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Rough Neutrosophic Multisets Geometric Aggregation Operator with Entropy Weight Combined Roughness Dice Similarity Measure and Its Application

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Abstract. Rough neutrosophic multisets (RNM) is an uncertainty set theory generalized from the rough neutrosophic set. In the same equivalence relation, the universal set is a neutrosophic multisets with boundary regions involving lower and upper approximation. To date, to handle the multiplicity of information collected, the rough neutrosophic multisets geometric aggregation operator (RNMGAO) is introduced. The algebraic operations of RNM used in the derivation of RNMGAO are defined. The entropy measure of RNM is also discussed as a weighted assign for each criterion simultaneously with the geometric aggregation operator. The roughness Dice similarity measure of RNM is combined in methodology for ranking purposed. The application in medical diagnosis of three epidemic diseases Coronavirus, Influenza, and Pneumonia is implemented as a case study.

1 Introduction

In data collection, the relation of information collected raises the uncertainty issue when critical opinions always come with uncertainties that are dependent upon observations and decisions made by experts [1]. Such as for games (win, fair, or losing), voting (vote, blank vote, and against), and decision-making (right, neutral, or wrong). Since there are uncertainties, the collected information is vague, imprecise, ambiguous, and inconsistent condition [2]. In addition, the data collection can be categorized into single valued data and multiset data. For example, in the medical field, the observation of the symptoms appearing in the patient needs to be taken at least three days before. Also, in the selection process, the multiple-phase meeting needs to be done before the finalized result is taken. To handle the

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single valued data collected, various set theory have been proposed by many researcher like probablity theory [3], game theory [4], fuzzy logic and set [5], soft set [6], rough set [7], vague set or intuitionistic fuzzy set [8], and neutrosophic set [9]. As rapid research, most of the single valued data theory is extended to handle the multisets condition, such as multiset theory [10], fuzzy multisets [11], soft multisets [12], intuitionistic fuzzy multisets [13], neutrosophic multiset [14], and n -refined neutrosophic set [15]. All the existing set theories have successfully developed and applied accordingly to suitable uncertainty situations. Also, the hybridization theories are introduced to compromise the interval and parametrized different attributes such as interval-valued fuzzy set [16], interval-valued neutrosophic rough set [17], and interval-valued neutrosophic soft set [18]. Besides that, to cater to the uncertain information in the same equivalence relation, such as in medical diagnosis, the most common symptoms and less common symptom is an important information need to be consider. Since the rough set introduced by Pawlak's can handle it, there was succesfully hybrization set theory solving the uncertainty information in the same equivalence relation such as rough fuzzy set [19]. Next, the extended rough theory becomes rapidly introduced such as rough intuitionistic fuzzy set [20] and rough neutrosophic set [21]. Next, to handle the multisets condition, rough neutrosophic multisets is introduced [22].

Then, rapidly significant properties cater for uncertainty information which is vague, imprecise, ambiguous, and inconsistent condition are introduced such as relation operations [23]–[29], entropy measure [30]–[36], aggregation operators [37]–[40], distance and similarity measure [41]–[49], and algorithms for decision making is introduced [50], [51]–[56]. Therefore, the objectives of this research are to introduce the algebraic operation over rough neutrosophic multisets (RNM) in the derivation of the rough neutrosophic multisets geometric aggregation operator (RNMGAO). Next, the flowchart of the process for solving the medical diagnosis is presented by assigning the entropy weight combined with the roughness Dice similarity measure of RNM.

The paper is organized as follows: The Section 2 presents some of the preliminaries of the study. The Section 3 presents the novel algebraic operation of RNM and the derivation of RNMGAO with proven properties. In Section 4, the implementation study in medical diagnosis is presented as verifying the RNM operators. Lastly, Section 5 concludes the research summary and future work.

2 Preliminaries

In this section, presents some of the preliminaries of the study are reviewed.

Definition 1. [22] Rough Neutrosophic Multisets
Let U be a non-null set with the generic elements in U denoted by x_j and R be an equivalence relation on U. Let

$$A = \left\{ \left\langle \begin{array}{c} \left(T_A^1(x_j), T_A^2(x_j), \dots T_A^p(x_j)\right), \\ \left(I_A^1(x_j), I_A^2(x_j), \dots I_A^p(x_j)\right), \left(F_A^1(x_j), F_A^2(x_j), \dots F_A^p(x_j)\right) \end{array} \right\rangle : i, j \in \mathbb{Z}^+, x_j \in U, \atop j = 1, 2, \dots, q \text{ and } i = 1, 2, \dots, p \right\}$$

be neutrosophic multisets in U with the truth-membership sequence $(T_A^1, T_A^2, \dots T_A^p)$, indeterminacy-membership sequences $(I_A^1, I_A^2, \dots I_A^p)$ and falsity-membership sequences $(F_A^1, F_A^2, \dots F_A^p)$. The lower and the upper approximations of A in the approximation (U, R) are denoted by $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are respectively defined as follows:

$$\underline{Nm}(A) = \left\{\langle x_j, \left(T^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j)\right)\rangle \,\middle|\, y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U\right\},$$

and

$$\overline{Nm}(A) = \left\{\langle x_j, \left(T^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j)\right)\rangle \,\middle|\, y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U\right\}.$$

where

$j = 1, 2, \dots, q$ and $i = 1, 2, \dots, p$ are positive integers,

$T^i_{\underline{Nm}(A)}(x_j) = \wedge_{y\in[x_j]_R} T^i_A(y_j)$, $I^i_{\underline{Nm}(A)}(x_j) = \vee_{y\in[x_j]_R} I^i_A(y_j)$,

$F^i_{\underline{Nm}(A)}(x_j) = \vee_{y\in[x_j]_R} F^i_A(y_j)$, $T^i_{\overline{Nm}(A)}(x_j) = \vee_{y\in[x_j]_R} T^i_A(y_j)$,

$I^i_{\overline{Nm}(A)}(x_j) = \wedge_{y\in[x_j]_R} I^i_A(y_j)$, $F^i_{\overline{Nm}(A)}(x_j) = \wedge_{y\in[x_j]_R} F^i_A(y_j)$.

