchers have also explored the application of nonlinear demand functions in the circularity index, utilizing logarithmic and logistic forms. Additionally, certain studies have considered the constant per unit gross profit or a function of unit gross profit dependent on factors such as price, circularity index, modelled in linear, exponential, or logistic forms.

Based on our recent literature reviews, those literature sources do not simultaneously integrate both the circular economy indicator and the demands of the two markets. We employ demand functions that depend on the circularity index in logarithmic form and unit gross profit functions based on the circularity index in exponential form. Moreover, our unit disposal cost exhibits a reciprocal function that experiences exponential growth with an increase in the circularity index.

3 Mathematical modelling

3.1 Notations

The notations that are used in the study are shown in Table 2.

Table 2. Notations

Decision variables	
T	Time length of the common cycle [unit of time].
C_i	Circularity index of the repaired items, $0 \leq C_i \leq 1$.
Q	Production quantity of high-quality new items for selling in the primary market [units].
Q_o	Extra procurement batch size of repaired items from third party supplier for selling in the secondary market if not enough of the moderate-quality returned items [units].
Q_{R1}	Repaired quantity for selling in the primary market [units].
Q_{R2}	Repaired quantity for selling in the secondary market [units].
Input functions that are depending on circularity index	
$D_1(C_i)$	The primary market demand rate function of the repaired high-quality returned items and produced new items from production process, which is depending on the circularity index [units/unit of time].
$D_2(C_i)$	The secondary market demand rate function of the repaired moderate-quality returned items, which is depending on the circularity index [units/unit of time].
$g_p(C_i)$	The gross profit per unit sold for new items in the primary market [$/unit].
$P_1(C_i)$	The gross profit per unit repaired for high-quality returned items sold in the primary market [$/unit].
$P_2(C_i)$	The gross profit per unit repaired for moderate-quality returned items sold in the secondary market [$/unit].
$c_d(C_i)$	The disposal cost function per unit for surplus returned items of moderate-quality and low-quality returned items.
$g_o(C_i)$	The gross profit function per unit (after subtracting the unit ordering cost) for selling an additional procurement quantity from a third-party supplier in the secondary market. [$/unit].

Input parameters	
α	Proportion of high-quality returned items.
β	Proportion of moderate-quality returned items.
γ	Proportion of low-quality returned items.
d_1	Fixed demand rate in primary market when circularity index is equal to zero [units/unit of time].
d_2	Fixed demand rate in secondary market when circularity index is equal to zero [units/unit of time].
P	Production rate [units/unit of time].
R_1	Repair rate of high-quality returned items [units/unit of time].
R_2	Repair rate of moderate-quality returned items [units/unit of time].
k_0	Setup cost of an extra procurement cycle of repaired items from third party supplier for selling in the secondary market if not enough of the moderate-quality returned items [$].
k_1	Setup cost of a production cycle for new good items [$].
k_2	Setup cost of a repairing cycle of returned items [$].
h_0	Unit holding cost per unit time of extra procurement from third party supplier [$/unit/unit of time].
h_1	Unit holding cost per unit time of good items [$/unit/unit of time].
h_2	Unit holding cost per unit time of returned items [$/unit/unit of time].
h_3	Unit holding cost per unit time of repaired items [$/unit/unit of time].

3.2 Assumptions

The following assumptions are used in the model development:

a) The planning horizon is infinite in length.
b) Shortages are not allowed.
c) The returned used items have three quality levels – high, moderate, and low.
d) The repaired high-quality used items and produced items from production are to satisfy the demand in the primary market.
e) The repaired moderate-quality used items are to satisfy the demand in the secondary market.
f) The inspection process is conducted simultaneously during the production process and collection of used items.
g) The return rate of used items is depended on the circularity index of repaired items and the primary market demand, i.e. $(\alpha+\beta+\gamma)C_iD_1(C_i)$.
h) All the high-quality used items are expected to be repaired and sold in primary market.
i) Two cases are expected to happen for the moderate-quality used items, which are as follows:
   (i) Case 1: Just enough or more than enough moderate-quality returned items are available. If the moderate-quality returned items are more than enough, then there is a need to dispose the surplus items. This case occurs when $\beta C_iD_1(C_i) \geq D_2(C_i)$.
   (ii) Case 2: Not enough moderate-quality used items are available and there is a need for extra procurement quantity from a third-party supplier. This case occurs when $\beta C_iD_1(C_i) < D_2(C_i)$.
j) The low-quality returned items are continuously disposed.

k) Production rate must be more than demand rate in primary market, $P > D_1(C_i)$.

l) Repair rate of high-quality returned items must be more than demand rate in primary market, $R_1 > D_1(C_i)$.

m) Repair rate of moderate-quality returned items must be more than demand rate in secondary market, $R_2 > D_2(C_i)$.

