Neutrosophic Bézier Curve Model for Uncertainty Problem Using Approximation Approach

Siti Nur Idara Rosli[1] and *Mohammad Izat Emir* Zulkifly[1]

[1]Department of Mathematical Sciences, Faculty of Science, University of Technology Malaysia (UTM), 81310 Johor Bahru, Johor, Malaysia

Abstract. The problem of gathering data with uncertainty is difficult to address since certain values are eliminated owing to noise. Thus, the fundamental gap revealed is that fuzzy and intuitionistic fuzzy sets cannot deal with indeterminacy problems as compared to neutrosophic sets. This research demonstrates how to use a neutrosophic set to approximate the Bézier curve. The neutrosophic set and its qualities are used to identify the neutrosophic control point relation in the first stage. The control point and the Bernstein basis function are then combined to form a neutrosophic Bézier. The curve is then depicted using an approximation method involving truth membership, false membership, and indeterminacy membership curves. A numerical example and an algorithm for obtaining the neutrosophic Bézier curve are provided at the end of this work. As a result, this research can help data analysts acquire data without wasting any uncertain information data. Besides, this study can make a significant contribution to the scope of computational mathematics and modeling.

1 Introduction

Dealing with uncertain data presents significant challenges in a variety of industries, including finance, engineering, and research. Various strategies for dealing with uncertainty have been developed, including statistical inference, Bayesian analysis, fuzzy logic, and neural networks. "The fuzzy set (FS) model is a theoretical paradigm for ealing with imprecise and ambiguous data. Lotfi Zadeh developed it in the 1960s to address the inherent ambiguity and ambiguity of language itself and human intellect" [1]. Thus, FS just analyses truth membership and false membership data and ignores and does not assess uncertainty. As a result, Krassimir Atanassov [2] created the ntuitionistic uzzy set (IFS) theory in 1986, which is a generalization of FS that incorporates truth degree, false degree, and uncertainty information. It is perfectly suitable for coping with ambiguity. "Since FS theory only considers entire membership data, the IFS concept is an alternative technique for establishing FS when information is recorded is insufficient to classify and process" [3]. Several academics have used geometric modeling with an IFS method to deal with ambiguous data [4-6].

However, Florentin Smarandache [7] came up with the idea of neutrosophic to use the idea of neutrality in math. The neutrosophic set (NS) idea is described by degrees of belonging, not belonging, and not knowing. The IFS idea is different from the NS idea in

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the following ways: IFS are described by the truth degree, false degree, and the degree of uncertainty. On the other hand, NS looks at the degree of uncertainty or indeterminacy separately from the degree of membership or truth and the degree of false, and all the degrees are looked at separately. It all depends on the topic or subject space (discourse universe). In this way, an NS is a way to solve and show problems that involve more than one field. In NS theory, on the other hand, an element can belong to more than a set at the same time. This makes it possible to show more complicated kinds of uncertainty and ambiguity, like when a statement can be both true and untrue. Some studies have also used geometric modeling and neutrosophic set techniques [8,9].

In terms of showing curves, the set of data is a crucial component. If there is some ambiguity in a data collection, this needs to be sorted before being used to build curve models. Using an NS to make geometric models is a good way to deal with the problem of making data clear when there is uncertainty. Several studies have been done to make sure that curves and surfaces in geometric modeling with certainty data can be used easily [23-25]. However, there are some academic studies on fuzzy geometry modeling [10–15]. Recently, Rosli and Zulkifly [27-32] proposed neutrosophic geometric modeling for B-splin curve interpolation, B-spline surface nterpolation, 3-dimensional quartic Bézier curve approximation model, bicubic Bezier surface approximation, 3-dimensional B-spline approximation model, and an interval neutrosophic cubic Bézier curve approximation model respectively. Meanwhile, this study will propose the 2-dimensional cubic Bézier curve by using the approximation method. When neutrosophic elements are present, the process of analyzing, visualizing, and modeling data into curves and surfaces becomes more challenging. When dealing with uncertain data, there is a high likelihood of data being rejected. That is the main problem of this study. As a motivation for this study, uncertain data must be treated so that the data's nature and behaviors can provide an accurate picture of a modeled study.