Here $\wedge$ and $\vee$ denote "min" and "max'' operators, respectively, and $[x_j]_R$ is the equivalence class of the x_j. $T^i_A(y_j)$, $I^i_A(y_j)$ and $F^i_A(y_j)$ are the membership sequences, indeterminacy sequences, and non-membership sequences of y with respect to A. The pair of $\left(\underline{Nm}(A), \overline{Nm}(A)\right)$ is called the RNM in (U, R), respectively denoted by:

$$RNM(A) = \left(\underline{Nm}(A), \overline{Nm}(A)\right)$$

$$= \left\{\left\langle x_j, \left(\begin{matrix}\left[T^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j)\right], \\ \left[T^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j)\right]\end{matrix}\right)\right\rangle \,\middle|\, y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U\right\} \quad (1)$$

The lower and upper approximation of RNM is not in order. Also, only the same repeated numbers of components $1, 2, \dots, p$ is consider for RNM.

Definition 2. [48] Roughness Dice Similarity Measure of Rough Neutrosophic Multisets
The roughness Dice similarity measure $S^D_{RNM}(A, B)$ between $RNM(A)$ and $RNM(B)$ is formulated as:

$$S^D_{RNM}(A, B) = \frac{2}{p}\sum_{i=1}^{p}\frac{1}{q}\left[\sum_{j=1}^{q}\frac{\beta}{(\delta_1)^2 + (\delta_2)^2}\right] \quad (2)$$

where

$\beta = \Delta T^i_{Nm(A)}(x_j)\Delta T^i_{Nm(B)}(x_j) + \Delta I^i_{Nm(A)}(x_j)\Delta I^i_{Nm(B)}(x_j) + \Delta F^i_{Nm(A)}(x_j)\Delta F^i_{Nm(B)}(x_j)$,

$(\delta_1)^2 = \left(\Delta T^i_{Nm(A)}(x_j)\right)^2 + \left(\Delta I^i_{Nm(A)}(x_j)\right)^2 + \left(\Delta F^i_{Nm(A)}(x_j)\right)^2$,

$(\delta_2)^2 = \left(\Delta T^i_{Nm(B)}(x_j)\right)^2 + \left(\Delta I^i_{Nm(B)}(x_j)\right)^2 + \left(\Delta F^i_{Nm(B)}(x_j)\right)^2$,

$\Delta T^i_{Nm(A)}(x_j)$ and $\Delta T^i_{Nm(B)}(x_j)$, $\Delta I^i_{Nm(A)}(x_j)$ and $\Delta I^i_{Nm(B)}(x_j)$, $\Delta F^i_{Nm(A)}(x_j)$ and $\Delta F^i_{Nm(B)}(x_j)$ are roughness approximation for truth, indeterminate, and falsity membership sequence for $RNM(A)$ and $RNM(B)$, respectively,

$$\Delta T^i_{Nm(A)}(x_j) = 1 - \left(\frac{T^i_{\underline{Nm}(A)}(x_j) + \left(T^i_{\overline{Nm}(A)}(x_j)\right)^C}{|X|}\right),$$

$$\Delta I^i_{Nm(A)}(x_j) = 1 - \left(\frac{I^i_{\underline{Nm}(A)}(x_j) + \left(I^i_{\overline{Nm}(A)}(x_j)\right)^C}{|X|}\right),$$

$$\Delta F^{i}_{Nm(A)}(x_j) = 1 - \left(\frac{F^{i}_{\underline{Nm}(A)}(x_j) + \left(F^{i}_{\overline{Nm}(A)}(x_j) \right)^{C}}{|X|} \right),$$

$\left[\Delta T^{i}_{Nm(A)}(x_j), \Delta I^{i}_{Nm(A)}(x_j), \Delta F^{i}_{Nm(A)}(x_j)\right] \neq [0,0,0]$,
$\left[\Delta T^{i}_{Nm(B)}(x_j), \Delta I^{i}_{Nm(B)}(x_j), \Delta F^{i}_{Nm(B)}(x_j)\right] \neq [0,0,0]$,
$\Delta T^{i}_{Nm(A)}(x_j), \Delta I^{i}_{Nm(A)}(x_j), \Delta F^{i}_{Nm(A)}(x_j) \in [0,1]$,
$\Delta T^{i}_{Nm(B)}(x_j), \Delta I^{i}_{Nm(B)}(x_j), \Delta F^{i}_{Nm(B)}(x_j) \in [0,1]$,
$0 \leq \Delta T^{i}_{Nm(A)}(x_j) + \Delta I^{i}_{Nm(A)}(x_j) + \Delta F^{i}_{Nm(A)}(x_j) \leq 3$, and
$0 \leq \Delta T^{i}_{Nm(B)}(x_j) + \Delta I^{i}_{Nm(B)}(x_j) + \Delta F^{i}_{Nm(B)}(x_j) \leq 3$
for $i = 1,2,\dots,p$ and $j = 1,2,\dots,q$.

Definition 3. [34] Entropy measure for Rough Neutrosophic Multisets
An entropy $E_{RNM}(A)$ on RNM is a function $E: RNM(X) \to [0,1]$ is formulated as:

$$E_{RNM}(A) = 1 - \left(\frac{1}{2pq} \sum_{i=1}^{p} \sum_{j=1}^{q} \left\{ \left(\left(T^{i}_{\underline{Nm}(A)}(x_j) + F^{i}_{\underline{Nm}(A)}(x_j) \right) \left| 2I^{i}_{\underline{Nm}(A)}(x_j) - 1 \right| \right) + \left(\left(T^{i}_{\overline{Nm}(A)}(x_j) + F^{i}_{\overline{Nm}(A)}(x_j) \right) \left| 2I^{i}_{\overline{Nm}(A)}(x_j) - 1 \right| \right) \right\} \right) \quad (3)$$

Definition 4. [57] Entropy weight
The entropy weight of the j-th criteria is formulated as:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{q}(1 - E_j)} \quad (4)$$

where E_j is entropy weight of criteria. Here, a weight vector $W = (w_1, w_2, \dots, w_q)^T$ with $w_j \in [0,1]$ and $\sum_{j=1}^{q} w_j = 1$.