3.3 Inventory level movement

The inventory level movement of the two cases are shown in the following figures:

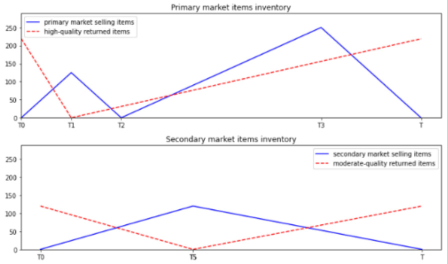


Fig. 1a. Case 1 involving just enough moderate-quality returned items.

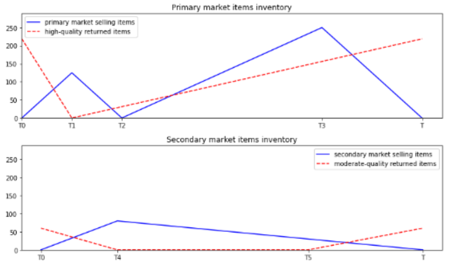


Fig. 1b. Case 1 involving more than enough moderate-quality returned items (need to dispose surplus moderate-quality returned items during the interval T_4 to T_5).

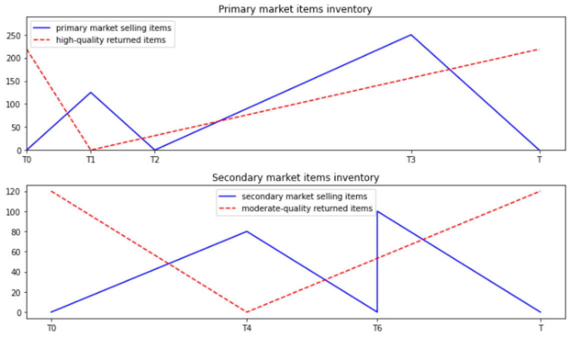


Fig. 2. Case 2 involving not enough moderate-quality returned items (need extra procurement quantity from third party).

3.4 Functions involved

This section presents the mathematical formulation of the model, utilizing the notations and assumptions established in the preceding sections.

The quantity of newly produced, high-quality items sold in the primary market is

$$Q = D_1(C_i)(1 - \alpha C_i)T. \tag{1}$$

The predicted cycle time for combined repair and subsequent production, based on the production model, is

$$T = \frac{Q}{D_1(C_i)(1-\alpha C_i)}. \tag{2}$$

The quantity of repaired high-quality returned items sold in the primary market is

$$Q_{R1} = \alpha C_i D_1(C_i)T. \tag{3}$$

The quantity of repaired moderate-quality returned items sold in the secondary market is

$$Q_{R2} = D_2(C_i)T. \tag{4}$$

The primary market demand rate for repaired high-quality returned items and newly produced items from the new production process, which is dependent on the circularity index of repair items, is defined by the following equation,

$$D_1(C_i) = d_1 + c_1 \ln(1 + \iota_1 C_i), \tag{5}$$

where c_1 and ι_1 are positive constants. The demand rate in primary market is fixed when the circularity index is equal to zero, i.e. $D_1(0) = d_1$, where $C_i = 0$.

The secondary market demand rate for repaired moderate-quality returned items, which is dependent on the circularity index of repair items, is defined by the following equation,

$$D_2(C_i) = d_2 + c_2 \ln(1 - \iota_2 C_i), \tag{6}$$

where c_2 and ι_2 is are positive constants. The demand rate in secondary market is fixed when circularity index is equal to zero, i.e. $D_2(0) = d_2$, where $C_i = 0$.

Figure 3 depicts the relationship between the demand rates of the primary, $D_1(C_i)$ and secondary markets, $D_2(C_i)$ and the circularity index, C_i. Figure 4 depicts the gross profit functions of new production, $g_p(C_i)$, high-quality returned item repair, $P_1(C_i)$, moderate-quality returned item repair, $P_2(C_i)$, and additional procurement, $g_0(C_i)$, all as functions of the circularity index, C_i. It is clear that both demand functions increase with the circularity index, but the rate of increase decrease with the circularity index. This is supported by the assumption that customers react positively to higher circularity index.