The purpose of this research is to develop a modeling tool that can handle ambiguous data and is based on the neutrosophic Bézier curve (NBC) approximation. Before constructing NBC, the NS, and its attributes must be used to define the neutrosophic control point relation (NCPR). The Bernstein basis function is used with these control points to make models of NBCs, which are then shown using an approximation method. The following is how this paper is structured. Section 1 addressed some background information about this study. Section 2 demonstrates the essential concept of neutrosophic and the NCPR. Section 3 presents the use of NCPR to approximate the neutrosophic Bézier curve (NBC). Section 4 provides a numerical example as well as a visualization of NBC. The characteristics of the curve, as well as the technique used to produce them, are also presented. Finally, part 5 will bring this investigation to a close.

2 Preliminaries

This section defines NSs fundamentally, including the concept of NS and the NCPR. The inconsistent Intuitionistic Set (IS) is a variant of the Neutrosophic Set, which is also known as the Pictorial FS, Pythagoras FS, Sphere FS, and q-Rung Orthopair FS [16]. In addition, none of these sets are more general than the Intuitionistic Set (IS) [16]. In fuzzy systems, the intuitionistic set can accommodate partial information but not unclear or distorted data [16]. “A neutrosophic set has three membership functions. There are three of them: a truth degree, ‘*T*’, an indeterminacy degree, ‘*I*’ and a falsity degree, ‘*F*’ which adds the new term “indeterminacy" to the NS specification” [16].

Definition 1 [7]

Let Y be the main of conversation, with elements in Y denoted as y. The neutrosophic set is an object in the form.

$$\hat{A}=\{\left\langle y:T_{\hat{A}(y)},I_{\hat{A}(y)},F_{\hat{A}(y)}\right\rangle \mid y\in Y\} \tag{1}$$

where, the functions $T,I,F:Y\rightarrow]^{-}0,1^{+}[$ define, respectively, the degree of truth membership, the degree of indeterminacy, and the degree of false membership of the element $y\in Y$ to the set $\hat{A}$ with the condition;

$$0^{-}\leq T_{\hat{A}}(y)+I_{\hat{A}}(y)+F_{\hat{A}}(y)\leq 3^{+} \tag{2}$$

There is no limit to the amount of $T_{\hat{A}}(y),I_{\hat{A}}(y)$ and $F_{\hat{A}}(y)$
A value is chosen by NS from one of the real standard subsets or one of the non-standard subsets of $]^{-}0,1^{+}[$. The actual value of the interval $[0,1]$, on the other hand, $]^{-}0,1^{+}[$ will be utilized in technical applications since its utilization in real data such as the resolution of scientific challenges, will be physically impossible. As a direct consequence of this, membership value utilization is increased.

$$\hat{A}=\{\left\langle y:T_{\hat{A}(y)},I_{\hat{A}(y)},F_{\hat{A}(y)}\right\rangle \mid y\in Y\} \text{ and } T_{\hat{A}}(y),I_{\hat{A}}(y),F_{\hat{A}}(y)\in[0,1] \tag{3}$$

There is no restriction on the sum of $T_{\hat{A}}(y),I_{\hat{A}}(y),F_{\hat{A}}(y)$ .Therefore,

$$0\leq T_{\hat{A}}(y)+I_{\hat{A}}(y)+F_{\hat{A}}(y)\leq 3 \tag{4}$$

Definition 2 [8,9]
Let $\hat{N}=\{\left\langle z:T_{\hat{N}(z)},I_{\hat{N}(z)},F_{\hat{N}(z)}\right\rangle \mid z\in Z\}$ and $\hat{M}=\{\left\langle y:T_{\hat{M}(y)},I_{\hat{M}(y)},F_{\hat{M}(y)}\right\rangle \mid y\in Y\}$ be neutrosophic elements. Thus, $NR=\{\left\langle (z,y):T_{(z,y)},I_{(z,y)},F_{(z,y)}\right\rangle \mid z\in\hat{N},y\in\hat{M}\}$ is a Neutrosophic Relation (NR) on $\hat{N}$ and $\hat{M}$.