Definition 5. [58] Rough Neutrosophic Geometric Aggregation Operator
Let $N(P_i) = \left(\underline{N}(P_i), \overline{N}(P_i)\right)$ in (X, R) $(i = 1,2,\dots,n)$ be a collection of RNM numbers. Then, the rough neutrosophic geometric aggregation operator (RNGMO) is formulated as:

$$\text{RNGMO}(N(P_1), N(P_2), \dots, N(P_n)) = \left(\otimes_{i=1}^{n} \left[\underline{N}(P_i)\right]^{\frac{1}{n}}, \otimes_{i=1}^{n} \left[\overline{N}(P_i)\right]^{\frac{1}{n}} \right) \quad (5)$$

3 Methodology: rough neutrosophic multisets geometric aggregation operator

The definition of rough neutrosophic multisets geometric aggregation operator (RNMGAO) based on the derivation of algebraic operations over RNM is introduced in this section. The definition of RNMGAO is derived as extended from [58].

3.1 Algebraic operations on rough neutrosophic multisets

Algebraic operations which is addition, multiplication, scalar multiplication, and power on RNM are defined in this section.

Let $RNM(A)$ and $RNM(B)$ be two RNM over (U,R) with truth membership sequence $\left[T^{i}_{\underline{Nm}(A)}(x_j), T^{i}_{\overline{Nm}(A)}(x_j)\right]$, $\left[T^{i}_{\underline{Nm}(B)}(x_j), T^{i}_{\overline{Nm}(B)}(x_j)\right]$, indeterminate membership sequence $\left[I^{i}_{\underline{Nm}(A)}(x_j), I^{i}_{\overline{Nm}(A)}(x_j)\right]$, $\left[I^{i}_{\underline{Nm}(B)}(x_j), I^{i}_{\overline{Nm}(B)}(x_j)\right]$ and falsity of membership sequence $\left[F^{i}_{\underline{Nm}(A)}(x_j), F^{i}_{\overline{Nm}(A)}(x_j)\right]$, $\left[F^{i}_{\underline{Nm}(B)}(x_j), F^{i}_{\overline{Nm}(B)}(x_j)\right]$for lower and upper approximation of $RNM(A)$ and $RNM(B)$, respectively.

Definition 6. (Addition) The addition between $RNM\ (A)$ and $RNM\ (B)$ denoted by $RNM\ (A) \oplus RNM\ (B) = \langle \underline{Nm}(A) \oplus \underline{Nm}\ (B), \overline{Nm}(A) \oplus \overline{Nm}\ (B) \rangle$ is formulated as:

$$\underline{Nm}(A) \oplus \underline{Nm}\ (B) = \left\{ \left\langle x_j, \begin{pmatrix} \left(T^{i}_{\underline{Nm}(A)}(x_j) + T^{i}_{\underline{Nm}(B)}(x_j) - \left(T^{i}_{\underline{Nm}(A)}(x_j)\, T^{i}_{\underline{Nm}(B)}(x_j)\right)\right), \\ \left(I^{i}_{\underline{Nm}(A)}(x_j)\, I^{i}_{\underline{Nm}(B)}(x_j)\right), \left(F^{i}_{\underline{Nm}(A)}(x_j)\, F^{i}_{\underline{Nm}(B)}(x_j)\right) \end{pmatrix} \right\rangle : x_j \in U, \\ i,j \in \mathbb{Z}^+ \right\}, \text{ and}$$

$$\overline{Nm}(A) \oplus \overline{Nm}\ (B) = \left\{ \left\langle x_j, \begin{pmatrix} \left(T^{i}_{\overline{Nm}(A)}(x_j) + T^{i}_{\overline{Nm}(B)}(x_j) - \left(T^{i}_{\overline{Nm}(A)}(x_j)\, T^{i}_{\overline{Nm}(B)}(x_j)\right)\right), \\ \left(I^{i}_{\overline{Nm}(A)}(x_j)\, I^{i}_{\overline{Nm}(B)}(x_j)\right), \left(F^{i}_{\overline{Nm}(A)}(x_j)\, F^{i}_{\overline{Nm}(B)}(x_j)\right) \end{pmatrix} \right\rangle : x_j \in U, \\ i,j \in \mathbb{Z}^+ \right\}. \quad (6)$$

Definition 7. (Multiplication) The multiplication between $RNM\ (A)$ and $RNM\ (B)$ denoted by $RNM\ (A) \otimes RNM\ (B) = \langle \underline{Nm}(A) \otimes \underline{Nm}\ (B), \overline{Nm}(A) \otimes \overline{Nm}\ (B) \rangle$ is formulated as:

$$\underline{Nm}(A) \otimes \underline{Nm}\ (B) = \left\{ \left\langle x_j, \begin{matrix} \left(T^{i}_{\underline{Nm}(A)}(x_j)\, T^{i}_{\underline{Nm}(B)}(x_j)\right), \\ \left(I^{i}_{\underline{Nm}(A)}(x_j) + I^{i}_{\underline{Nm}(B)}(x_j) - \left(I^{i}_{\underline{Nm}(A)}(x_j)\, I^{i}_{\underline{Nm}(B)}(x_j)\right)\right), \\ \left(F^{i}_{\underline{Nm}(A)}(x_j) + F^{i}_{\underline{Nm}(B)}(x_j) - \left(F^{i}_{\underline{Nm}(A)}(x_j) \cdot F^{i}_{\underline{Nm}(B)}(x_j)\right)\right) \end{matrix} \right\rangle : x_j \in U, \\ i,j \in \mathbb{Z}^+ \right\}, \text{ and}$$

$$\overline{Nm}(A) \otimes \overline{Nm}\ (B) = \left\{ \left\langle x_j, \begin{pmatrix} \left(T^{i}_{\overline{Nm}(A)}(x_j)\, T^{i}_{\overline{Nm}(B)}(x_j)\right), \\ \left(I^{i}_{\overline{Nm}(A)}(x_j) + I^{i}_{\overline{Nm}(B)}(x_j) - \left(I^{i}_{\overline{Nm}(A)}(x_j)\, I^{i}_{\overline{Nm}(B)}(x_j)\right)\right), \\ \left(F^{i}_{\overline{Nm}(A)}(x_j) + F^{i}_{\overline{Nm}(B)}(x_j) - \left(F^{i}_{\overline{Nm}(A)}(x_j)\, F^{i}_{\overline{Nm}(B)}(x_j)\right)\right) \end{pmatrix} \right\rangle : x_j \in U, \\ i,j \in \mathbb{Z}^+ \right\}. \quad (7)$$