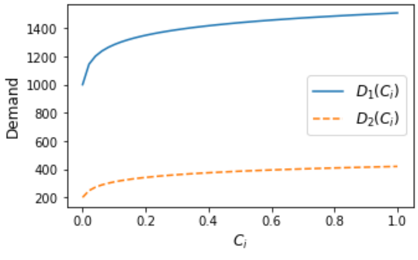


Fig. 3. Demand rate of primary market, $D_1(C_i)$ and secondary market, $D_2(C_i)$ vs. circularity index, C_i.

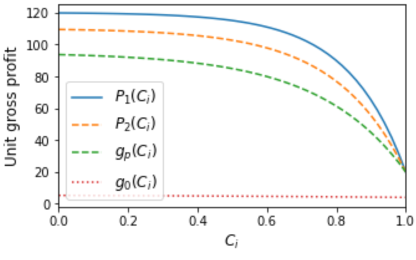


Fig. 4. Gross profit function of new production items, $g_p(C_i)$, gross profit function of repair high-quality returned items, $P_1(C_i)$, gross profit function of repaired moderate-quality returned items, $P_2(C_i)$, and gross profit function of extra procurement items, $g_0(C_i)$ vs. circularity index, C_i.

The gross profit per unit sold for new items in the primary market is

$$g_p(C_i) = p_0 - a_1 e^{\delta_1(C_i - 1)}, \tag{7}$$

where $g_p(C_i)$ is positive, a_1 and δ_1 are positive constants and $g_p(0) \approx p_0$ when $C_i = 0$. The gross profit per unit of repaired high-quality returned items sold in the primary market is

$$P_1(C_i) = p_1 - a_2 e^{\delta_2(C_i - 1)}, \tag{8}$$

where $P_1(C_i)$ is positive, a_2 and δ_2 are positive constants and $P_1(0) \approx p_1$ when $C_i = 0$. The gross profit per unit of repaired moderate-quality returned items sold in the secondary market is

$$P_2(C_i) = p_2 - a_3 e^{\delta_3(C_i - 1)}, \tag{9}$$

where $P_2(C_i)$ is positive, a_3 and δ_3 are positive constants and $P_2(0) \approx p_2$ is positive fixed unit gross profit when $C_i = 0$.

The unit gross profit function (after deducting unit ordering cost) for selling extra procurement quantity from third party supplier in the secondary market is

$$g_o(C_i) = p_3 - a_4 e^{\delta_4(C_i - 1)}, \tag{10}$$

where $g_o(C_i)$ is positive, a_4 and δ_4 are positive constants and $g_o(0) \approx p_3$ when $C_i = 0$.

It is clear that the gross profit functions decrease with the circularity index, and the rate of decrease increase with the circularity index. This is supported by the assumption that costs go up to support higher circularity level.

The unit disposal cost function for low-quality returned items and surplus returned items of moderate-quality is

$$c_d(C_i) = \frac{L_0}{\kappa - C_i}, \tag{11}$$

where L_0 is a positive constant and κ, is a constant greater than maximum value of circularity index, $C_i = 1$, that is $\kappa > 1$. Figure 5 depicts the relationship between unit disposal cost, $c_d(C_i)$ and circularity index, C_i. It illustrates that the unit disposal cost exhibits exponential growth with an increase in the circularity index.

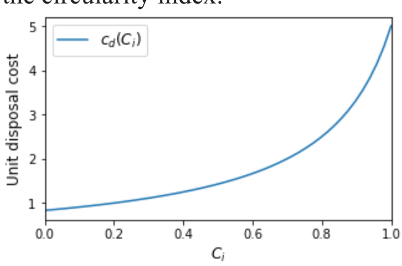


Fig. 5. The unit disposal cost, $c_d(C_i)$ vs. circularity index, C_i.

3.5 Mathematical formulation

Using the functions from Section 3.4, we formulate the total profit functions in this section. In Case 1 (without procurement), where the quantity of moderate-quality returned items is sufficient or exceeds the amount required for repair and sale in the secondary market, the total profit function, $TP_1(T, C_i)$ is determined by subtracting setup cost, holding cost, and disposal cost of surplus moderate-quality and low-quality returned items from the gross profit. The total profit earned per unit time, denoted by $TPUT(T, C_i)$, is obtained by dividing the total profit function by the duration of a single cycle within the system.