Definition 3 [8,9] NS of $\hat{N}$ in space Z is Neutrosophic Point (NP) and $\hat{N}=\{\hat{N}_i\}$ where $i=0,...,n$ is a collection of NPs where the existence $T_{\hat{N}}:Z\rightarrow[0,1]$ as truth degree, $I_{\hat{N}}:Z\rightarrow[0,1]$ as indeterminacy degree and $F_{\hat{N}}:Z\rightarrow[0,1]$ as false degree with

$$T_{\hat{N}}(\hat{N})=\begin{cases}0 & \text{if } \hat{N}_i\notin\hat{N}\\ a\in(0,1) & \text{if } \hat{N}_i\,\hat{\in}\,\hat{N}\\ 1 & \text{if } \hat{N}_i\in\hat{N}\end{cases}$$

$$I_{\hat{N}}(\hat{N})=\begin{cases}0 & \text{if } \hat{N}_i\notin\hat{N}\\ b\in(0,1) & \text{if } \hat{N}_i\,\hat{\in}\,\hat{N}\\ 1 & \text{if } \hat{N}_i\in\hat{N}\end{cases} \tag{5}$$

$$F_{\hat{N}}(\hat{N}) = \begin{cases} 0 & \text{if } \hat{N}_i \notin \hat{N} \\ c \in (0,1) & \text{if } \hat{N}_i \hat{\in} \hat{N} \\ 1 & \text{if } \hat{N}_i \in \hat{N} \end{cases}$$

2.1 Neutrosophic point relation (NPR)

The concept of the NS, which was discussed in the previous section, serves as the cornerstone for NPR. If *N* and *M* are a group of Euclid eternal space points and then, the following is how NPR is described:

Definition 4. [27] Let N, M be a grouping of elements in a global area that are part of a set that is not null and $N, M, O \in \mathbf{R} \times \mathbf{R} \times \mathbf{R}$, then the term "NPR" refers to

$$\hat{R} = \left\{ \begin{array}{l} \left\langle \left(n_i, m_j\right), T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \right\rangle \\ \mid T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j) \in I \end{array} \right\} \tag{6}$$

where $\left(n_i, m_j\right)$ is a set of ordered positions and $\left(n_i, m_j\right) \in N \times M$ while $T_R(n_i, m_j), I_R(n_i, m_j), F_R(n_i, m_j)$ are the truth membership, the indeterminacy membership, and the false membership that follows the condition of the neutrosophic set which is respectively, $0 \le T_{\hat{N}}(z) + I_{\hat{N}}(z) + F_{\hat{N}}(z) \le 3$.

2.2 Neutrosophic control point relation (NCPR)

"A control point (CP) is a single point or set of points utilized in graphic design and mathematical simulation that impacts the shape or behavior of a curve, surface, or other geometric object" [27]. In this study, CPs are a set of points utilized to determine the outlines of NBC. It also is crucial to the generation and production of smooth curves in geometric modeling. The concept of NS and its attributes are used in this section to define NCPR. Based on research in [17-19] the FS concept is used to define fuzzy control points.

Definition 5 [27]
Let $\hat{R}$ be an NPR, then NCPR is viewed as a group of points $n+1$ that denotes locations and coordinates is used to describe the curve and is indicated by

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_0^T, \hat{p}_1^T, ..., \hat{p}_n^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_0^I, \hat{p}_1^I, ..., \hat{p}_n^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_0^F, \hat{p}_1^F, ..., \hat{p}_n^F \right\} \end{aligned} \tag{7}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are NCP for truth, false, and indeterminacy membership function and i is one less than n.