Definition 8. (Scalar) Consider the scalar multiplication of $RNM\ (A)$ denoted by $\lambda(RNM\ (A))$, for $\lambda > 0$ is formulated as:

$$\lambda\big(RNM\ (A)\big) = \left\{ \left\langle x_j, \left(\begin{matrix} \left[1-(1-T^i_{\underline{Nm}(A)}(x_j))^\lambda, \left(I^i_{\underline{Nm}(A)}(x_j)\right)^\lambda, \left(F^i_{\underline{Nm}(A)}(x_j)\right)^\lambda\right] \\ \left[1-(1-T^i_{\overline{Nm}(A)}(x_j))^\lambda, \left(I^i_{\overline{Nm}(A)}(x_j)\right)^\lambda, \left(F^i_{\overline{Nm}(A)}(x_j)\right)^\lambda\right] \end{matrix} \right) \right\rangle : x_j \in U, \atop i,j \in \mathbb{Z}^+, \lambda > 0 \right\} \tag{8}$$

Definition 9. (Power) Consider the power of $RNM\ (A)$ denoted by $(RNM\ (A))^\lambda$, for $\lambda > 0$ is formulated as:

$$(RNM\ (A))^\lambda = \left\{ \left\langle x_j, \left(\begin{matrix} \left[\left(T^i_{\underline{Nm}(A)}(x_j)\right)^\lambda, 1-(1-I^i_{\underline{Nm}(A)}(x_j))^\lambda, 1-(1-F^i_{\underline{Nm}(A)}(x_j))^\lambda\right] \\ \left[\left(T^i_{\overline{Nm}(A)}(x_j)\right)^\lambda, 1-(1-I^i_{\overline{Nm}(A)}(x_j))^\lambda, 1-(1-F^i_{\overline{Nm}(A)}(x_j))^\lambda\right] \end{matrix} \right) \right\rangle : x_j \in U, \atop i,j \in \mathbb{Z}^+, \lambda > 0 \right\} \tag{9}$$

3.2 Rough neutrosophic multisets geometric aggregation operator

In this section, the geometric aggregation operator for RNM is defined as extended from Definition 4, and the proof for a proposition based on Definitions 6-9 is shown as follows.

Definition 10. Let A be neutrosophic multisets in X with the truth membership sequence T^i_A, indeterminacy membership sequence I^i_A, and falsity of membership sequence F^i_A in the equivalence relation R. The lower and upper approximations of A in (X, R) are denoted by $\left(T^i_{\underline{Nm}(A)}, I^i_{\underline{Nm}(A)}, F^i_{\underline{Nm}(A)}\right)$ and $\left(T^i_{\overline{Nm}(A)}, I^i_{\overline{Nm}(A)}, F^i_{\overline{Nm}(A)}\right)$. Then, the RNM number is defined as:
$\left\langle \left(T^i_{\underline{Nm}(A)}, I^i_{\underline{Nm}(A)}, F^i_{\underline{Nm}(A)}\right), \left(T^i_{\overline{Nm}(A)}, I^i_{\overline{Nm}(A)}, F^i_{\overline{Nm}(A)}\right) \right\rangle = \langle \underline{Nm}(A_j), \overline{Nm}(A_j) \rangle$.

Definition 11. Let $Nm(A_j) = \left(\underline{Nm}(A_j), \overline{Nm}(A_j)\right)$ in (X, R) $(j = 1,2, \dots, n)$ be a collection of RNM numbers. Then, the RNM geometric mean operator $\mathcal{G}_{RNM}(A)$ is formulated as:

$$\mathcal{G}_{RNM}(A) = \mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big) = \left(\prod_{j=1}^n \left(\underline{Nm}(A_j)\right)^{\frac{1}{n}}, \prod_{j=1}^n \left(\overline{Nm}(A_j)\right)^{\frac{1}{n}}\right) \tag{10}$$

where

$$\prod_{j=1}^n \left(\underline{Nm}(A_j)\right)^{\frac{1}{n}} = \left(\prod_{j=1}^n \left(T^i_{\underline{Nm}(A_j)}\right)^{\frac{1}{n}}, 1-\prod_{j=1}^n \left(1-I^i_{\underline{Nm}(A_j)}\right)^{\frac{1}{n}}, 1-\prod_{j=1}^n \left(1-F^i_{\underline{Nm}(A_j)}\right)^{\frac{1}{n}}\right)$$ and

$\prod_{j=1}^{n}\left(\overline{Nm}(A_j)\right)^{\frac{1}{n}}$
$=\left(\prod_{j=1}^{n}\left(T^{i}_{\overline{Nm}(A_j)}\right)^{\frac{1}{n}}, 1-\prod_{j=1}^{n}\left(1-I^{i}_{\overline{Nm}(A_j)}\right)^{\frac{1}{n}}, 1-\prod_{j=1}^{n}\left(1-F^{i}_{\overline{Nm}(A_j)}\right)^{\frac{1}{n}}\right).$

Theorem 1. Let $Nm(A_j)=\left(\underline{Nm}(A_j),\overline{Nm}(A_j)\right)$ $(j=1,2,\dots,n)$ be a collection of RNM numbers. The aggregated value $\mathcal{G}_{RNM}(\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)$ is also a RNM number.