Case 1: $TPUT_1(T, C_i) = \frac{TP_1(T, C_i)}{T}$

$$
\begin{aligned}
&= P_1(C_i)[\alpha C_i D_1(C_i)] + g_p[D_1(C_i)(1-\alpha C_i)] + P_2(C_i)[D_2(C_i)] - \frac{(k_1+k_2)}{T} \qquad (12)\\
&\quad - \frac{h_1 T}{2}(1-\alpha C_i)^2\left[D_1(C_i) - \frac{\left(D_1(C_i)\right)^2}{P}\right]\\
&\quad - \frac{h_2 T}{2}\left\{\alpha C_i D_1(C_i)\left[1 - \frac{\alpha C_i D_1(C_i)}{R_1}\right]\right.\\
&\quad \left. + \frac{[D_2(C_i)]^2}{\beta C_i D_1(C_i)}\left[1 - \frac{\beta C_i D_1(C_i)}{R_2}\right]\right\}\\
&\quad - \frac{h_3 T}{2}\left\{(\alpha C_i)^2\left[D_1(C_i) - \frac{[D_1(C_i)]^2}{R_1}\right]\right.\\
&\quad \left. + \left[D_2(C_i) - \frac{[D_2(C_i)]^2}{R_2}\right]\right\}\\
&\quad - c_d(C_i)\left\{\gamma C_i D_1(C_i) + \beta C_i D_1(C_i)\left[1 - \frac{D_2(C_i)}{R_2}\right]\right.\\
&\quad \left. - D_2(C_i)\left[1 - \frac{\beta C_i D_1(C_i)}{R_2}\right]\right\}
\end{aligned}
$$

In Case 2 (with procurement), where the quantity of moderate-quality returned items is insufficient for secondary market repair, necessitating the procurement of additional moderate-quality items from a third-party supplier, the total profit function, $TP_2(T, C_i)$ is calculated by subtracting setup cost, holding cost, and disposal cost of low-quality returned items from the gross profit.

Case 2: $TPUT_2(T, C_i) = \frac{TP_2(T, C_i)}{T}$

$$
\begin{aligned}
&= P_1(C_i)[\alpha C_i D_1(C_i)] + g_p[D_1(C_i)(1-\alpha C_i)] + P_2(C_i)[D_2(C_i)] \qquad (13)\\
&\quad + g_o(C_i)[D_2(C_i) - \beta C_i D_1(C_i)]\\
&\quad - \frac{h_0 T}{2} D_2(C_i)\left[1 - \frac{\beta C_i D_1(C_i)}{D_2(C_i)}\right]^2 - \frac{(k_0+k_1+k_2)}{T}\\
&\quad - \frac{h_1 T}{2}(1-\alpha C_i)^2\left[D_1(C_i) - \frac{\left(D_1(C_i)\right)^2}{P}\right]\\
&\quad - \frac{h_2 T}{2} C_i D_1(C_i)\left\{\alpha\left[1 - \frac{\alpha C_i D_1(C_i)}{R_1}\right] + \beta\left[1 - \frac{\beta C_i D_1(C_i)}{R_2}\right]\right\}\\
&\quad - \frac{h_3 T}{2}[C_i D_1(C_i)]^2\left\{\alpha^2\left[\frac{1}{D_1(C_i)} - \frac{1}{R_1}\right] + \beta^2\left[\frac{1}{D_2(C_i)} - \frac{1}{R_2}\right]\right\}\\
&\quad - c_d \gamma C_i D_1(C_i)
\end{aligned}
$$

Our objective is to maximise the total profit per unit time $TPUT(T, C_i)$ that is subject to $0 \le C_i \le 1$ and $T \ge 0$. Let θ be the threshold circularity index of the repaired items. If $C_i >$

θ, then there are sufficient moderate-quality used items for the secondary market. If $C_i < 0$, the quantity of items is insufficient. When $C_i = \theta$, then there are just enough items. In other words, if $C_i < \theta$, Case 2 is implemented; otherwise, Case 1 is applied. Hence the problem is

$$\begin{cases} \max\limits_{T,C_i} TPUT(T,C_i) \\ s.t. \\ \theta \le C_i \le 1 => TPUT_1(T,C_i)\ Case\ 1\ (without\ extra\ procurement) \\ 0 < C_i \le \theta => TPUT_2(T,C_i)\ Case\ 2\ (with\ extra\ procurement) \\ 0 \le C_i \le 1 \\ T, C_i, \theta \ge 0 \end{cases} \tag{14}$$