3 Approximation of neutrosophic bézier curve (NBC)

Geometric simulation often makes use of Bézier curves, which are parameterized curves whose shape is governed by a control polygon [20, 21]. The number of data points used to build the curve is equivalent to the degree of the polynomial [22]. The following definition gives an instance of a curve for Bézier, which is generated by integrating the Bernstein polynomial or basis function with fuzzy control points. NCPR and Definition 1 are used to construct the NBC, which is then mixed with the Bézier blending function in a geometric model. Next, it discusses the characteristics of the NBC model. Mathematically, NBC for approximation method is represented as follows:

Definition 6
Let

$$\hat{B}_i^T = \left\{\hat{b}_0^T, \hat{b}_1^T, \ldots, \hat{b}_n^T\right\}; \hat{B}_i^I = \left\{\hat{b}_0^I, \hat{b}_1^I, \ldots, \hat{b}_n^I\right\}; \hat{B}_i^F = \left\{\hat{b}_0^F, \hat{b}_1^F, \ldots, \hat{b}_n^F\right\}$$

where $i = 0,1,\ldots,n$ is NCPR. NBC is defined as $BC(t)$ with the curve position vector depending on the value of the value t, then blending with J_i by Bézier curves are represented by the blending function as

$$BC(t) = \sum_{i=0}^{n} \hat{B}_i^T J_{n,i}(t) \qquad 0 \le \mathrm{t} \le 1 \tag{8}$$

$$BC(t) = \sum_{i=0}^{n} \hat{B}_i^I J_{n,i}(t) \qquad 0 \le \mathrm{t} \le 1 \tag{9}$$

$$BC(t) = \sum_{i=0}^{n} \hat{B}_i^F J_{n,i}(t) \qquad 0 \le \mathrm{t} \le 1 \tag{10}$$

where $0 \le t \le 1$ and the blending function is a Bézier or Bernstein basis, J_i:

$$J_{(n,i)}(t) = \binom{n}{i} t^i (1-t)^{n-i} \qquad (0)^0 \equiv 1 \tag{11}$$

with

$$\binom{n}{i} = \frac{n!}{i!(n-1)!} \qquad (0)^0 \equiv 1 \tag{12}$$

3.1 Properties of neutrosophic bézier curve (NBC)

In the context of NURBS curves, a Bézier curve is a specific case that is determined by a control polygon. Since the Bézier basis is the same as the Bernstein basis, it is easy to recognize several characteristics of Bézier curves. Therefore, the neutrosophic Bézier curve (NBC) possesses the following essential features:

- The NBC's fundamental features are genuine.
- There are fewer control polygon points than the degree of the polynomial defining the curve segment, NBC.
- In most cases, the NBC will conform to the outline of the control polygon.

- The NBC's starting and ending positions also happen to be the start and end points of the control polygon.
- The initial and last polygon spans correspond in direction to the tangent vectors at the ends of the NBC.
- The NBC is located inside the largest convex polygon specified by the vertices of the control polygon, also known as the convex hull of the control polygon.
- The NBC displays the phenomenon of declining variance. This means that the curve doesn't sway more frequently than the control polygon does around any given straight line.
- Affine transformations do not affect the NBC.

4 Numerical example and visualization

Let's consider a numerical example to demonstrate NBC using the approximation method. The example only employs numerical examples at random and will use the approximation method. A neutrosophic cube Bézier curve consists of four NCPs with a degree of polynomial three $n=3$ will be shown.

4.1 Application of neutrosophic bézier curve (NBC)

Let's consider 2D-shape of NBC with four control points which are $\hat{B}_0=2,\hat{B}_1=7,\hat{B}_2=11,\hat{B}_3=17$ with degree three $n=3$ as in **Table 1**. According to the example below, all data should satisfy the NS characteristics as in **Definition 1**.

Table 1. NCPR with its respective degrees

NCPR $\hat{B}_i$	Truth Membership $\hat{B}_i^T$	Indeterminacy Membership $\hat{B}_i^I$	False Membership $\hat{B}_i^F$
$\hat{B}_0=2$	0.4	0.4	0.4
$\hat{B}_1=7$	0.2	0.6	0.7
$\hat{B}_2=11$	0.3	0.3	0.6
$\hat{B}_3=17$	0.5	0.5	0.5