Proof

$\underline{Nm}(A_j)$ and $\overline{Nm}(A_j)$ are NM numbers. From Definition 10, $\prod_{j=1}^{n}\left(\underline{Nm}(A_j)\right)^{\frac{1}{n}}$ and $\prod_{j=1}^{n}\left(\overline{Nm}(A_j)\right)^{\frac{1}{n}}$ are NM numbers. Hence, $\mathcal{G}_{RNM}(Nm(A_1),Nm(A_2),\dots,Nm(A_n)$ is also a RNM number in (X,R). ■

Proposition 1. The $\mathcal{G}_{RNM}(A)$ operator satisfies the following properties:

(A1): *Idempotent law*: If $Nm(A_j)=Nm(A)$ for $j=1,2,\dots,n$, then there exists $\mathcal{G}_{RNM}(Nm(A_1),Nm(A_2),\dots,Nm(A_n)=Nm(A))$.

(A2): *Boundedness*: If $Nm(A)^-=\min\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)$ and $Nm(A)^+=\max\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)$ for $j=1,2,\dots,n$, then there exists $Nm(A)^-\subseteq\mathcal{G}_{RNM}\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)\subseteq Nm(A)^+$. Both operators are bounded.

(A3): *Monotonicity*: If $Nm(A_j)\subseteq Nm(B_j)$ for $j=1,2,\dots,n$ be two collections of rough neutrosophic multisets, then $\mathcal{G}_{RNM}\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)\subseteq \mathcal{G}_{RNM}\left(Nm(B_1),Nm(B_2),\dots,Nm(B_n)\right)$ holds.

(A4): *Commutativity*: If $\left(Nm(A_1{}^{\circ}),Nm(A_2{}^{\circ}),\dots,Nm(A_n{}^{\circ})\right)$ is any permutation of $\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)$, then
$\mathcal{G}_{RNM}\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)$
$=\mathcal{G}_{RNM}\left(Nm(A_1{}^{\circ}),Nm(A_2{}^{\circ}),\dots,Nm(A_n{}^{\circ})\right)$ holds.

Proof

(A1): For $Nm(A_j)=Nm(A)$,
$\mathcal{G}_{RNM}\left(Nm(A_1),Nm(A_2),\dots,Nm(A_n)\right)$
$=\left(\prod_{j=1}^{n}\left(\underline{Nm}(A_j)\right)^{\frac{1}{n}},\prod_{j=1}^{n}\left(\overline{Nm}(A_j)\right)^{\frac{1}{n}}\right)$
$=\left(\underline{Nm}(A_j),\overline{Nm}(A_j)\right)=Nm(A_j)=Nm(A).$

(A2): Let $Nm(A_j)=\left(\underline{Nm}(A_j),\overline{Nm}(A_j)\right)$ $(j=1,2,\dots,n)$ be a collection of RNM numbers and let
$Nm(A)^-=\left(\min_j T^{i}_{\underline{Nm}(A_j)},\max_j I^{i}_{\underline{Nm}(A_j)},\max_j F^{i}_{\underline{Nm}(A_j)}\right),$
$\left(\min_j T^{i}_{\overline{Nm}(A_j)},\max_j I^{i}_{\overline{Nm}(A_j)},\max_j F^{i}_{\overline{Nm}(A_j)}\right)$ and
$Nm(A)^+=\left(\max_j T^{i}_{\underline{Nm}(A_j)},\min_j I^{i}_{\underline{Nm}(A_j)},\min_j F^{i}_{\underline{Nm}(A_j)}\right),$
$\left(\max_j T^{i}_{\overline{Nm}(A_j)},\min_j I^{i}_{\overline{Nm}(A_j)},\min_j F^{i}_{\overline{Nm}(A_j)}\right).$

Then there exist
$Nm(A)^- \subseteq \mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big) \subseteq Nm(A)^+$.

(A3): Since $Nm(A_j) \subseteq Nm(B_j)$ for $j = 1,2,\dots,n$,
$\mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big) \subseteq$
$\mathcal{G}_{RNM}\big(Nm(B_1), Nm(B_2), \dots, Nm(B_n)\big)$.
It proves the monotonicity of the function $\mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big)$.

(A4): Since $\big(Nm(A_1{}^\circ), Nm(A_2{}^\circ), \dots, Nm(A_n{}^\circ)\big)$ is any permutation of $\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big)$,
$\mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big)$
$\cup\, \mathcal{G}_{RNM}\big(Nm(A_1{}^\circ), Nm(A_2{}^\circ), \dots, Nm(A_n{}^\circ)\big)$
$= \mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big)$ or
$\mathcal{G}_{RNM}\big(Nm(A_1{}^\circ), Nm(A_2{}^\circ), \dots, Nm(A_n{}^\circ)\big)$
Hence,
$\mathcal{G}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big) =$
$\mathcal{G}_{RNM}\big(Nm(A_1{}^\circ), Nm(A_2{}^\circ), \dots, Nm(A_n{}^\circ)\big)$.

Therefore, the proof is complete. ■

Definition 12. Let $Nm(A_j) = \big(\underline{Nm}(A_j), \overline{Nm}(A_j)\big)$ in (X, R) $(j = 1,2,\dots,n)$ be a collection of RNM numbers and $[\mathcal{w}_1, \mathcal{w}_2, \dots, \mathcal{w}_n]$ be the weight of RNM numbers $Nm(A_1), Nm(A_2), \dots, Nm(A_n)$. Then the weighted RNM geometric aggregation operator $\mathcal{G}^{\mathcal{w}}_{RNM}(A)$ is formulated as:

$$\mathcal{G}^{\mathcal{w}}_{RNM}(A) = \mathcal{G}^{\mathcal{w}}_{RNM}\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big)$$
$$= \left(\prod_{j=1}^{n}\left(\underline{Nm}(A_j)\right)^{\mathcal{w}_j}, \prod_{j=1}^{n}\left(\overline{Nm}(A_j)\right)^{\mathcal{w}_j}\right) \qquad (12)$$