When $\beta C_i D_1(C_i) \ge D_2(C_i)$, we have sufficient moderate-quality used items for repairing process. To find the C_i threshold (θ), we solve the following equation:

$$\beta C_i D_1(C_i) = D_2(C_i) \tag{15}$$

Case 1 ($\theta \le C_i \le 1$): The Lagrangian function is

$$L(T, C_i, \mu_1, \mu_2) = TPUT(T, C_i) - \mu_1(\theta - C_i) - \mu_2(C_i - 1). \tag{16}$$

Karush-Kuhn-Tucker (KKT) necessary conditions for a solution to be optimal are given by:

$$\frac{\partial L(T, C_i, \mu_1, \mu_2)}{\partial T} = \frac{\partial TPUT(T, C_i)}{\partial T} = 0 \tag{17}$$

$$\frac{\partial L(T, C_i, \mu_1, \mu_2)}{\partial C_i} = \frac{\partial TPUT(T, C_i)}{\partial C_i} + \mu_1 - \mu_2 = 0 \tag{18}$$

$$\mu_1(\theta - C_i) = 0 \tag{19}$$

$$\mu_2(C_i - 1) = 0 \tag{20}$$

$$\mu_1, \mu_2 \ge 0 \tag{21}$$

Case 2 ($0 < C_i \le \theta$): The Lagrangian function is

$$L(T, C_i, \mu_1, \mu_2) = TPUT(T, C_i) - \mu_1(\theta - C_i) - \mu_2 C_i. \tag{22}$$

Karush-Kuhn-Tucker (KKT) necessary conditions for a solution to be optimal are given by:

$$\frac{\partial L(T, C_i, \mu_1, \mu_2)}{\partial T} = \frac{\partial TPUT(T, C_i)}{\partial T} = 0 \tag{23}$$

$$\frac{\partial L(T, C_i, \mu_1, \mu_2)}{\partial C_i} = \frac{\partial TPUT(T, C_i)}{\partial C_i} + \mu_1 - \mu_2 = 0 \tag{24}$$

$$\mu_1(C_i - \theta) = 0 \tag{25}$$

$$\mu_2 C_i = 0 \tag{26}$$

$$\mu_1, \mu_2 \ge 0 \tag{27}$$

4 Solution procedure and numerical example

Following the presentation of two distinct scenarios in the previous section, this section elaborates on the employed methodologies for achieving optimal solutions within each scenario.

4.1 Solution procedure

The solution procedure that we used to find the optimal common cycle length, T^*, circularity index, C_i^*, and the total profit per unit time, $TPUT^*(T^*, C_i^*)$, are shown as below:

Step 1. Find the threshold value θ by solving C_i that satisfy $\beta C_i D_1(C_i) = D_2(C_i)$.

Step 2. Solve the KKT equations (17) to (21) to find the optimal $TPUT_1$.

Step 2a. Set $C_i = 1$. Then $\mu_1 = 0$. Find μ_2 from (18) and T^* from (17). The solution is feasible and optimal if $\mu_2 \geq 0$.

Step 2b. Set $C_i = \theta$. Then $\mu_2 = 0$. Find μ_1 from (18) and T^* from (17). The solution is feasible and optimal if $\mu_1 \geq 0$.

Step 2c. Set $\mu_1 = \mu_2 = 0$. Find T^* and μ^* by solving (17) and (18) simultaneously. The solution is feasible and optimal if $T^* \geq 0$ and $\theta \leq C_i^* \leq 1$.

Step 3. Solve the KKT equations (23) to (27) to find the optimal $TPUT_2$.

Step 3a. Set $C_i = 0$. Then $\mu_1 = 0$. Find μ_2 from (24) and T^* from (23). The solution is feasible and optimal if $\mu_2 \geq 0$.

Step 3b. Set $C_i = \theta$. Then $\mu_2 = 0$. Find μ_1 from (24) and T^* from (23). The solution is feasible and optimal if $\mu_1 \geq 0$.

Step 3c. Set $\mu_1 = \mu_2 = 0$. Find T^* and μ^* by solving (23) and (24) simultaneously. The solution is feasible and optimal if $T^* \geq 0$ and $\theta \leq C_i^* \leq 1$.

Step 4. Compare $TPUT_1^*$ and $TPUT_2^*$ to determine the optimal of T^* and C_i^*.