Table 1 shows four NCPRs with their respective degrees. First, from the Bernstein polynomials that have been shown in **Definition 6** blending with NCPR with Truth Membership degree can be expressed as follows:

$$\hat{B}_i^T(t)=\hat{B}_0J_{3,0}(t)+\hat{B}_1J_{3,1}(t)+\hat{B}_2J_{3,2}(t)+\hat{B}_3J_{3,3}(t)$$

$$\hat{B}_i^T(t)=(2,0.4)(1-t)+(7,0.2)3t(1-t)^2+(11,0.3)3t^2(1-t)+(17,0.5)t^3$$

It's also can be represented in matrix form as follows:

$$\hat{B}_i^T(t)=\hat{B}_0J_{3,0}(t)+\hat{B}_1J_{3,1}(t)+\hat{B}_2J_{3,2}(t)+\hat{B}_3J_{3,3}(t)$$
$$=\left[J_{3,0},J_{3,1},J_{3,2},J_{3,3}\right]\left[\hat{B}_0,\hat{B}_1,\hat{B}_2,\hat{B}_3\right]$$

$$= [(1-t), 3t(1-t)^2, 3t^2(1-t), t^3] \begin{bmatrix} (2,0.4) \\ (7,0.2) \\ (11,0.3) \\ (17,0.5) \end{bmatrix}$$

$$= [-t^3 + 3t^2 - 3t + 1, 3t^3 - 6t^2 + 3t, -3t^3 + 3t^2, t^3] \begin{bmatrix} (2,0.4) \\ (7,0.2) \\ (11,0.3) \\ (17,0.5) \end{bmatrix}$$

$$= [t^3, t^2, t^1, 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} (2,0.4) \\ (7,0.2) \\ (11,0.3) \\ (17,0.5) \end{bmatrix}$$

Figure 2 shows the neutrosophic cubic Bézier curve for the Truth degree.

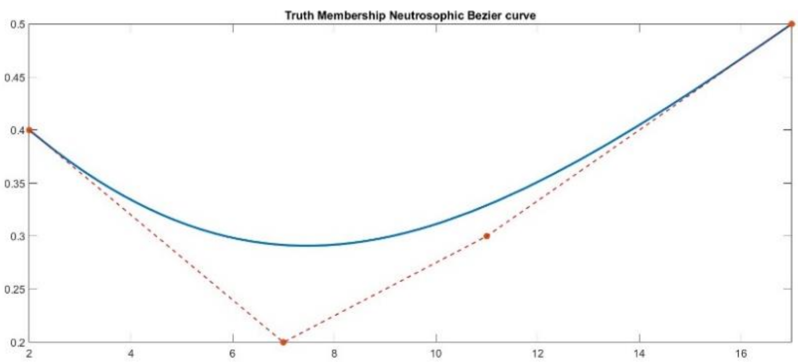

Figure 2 NBC approximation for Truth Membership

This method is used more than once to see the indeterminacy and falsity of membership curves for the degree. Therefore, the visualization of NBCs for indeterminacy and falsity membership is as follows:

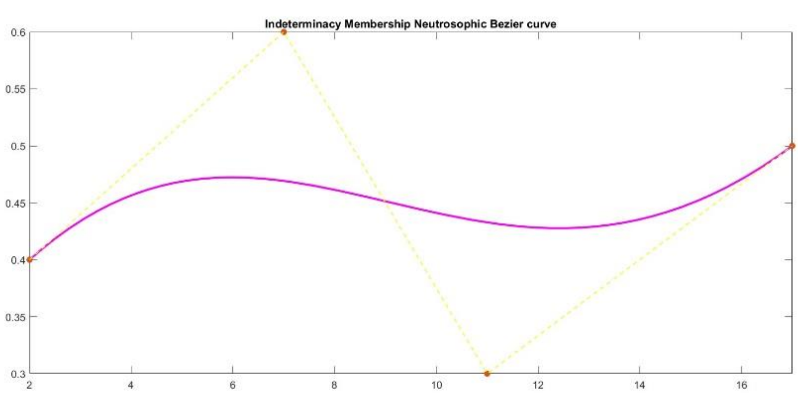

Figure 3 NBC approximation for Indeterminacy Membership

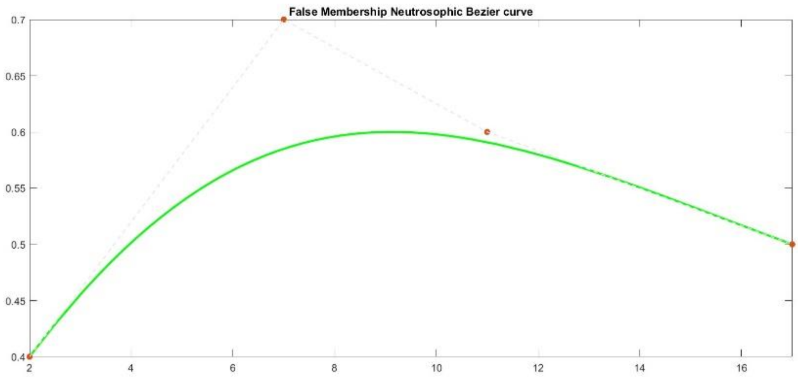

Figure 4 NBC approximation for False Membership

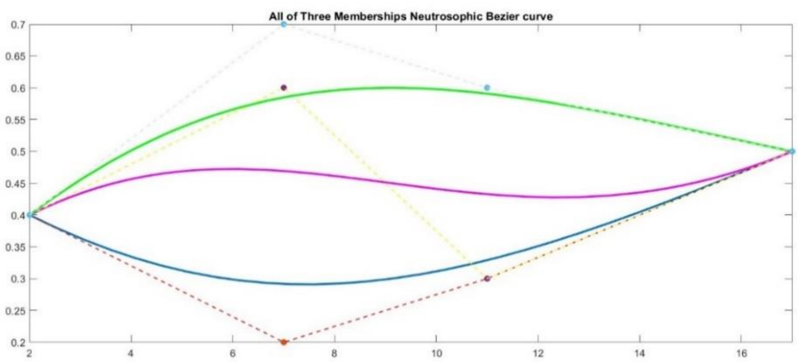

Figure 5 NBC Approximation (Truth, indeterminacy, and falsity memberships)

As part of a discussion, **Figure 5** depicts NBC to see the pattern and behavior of the Bézier curves for all memberships in a single figure. From **Figures 2** to **5**, the blue curve represents the NBC's truth membership with its control polygon shown by the red dotted line, the purple curve represents its indeterminacy membership, the yellow dotted line shows its control polygon and the green curve represents its false membership, while the grey dotted line indicates its control polygon. The neutrosophic cubic Bézier curve approximation is shown in clear detail in **Figure 5** together with the appropriate control points and control polygons for all membership. **Table 1** and all the figures show that all degrees in the interval $[0,1]$ separately since they are independent. However, the sensitivity of this analysis is the total of truth, indeterminacy, and falsity membership must be less than 3 and follow the condition of NS which is $0 \leq T_{\hat{A}}(y) + I_{\hat{A}}(y) + F_{\hat{A}}(y) \leq 3$. An algorithm for constructing the NBC as well as the flowchart of this study continues to follow:

Step 1: Describe the NCPR using the NS approach and its attributes.
Step 2: Determine the degree of polynomials, n using the amount of NCPR from Step 1.
Step 3: Based on **Definition 6**, figure out the Bernstein basis function.
Step 4: The coefficients of the terms in the parameter are collected and rewritten as shown in the example.
Step 5: Steps 1–4 are followed for memberships in the truth, indeterminacy, and falsehood.
Step 6: The NBCs will be visualized using the approximation method for all three memberships.

5 Conclusions

This study defined NBC approximation for cubic cases based on the definition of NCPR. The NBC model approximation is a good way to describe uncertainty data that includes neutrosophic properties because it has three distinct memberships. All data may be analyzed and processed using these tools. The advantage of this approach is its capacity to visualize neutrosophic data in the form of a Bézier curve, which is simple for data analysts to grasp and analyze. The neutrosophic uncertainty data problem may be handled using the NBC model, as shown in **Figures 1** to **5**. To tackle the neutrosophic data problem, this model may be expanded to the Bézier curve with interpolation technique, and Bézier surface with approximation and interpolation method for future studies. The same applies to another geometric model, such as a B-spline curve and surface that uses both approximation and interpolation methods, as well as a NURBS surface and curve that uses both approximation and interpolation methods to extend this study. This model may be utilized in a variety of applications, including bathymetry data, predictive in medical disciplines such as cancer-level forecasting, image blur identification, and catastrophe alert systems.

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