where

$\prod_{j=1}^{n}\left(\underline{Nm}(A_j)\right)^{\mathcal{w}_j}$
$= \left(\prod_{j=1}^{n}\left(T^i_{\underline{Nm}(A_j)}\right)^{\mathcal{w}_j}, 1 - \prod_{j=1}^{n}\left(1 - I^i_{\underline{Nm}(A_j)}\right)^{\mathcal{w}_j}, 1 - \prod_{j=1}^{n}\left(1 - F^i_{\underline{Nm}(A_j)}\right)^{\mathcal{w}_j}\right)$ and
$\prod_{j=1}^{n}\left(\overline{Nm}(A_j)\right)^{\mathcal{w}_j}$
$= \left(\prod_{j=1}^{n}\left(T^i_{\overline{Nm}(A_j)}\right)^{\mathcal{w}_j}, 1 - \prod_{j=1}^{n}\left(1 - I^i_{\overline{Nm}(A_j)}\right)^{\mathcal{w}_j}, 1 - \prod_{j=1}^{n}\left(1 - F^i_{\overline{Nm}(A_j)}\right)^{\mathcal{w}_j}\right)$

Theorem 2. Let $Nm(A_j) = \big(\underline{Nm}(A_j), \overline{Nm}(A_j)\big)$ $(j = 1,2,\dots,n)$ be a collection of RNM numbers. The aggregated value $\mathcal{G}^{\mathcal{w}}_{RNM}(\big(Nm(A_1), Nm(A_2), \dots, Nm(A_n)\big)$ is also a RNM number.

Proof

$\underline{Nm}(A_j)$ and $\overline{Nm}(A_j)$ are NM numbers. From Definition 10, $\prod_{j=1}^{n}\left(\underline{Nm}(A_j)\right)^{\mathcal{w}_j}$and $\prod_{j=1}^{n}\left(\overline{Nm}(A_j)\right)^{\mathcal{w}_j}$ are NM numbers. Hence, $\mathcal{G}_{RNM}(Nm(A_1), Nm(A_2), \dots, Nm(A_n)$ is also a RNM number in (X, R). ■

4 Implementation

This section described the used of each definition from preliminaries section and derivation of rough neutrosophic multisets geometric aggregation operator (RNMGAO). Next, the implemented of RNMGAO operator with entropy weight combined roughness Dice similarity measure in medical diagnosis is presented. The original data is adapted from [53].

4.1 Rough neutrosophic multisets geometric aggregation operator with entropy weight combined roughness dice similarity measure

All the operators used in this research is described as follows.
Let $RNM(A)$ and $RNM(B)$ are two set of rough neutrosophic multisets (RNM) in an equivalence relation.

i) To determine the entropy measure for $RNM(A)$ as referring Definition 3. The aim is to reduce the fuziness degree for data collected in RNM environment. Then, the result is used as entropy weight for j-th criteria for $RNM(A)$ as refering Definition 4.
ii) The entropy weight measure of $RNM(A)$ is assign for each criterion simultaneously with the RNMGAO. The weight RNMGAO ($\mathcal{G}_{RNM}^{w}(A)$) is determined as referring Definition 12. The aim is to overcome the multiplicity of the data collected.
iii) Then, the roughness measure for $RNM(A)$ and $RNM(B)$ is determined to increase the RNM-data accuracy.
iv) The roughness data of $RNM(A)$ and $RNM(B)$ is used in determined the Dice similarity measure of RNM is for ranking purposed. Therefore, the roughness Dice similarity measure $S_{RNM}^{D}(A,B)$ between $RNM(A)$ and $RNM(B)$ is determined as referring Definition 2.

From step i) until iv), all the operators used is in rough neutrosophic multisets environment.

4.2 Application in medical diagnosis

For the last three days, $(i = 1,2,3)$ patient, P is suffering from a cough, high fever, runny nose, sore throat, muscle pain, and headache. Based on the appearing symptoms, a doctor assumes epidemic diseases which are coronavirus (COVID-19), Influenza, and Pneumonia. The patient's description based on appearing symptoms is recorded. Meanwhile, a relation between three diseases, $D = \{ D_1, D_2, D_3\}$ and six symptoms, $S = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ is also collected. Here, D_1=Influenza, D_2= COVID-19, D_3= Pneumonia, x_1= Cough, x_2= Headache, x_3= Runny nose, x_4= Muscle pain, x_5= Sore throat, x_6= High fever.
The flowchart of the process for medical diagnosis used based on RNM operators is shown in Figure 1.

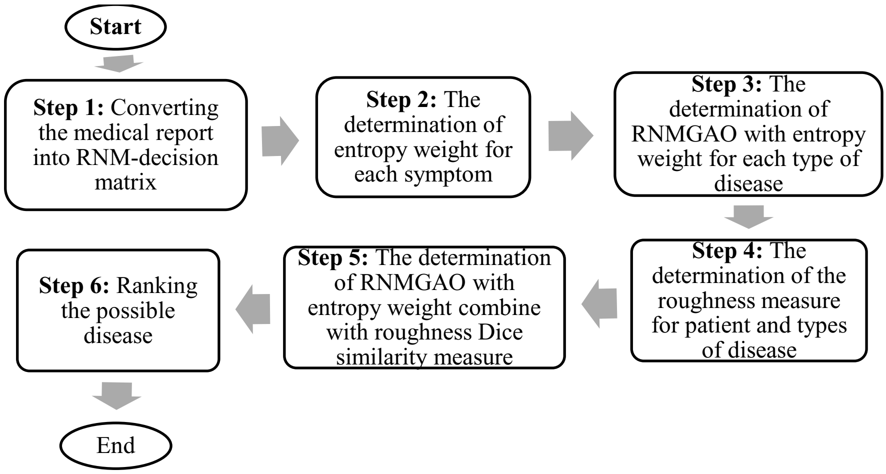

Fig. 1. The Flowchart of the Process for Medical Diagnosis based on Rough Neutrosophic Multisets Environment.