4.2 Numerical example

A numerical example is shown in this sub-section and will be used as the base case in the sensitivity analysis in the Section 5. The common parameter settings are as below:
$\alpha = 0.2,\ \beta = 0.7,\ \gamma = 1 - \alpha - \beta,\ P = 3000,\ R_1 = 1500,\ R_2 = 1000,\ k_0 = 150,\ k_1 = 100,\ k_2 = 80,\ h_0 = 10,\ h_1 = 5,\ h_2 = 3,\ h_3 = 2,\ p_0 = 120,\ p_1 = 110,\ p_2 = 95,\ p_3 = 6, a_1 = 100, a_2 = 90, a_3 = 75, a_4 = 2,\ \delta_1 = 6,\ \delta_2 = 90,\ \delta_3 = 4,\ \delta_4 = 1,\ L_0 = 1,\ \kappa = 1.2,\ d_1 = 1000, c_1 = 100, \iota_1 = 160, d_2 = 300, c_2 = 50, \iota_2 = 80.$

By using free and open source of Python version 3.8.5 in Jupyter© Notebook application (ananconda3), we obtain the solution in Table 3.

Table 3. Solution for numerical example.

T^* = 0.27726350287974255
C_i^* = 0.3939337521989038
θ = 0.4814282466029849
$TPUT^*(T^*, C_i^*)$= 206292.6180098717
Best policy with the common parameter settings is Case 2.

5 Sensitivity analysis and discussion

In this section, a numerical sensitivity analysis is performed to study how the optimal solution reacts to changes in parameter values. In each experiment, one set of parameter values are changed incrementally while all other parameters settings are retained. The common parameter settings are same as the Section 4.2.

The sensitivity analysis involves varying the demand functions of both the primary market and secondary market. This was achieved by systematically modifying the relevant parameters within each market's demand function. The percentage change is determined through the following equation:

$$\text{Percentage of change (PCTC)} = \frac{changed\ value - base\ value}{base\ value} \times 100\%. \quad (28)$$

Therefore, the percentage of change (PCTC) of the total profit per unit time (TPUT) is calculated as follows:

$$\frac{changed\ TPUT - base\ TPUT}{base\ TPUT} \times 100\%. \quad (29)$$

An investigation was undertaken to evaluate the responsiveness of the total profit per unit time ($TPUT^*$) to variations in the relevant parameters linked to the primary market demand function and the secondary market demand function. This analysis is aimed to: (1) identify the scenario (Case 1 or Case 2) yielding the maximum throughput $TPUT^*$, (2) quantify the percentage change in $TPUT^*$, (3) determine the optimal common cycle duration, (4) evaluate the circularity index of repaired items, C_i^*, and (5) the circularity index threshold, θ.

5.1 Sensitivity analysis for the primary market demand function depending on circularity index, $D_1(C_i) = d_1 + c_1 \ln(1 + \iota_1 C_i)$

The summary table of the sensitivity analysis for the primary market demand function is shown in Table 4(a).

Table 4(a). Sensitivity analysis for the primary market demand function depending on circularity index, $\boldsymbol{D_1(C_i) = d_1 + c_1 \ln(1 + \iota_1 C_i)}$.

No.	d_1	c_1	ι_1	$D_1(C_i^*)$	$TPUT^*$	PCTC of $TPUT^*$	T^*	C_i^*	θ	Policy
1.	600	60	96	819.69	137482.47	-33.36	0.28	0.39	0.85	Case 2
2.	800	80	128	1115.41	171597.56	-16.82	0.28	0.39	0.62	Case 2
3.	1000	100	160	1415.93	206292.62	0.00	0.28	0.39	0.48	Case 2
4.	1200	120	192	1724.78	241483.12	17.06	0.28	0.41	0.39	Case 1
5.	1400	140	224	2033.47	277416.34	34.48	0.29	0.41	0.33	Case 1

*Note for short form: The percent of change (PCTC), the optimal of total profit per unit time ($TPUT^*$), policy of Case 1 is the first scenario without procurement, and policy for Case 2 is the second scenario with procurement.

In Figure 6, we explore how changing the primary market demand rate, $D_1(C_i^*)$, affects the optimal circularity index, C_i^*, and the circularity index threshold, θ, and both of which are shown in relation to the optimal total profit per unit time, $TPUT^*$.

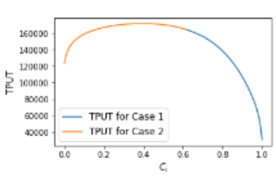


Fig. 6a. $d_1 = 600$, $c_1 = 60$, $\iota_1 = 96$.

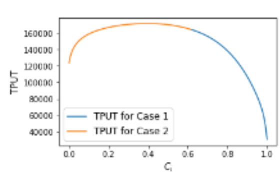


Fig. 6b. $d_1 = 800$, $c_1 = 80$, $\iota_1 = 128$.

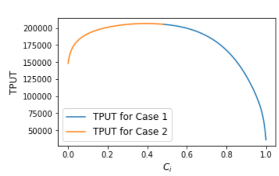
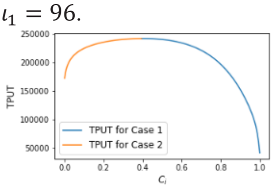


Fig. 6c. $d_1 = 1000$, $c_1 = 100$, $\iota_1 = 160$.



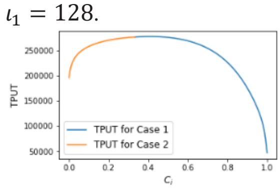


Fig. 6d. $d_1 = 1200$, $c_1 = 120$, $\iota_1 = 192$. **Fig. 6e.** $d_1 = 1400$, $c_1 = 140$, $\iota_1 = 224$. **Fig. 6.** The change of $D_1(C_i^*)$, affects C_i^* and θ in relation to $TPUT^*$.

A higher primary market demand rate, denoted as $D_1(C_i^*)$, corresponds to larger optimal values of total profit per unit time ($TPUT^*$), circularity index (C_i^*), and the optimal duration of the common cycle (T^*). However, the threshold C_i (θ) decreases with an increase in the primary market demand rate, $D_1(C_i^*)$. This means that as the primary market demand increases, the optimal $TPUT^*$ shifts from Case 2 to Case 1 (see how the peak of the graph changes in Figure 6).

For a smaller primary market size and lower values of the primary market demand rate, $D_1(C_i^*)$, Case 2 emerges as the preferred option. This is due to the necessity of additional procurement from a third-party supplier, given that the supply of moderate-quality returned items is insufficient.

In the event of a growing primary market size accompanied by higher values of the primary market demand rate, $D_1(C_i^*)$, Case 1 emerges as the preferable strategy, as the supply of moderate-quality returned items is adequate to meet the demands of the primary market.

5.2 Sensitivity Analysis for the secondary market demand function depending on circularity index, $D_2(C_i) = d_2 + c_2 \ln(1 - \iota_2 C_i)$

The summary table of the sensitivity analysis is shown in Table 4(b).

Table 4(b). Sensitivity Analysis for the secondary market demand function depending on circularity index, $\boldsymbol{D_2(C_i) = d_2 + c_2 \ln(1 - \iota_2 C_i)}$.

No.	d_2	c_2	ι_2	$D_2(C_i^*)$	$TPUT^*$	PCTC of $TPUT^*$	T^*	C_i^*	θ	Policy
1.	150	25	40	221.51	183390.78	-11.10	0.30	0.41	0.22	Case 1
2.	225	37.5	60	346.58	194494.08	-5.72	0.28	0.41	0.35	Case 1
3.	300	50	80	474.08	206292.62	0.00	0.28	0.39	0.48	Case 2
4.	375	62.5	100	606.08	218546.81	5.94	0.26	0.39	0.62	Case 2
5.	450	75	120	642.37	230999.60	11.98	0.25	0.39	0.76	Case 2

*Note for short form: The percent of change (PCTC), the optimal of total profit per unit time ($TPUT^*$), policy of Case 1 is the first scenario without procurement, and policy for Case 2 is the second scenario with procurement.

Figure 7 illustrates the variations in the optimal circularity index (C_i^*), the circularity index threshold (θ), and the maximum profit per unit time ($TPUT^*$) as the secondary market demand ($D_2(C_i^*)$) undergoes changes.

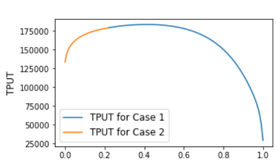


Fig. 7a. $d_2 = 150$, $c_2 = 25$, $\iota_2 = 40$.

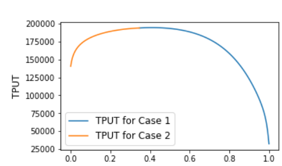


Fig. 7b. $d_2 = 225$, $c_2 = 37.5$, $\iota_2 = 60$.