Based on Figure 1, the medical diagnosis based on the RNM environment is discussed in the following steps.
Step 1: Converting the medical report into RNM-decision matrix
The rough neutrosophic multisets numbers for patient P with respect to criteria $S = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ according to three days of collected data $(i = 1,2,3)$ can be represented as the following form.

$$P = \left\{ \begin{matrix} \left\langle x_1, \begin{pmatrix} [(0.85, 0.85, 0.85), (0.05, 0.05, 0.05), (0.05, 0.05, 0.05)], \\ [(0.95, 0.95, 0.95), (0.15, 0.15, 0.15), (0.15, 0.15, 0.15)] \end{pmatrix} \right\rangle, \\ \left\langle x_2, \begin{pmatrix} [(0.25, 0.45, 0.85), (0.55, 0.35, 0.05), (0.65, 0.35, 0.05)], \\ [(0.45, 0.55, 0.95), (0.65, 0.45, 0.15), (0.75, 0.55, 0.15)] \end{pmatrix} \right\rangle, \\ \left\langle x_3, \begin{pmatrix} [(0.45, 0.45, 0.85), (0.35, 0.35, 0.05), (0.35, 0.35, 0.05)], \\ [(0.55, 0.55, 0.95), (0.45, 0.45, 0.15), (0.55, 0.55, 0.15)] \end{pmatrix} \right\rangle, \\ \left\langle x_4, \begin{pmatrix} [(0.25, 0.45, 0.85), (0.55, 0.35, 0.05), (0.65, 0.35, 0.05)], \\ [(0.45, 0.55, 0.95), (0.65, 0.45, 0.15), (0.75, 0.55, 0.15)] \end{pmatrix} \right\rangle, \\ \left\langle x_5, \begin{pmatrix} [(0.45, 0.45, 0.85), (0.35, 0.35, 0.05), (0.35, 0.35, 0.05)], \\ [(0.55, 0.55, 0.95), (0.45, 0.45, 0.15), (0.55, 0.55, 0.15)] \end{pmatrix} \right\rangle, \\ \left\langle x_6, \begin{pmatrix} [(0.85, 0.85, 0.85), (0.05, 0.05, 0.05), (0.05, 0.05, 0.05)], \\ [(0.95, 0.95, 0.95), (0.15, 0.15, 0.15), (0.15, 0.15, 0.15)] \end{pmatrix} \right\rangle. \end{matrix} \right\}$$

Let R be an equivalent relation on S as (R, S) indicated the same types of symptoms. The relation is composed of $(R, S) = \{\{x_1, x_6\}, \{x_2, x_3, x_4, x_5\}\}$. This relation indicated that cough (x_1) and high fever (x_6) are the most common symptoms and headaches (x_2), runny nose (x_3), muscle pain (x_4), and sore throat (x_5) are the less common symptom [43]. The relation between disease $D = \{D_1, D_2, D_3\}$ and symptoms $S = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ represent by rough neutrosophic multisets number is presented in the following form.

$$D_1(Influenza) = \left\{ \begin{array}{l} \left\langle x_1, \begin{pmatrix} [(0.45, 0.53, 0.67), (0.23, 0.33, 0.41), (0.67, 0.68, 0.72)], \\ [(0.52, 0.61, 0.68), (0.18, 0.23, 0.11), (0.21, 0.34, 0.17)] \end{pmatrix} \right\rangle, \\ \left\langle x_2, \begin{pmatrix} [(0.38, 0.47, 0.59), (0.41, 0.38, 0.41), (0.27, 0.41, 0.37)], \\ [(0.61, 0.61, 0.73), (0.13, 0.19, 0.11), (0.18, 0.13, 0.11)] \end{pmatrix} \right\rangle, \\ \left\langle x_3, \begin{pmatrix} [(0.38, 0.47, 0.59), (0.41, 0.38, 0.41), (0.27, 0.41, 0.37)], \\ [(0.61, 0.61, 0.73), (0.13, 0.19, 0.11), (0.18, 0.13, 0.11)] \end{pmatrix} \right\rangle, \\ \left\langle x_4, \begin{pmatrix} [(0.38, 0.47, 0.59), (0.41, 0.38, 0.41), (0.27, 0.41, 0.37)], \\ [(0.61, 0.61, 0.73), (0.13, 0.19, 0.11), (0.18, 0.13, 0.11)] \end{pmatrix} \right\rangle, \\ \left\langle x_5, \begin{pmatrix} [(0.38, 0.47, 0.59), (0.41, 0.38, 0.41), (0.27, 0.41, 0.37)], \\ [(0.61, 0.61, 0.73), (0.13, 0.19, 0.11), (0.18, 0.13, 0.11)] \end{pmatrix} \right\rangle, \\ \left\langle x_6, \begin{pmatrix} [(0.45, 0.53, 0.67), (0.23, 0.33, 0.41), (0.67, 0.68, 0.72)], \\ [(0.52, 0.61, 0.68), (0.18, 0.23, 0.11), (0.21, 0.34, 0.17)] \end{pmatrix} \right\rangle. \end{array} \right\},$$