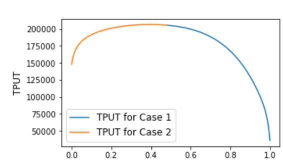
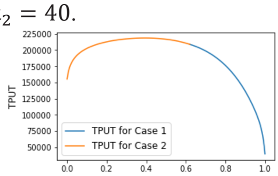


Fig. 7c. $d_2 = 300$, $c_2 = 50$, $\iota_2 = 80$.



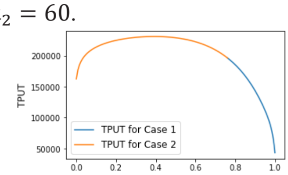


Fig. 7d. $d_2 = 375$, $c_2 = 62.5$, $\iota_2 = 100$

Fig. 7e. $d_2 = 450$, $c_2 = 75$, $\iota_2 = 120$

Fig. 7. The change of $D_2(C_i^*)$, affects C_i^* and θ in relation to $TPUT^*$.

An increased rate of secondary market demand, represented by $D_2(C_i^*)$, is associated with larger optimal values for total profit per unit time ($TPUT^*$), circularity index (C_i^*), and the threshold C_i (θ). However, the optimal duration of the common cycle, T^*, decreases as the secondary market demand rate, $D_2(C_i^*)$, increases.

In this sensitivity analysis, it is evident that for a smaller secondary market size and a lower value of the secondary market demand rate, $D_2(C_i^*)$, Case 1 emerges as the preferred policy. This is due to the fact that the supply of moderate-quality returned items is sufficient to meet the demands of the secondary market.

With the expansion of the secondary market and an increase in the value of the secondary market demand rate, $D_2(C_i^*)$, Case 2 proves to be a more favourable policy. This is because additional procurement becomes necessary, given that the supply of moderate-quality returned items is insufficient.

From the Table 4(a), for every 20% change in d_1, c_1, and ι_1 in the primary market demand rate, $D_1(C_i^*)$, the percentage of change (PCTC) of the total profit per unit time ($TPUT^*$) is about 16% to 17%. Whereas from the Table 4(b), for every 20% change in d_2, c_2, and ι_2 in the secondary market demand rate, $D_2(C_i^*)$, the percentage of change (PCTC) of the total profit per unit time ($TPUT^*$) is only about 5% to 6%. So, we can conclude that our model is more sensitive to the primary market demand rate, $D_1(C_i^*)$, as compared with the secondary market demand rate, $D_2(C_i^*)$.

6 Conclusion and future study

This paper has introduced an EPQ model with two market demands, repairable items, external procurement, and variable item return rates, along with a circular economy indicator. Two scenarios have been examined, and a model is devised to maximize the total profit per unit time (TPUT). In the first scenario, there is sufficient moderate-quality returned items available for repair, whereas in the second scenario, there is an insufficient of moderate-quality returned items for repair, that required an additional procurement from an external supplier. Items repaired from both scenarios will be marketed in the secondary market. Additionally, both scenarios have also involved the sale of repaired high-quality return items in the primary market. In the absence of a sufficient quantity of high-quality returned items, new production items are anticipated to meet the remaining demand of the primary market. Our study found that a higher moderate-quality return rate can benefit the first scenario (without external procurement) due to its positive impact on the circularity index. Furthermore, the study identified a positive correlation between the demand in both primary and secondary markets, and the total profit per unit time. Moreover, the analysis indicated that the first scenario is a more favourable policy in the context of a larger primary market demand and a smaller secondary market demand. In such circumstances, opting for a repair-only policy proves to be more advantageous. Looking at it differently, the sensitivity analysis highlights the second scenario as a more advantageous strategy, especially when addressing a smaller demand in the primary market and a larger demand in the secondary market. This requires additional procurement from a third-party supplier offering moderately quality used items, assuming that the return rate depends on the primary market demand rate. In our view, a higher demand rate in the primary market, indicative of the product's popularity and profitability, makes it more worthwhile to repair the used items.

This study primarily focused on dual-market demand, repairable items, external procurement, and variable item return rates, along with a circular economy indicator, while further research could explore the influence of circularity index in other aspect, for example, holding cost functions, setup cost functions, …, etc. Future research could investigate the potential link between the circularity index and unit selling prices for repairable goods, given the broader benefits of a circular economy, particularly in environmental sustainability.

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