$$D_2(COVID-19) = \left\{ \begin{array}{l} \left\langle x_1, \begin{pmatrix} [(0.73, 0.83, 0.89), (0.17, 0.31, 0.23), (0.36, 0.27, 0.31)], \\ [(0.87, 0.92, 0.97), (0.12, 0.11, 0.13), (0.23, 0.22, 0.21)] \end{pmatrix} \right\rangle, \\ \left\langle x_2, \begin{pmatrix} [(0.67, 0.73, 0.78), (0.23, 0.25, 0.27), (0.34, 0.37, 0.31)], \\ [(0.83, 0.89, 0.95), (0.15, 0.15, 0.23), (0.23, 0.25, 0.17)] \end{pmatrix} \right\rangle, \\ \left\langle x_3, \begin{pmatrix} [(0.67, 0.73, 0.78), (0.23, 0.25, 0.27), (0.34, 0.37, 0.31)], \\ [(0.83, 0.89, 0.95), (0.15, 0.15, 0.23), (0.23, 0.25, 0.17)] \end{pmatrix} \right\rangle, \\ \left\langle x_4, \begin{pmatrix} [(0.67, 0.73, 0.78), (0.23, 0.25, 0.27), (0.34, 0.37, 0.31)], \\ [(0.83, 0.89, 0.95), (0.15, 0.15, 0.23), (0.23, 0.25, 0.17)] \end{pmatrix} \right\rangle, \\ \left\langle x_5, \begin{pmatrix} [(0.67, 0.73, 0.78), (0.23, 0.25, 0.27), (0.34, 0.37, 0.31)], \\ [(0.83, 0.89, 0.95), (0.15, 0.15, 0.23), (0.23, 0.25, 0.17)] \end{pmatrix} \right\rangle, \\ \left\langle x_6, \begin{pmatrix} [(0.73, 0.83, 0.89), (0.17, 0.31, 0.23), (0.36, 0.27, 0.31)], \\ [(0.87, 0.92, 0.97), (0.12, 0.11, 0.13), (0.23, 0.22, 0.21)] \end{pmatrix} \right\rangle. \end{array} \right\} \text{ and}$$

$$D_3(Pneumonia) = \left\{ \begin{array}{l} \left\langle x_1, \begin{pmatrix} [(0.21, 0.31, 0.53), (0.71, 0.56, 0.51), (0.38, 0.39, 0.45)], \\ [(0.43, 0.51, 0.61), (0.51, 0.31, 0.41), (0.21, 0.37, 0.41)] \end{pmatrix} \right\rangle, \\ \left\langle x_2, \begin{pmatrix} [(0.27, 0.35, 0.41), (0.63, 0.71, 0.63), (0.81, 0.71, 0.58)], \\ [(0.37, 0.43, 0.52), (0.53, 0.25, 0.21), (0.27, 0.35, 0.14)] \end{pmatrix} \right\rangle, \\ \left\langle x_3, \begin{pmatrix} [(0.27, 0.35, 0.41), (0.63, 0.71, 0.63), (0.81, 0.71, 0.58)], \\ [(0.37, 0.43, 0.52), (0.53, 0.25, 0.21), (0.27, 0.35, 0.14)] \end{pmatrix} \right\rangle, \\ \left\langle x_4, \begin{pmatrix} [(0.27, 0.35, 0.41), (0.63, 0.71, 0.63), (0.81, 0.71, 0.58)], \\ [(0.37, 0.43, 0.52), (0.53, 0.25, 0.21), (0.27, 0.35, 0.14)] \end{pmatrix} \right\rangle, \\ \left\langle x_5, \begin{pmatrix} [(0.27, 0.35, 0.41), (0.63, 0.71, 0.63), (0.81, 0.71, 0.58)], \\ [(0.37, 0.43, 0.52), (0.53, 0.25, 0.21), (0.27, 0.35, 0.14)] \end{pmatrix} \right\rangle, \\ \left\langle x_6, \begin{pmatrix} [(0.21, 0.31, 0.53), (0.71, 0.56, 0.51), (0.38, 0.39, 0.45)], \\ [(0.43, 0.51, 0.61), (0.51, 0.31, 0.41), (0.21, 0.37, 0.41)] \end{pmatrix} \right\rangle. \end{array} \right\}$$

Step 2: The determination of entropy weight for each symptom
The entropy weight of RNM is computed for determining the weight for each symptom based on Definitions 3 and 4. As a result, the entropy weight for each symptom is shown as follows.

$$w_1{}^1 = w_6{}^1 = 0.1729, w_2{}^1 = w_3{}^1 = w_4{}^1 = w_5{}^1 = 0.1636$$

Step 3: The determination of RNMGAO with entropy weight for each type of disease
The entropies' weight in step 2 is used to determine the weight RNMGAO, $\mathcal{G}_{RNM}^{w}(A)$ based on Definitions 11 and 12.
Step 4: The determination of the roughness measure for patient and types of disease
The roughness measure is determined based on Definition 2.
Step 5: The determination of RNMGAO with entropy weight combine with roughness Dice similarity measure.
The roughness result in step 4 is used to determine the Dice similarity measure based on Definition 2. The result is shown in Table 1.

Table 1. The Dice Similarity Measures

Similarity measure	Influenza (D_1)	COVID-19 (D_2)	Pneumonia (D_3)
Dice similarity measure, $S_{RNM}^{D}(A, B)$	0.7829	0.9488	0.1745

Step 6: Ranking the possible disease.
Ranking the optimal solution based on the result in step 5. If the similarity measure is closer to one, the conclusion is that the alternatives are possibly selected. Based on Table 1, the ranking disease order is COVID-19 (D_2) $\prec$ Influenza (D_1) $\prec$ Pneumonia (D_3), and the result indicated that the patient is suffering from COVID-19.

5 Conclusion

In this research, an algebraic operation such as addition, multiplication, scalar, and power operation over rough neutrosophic multisets (RNM) is introduced. The use of the algebraic operation of RNM is derived from the basic definition of rough neutrosophic multisets geometric aggregation operator (RNMGAO). The idempotent law, boundedness, monotonicity, and commutativity properties of RNMGAO are completely proven. The aim is to handle the multiplicity of the information collected in the RNM environment. Next, the RNMGAO operator is presented with the entropy weight measure combined roughness Dice similarity measure for RNM. The objectives are to cater to the fuzziness degree of information by entropy measure, determine the entropy weight for each attribute, and assign the weight to RNMGAO for aggregate value in the alternative given. Lastly, the roughness Dice similarity measure is applied to rank the optimal solution. The flowchart of the process in medical diagnosis based on RNM operators was applied to show the applicability of the new operator approach. In the future, the other type of aggregation operator can be derived for RNM theory. Also, the RNMGAO and weight entropy with combination of roughness measure will be applied in others field such as investment, education and transportation selection problem.

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