Unsteady Dusty MHD Boundary Layer Flow Past A Sphere

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Abstract. The boundary layer problem for unsteady dusty Newtonian fluid flow through a sphere influenced by magnetic field has been investigated in this paper. The two fluid flow phases that will be discussed in this work are referred to as dusty. Including the fluid and dust phases in a two-phase flow can help us comprehend the interaction of dust particles in fluid flow better. Next, the impact of magnetohydrodynamics (MHD) on fluid flow will be important to researchers since it allows them to regulate systems like cooling systems by adjusting the magnetic field. Thus, by considering the influenced of magnetic field and the existence of dust particles will be examined in this paper. The mathematical model for fluid and dusty phases is developed using continuity and momentum equations. First order partial differential equations (PDEs) are obtained by applying suitable similarity transformations on higher order PDEs The Finite Difference Method (FDM), Newton's method, and the development of a block tridiagonal matrix are the main rules that are solved using the Keller Box method. The Keller Box procedure is programmed in MATLAB environment and analysed graphically. The results are discovered to be quite compatible with earlier research.

1 Introduction

Viscous studies have developed in academic circles due to the relevance of dusty viscous flows in the petroleum industry, crude purification, physiological flows, and other technological areas [1]. Then, the application of dust particles in boundary layers is followed by dust entrainment in a cloud created by a nuclear explosion and lunar surface degradation brought on by the exhaust of a landing vehicle [2]. Magnetohydrodynamics (MHD) is the study of the motion of electrically conducting fluids in magnetic fields. MHD, which studies the magnetic characteristics and behaviour of electrically conductive fluids, is commonly referred to as magneto-fluid dynamics or hydromagnetic [3]. Moreover, a magnetic field can affect the flow of an electrically conducting fluid. This holds significance in numerous technological and engineering fields, including the production of MHD electricity, flow metres, and pumps [4]. The innovative concept of using a magnetic field gradient that is beneficial to the direction of flow to reduce drag in fluids [5]. A significant drag decrease in laminar pipe flow has been observed, which could be attributed to the implementation of a controlled magnetic field gradient. Many researchers have studied the unsteady MHD Newtonian fluid flow past a variety of geometries, including sphere[6],[7], permeable shrinking sheet [8], an infinite annulus [9], laminar pipe [10], rectangular microchannel [11], square cavity [12], and couette flow between two plates [13].

Due to the growing significance in the fields of geophysics and engineering, studies regarding two-phase MHD fluid flow models are remained highly active. In many different applications, such as gas-freezing systems and nuclear-powered reactors, dusty fluids are utilised to minimise heat. [14]. The increasing strength of microscopic dust particles is strongly related to the increase of temperature in fluid and dust phase [15]. The magnetic field is responsible to control the velocity of fluid dust phase in circular pipe case [16].

This fluid flow problem provides in a better understanding of numerous types of manufacturing and industrial processes, such as the extraction of geothermal energy, the fluidization, sedimentation, crude oil fluidization, paint spraying, powder technology, and centrifugal separation of materials from fluid. Consequently, research on unsteady dusty Newtonian fluid flow under the effect of a magnetic field has been conducted in a variety of geometries including semi-infinite isothermal inclined plates [17], and porous medium [18] and stretching sheets [19]. However, no research has been done on unsteady dusty MHD Newtonian fluid flow past a sphere up until now, and that is the primary focus of this study.

2 Mathematical formulation

In the presence of a magnetic field, a boundary layer flow past a sphere of two-dimensional viscous incompressible fluid that is unsteady and dusty is studied. There will be two sets of equations of Navier Stokes which are fluid phase and dust phase. The particle phase is thought to consist of uniformly sized, spherical particles with a consistent range of densities. The dust particles travel from their starting point to their destination at a steady speed and temperature. The focus of this study will be on dust particle velocity and density, both of which are taken to be fixed. The force buoyancy and volume friction on dust particles is omitted. It demonstrates that pressure has no impact on the fluid phase of a dust particle. At a great distance from a sphere, the flow starts impulsively at rest with a uniform stream function U_∞ and subsequently flows vertically upward. Since the induced magnetic field is not taken into consideration, a low magnetic Reynolds number is assumed. Then, the polarization of charges is negligible due to the small Reynolds number. Since the force that the fluid exerts on the dust is equal to and opposite from the force that the dust exerts on the fluid, the fluid-particle interaction equation in the momentum equation will have the opposite sign. Equations (1)-(7) provides the governing equations for the Newtonian fluid flow problem past a sphere under the influence of a magnetic field.

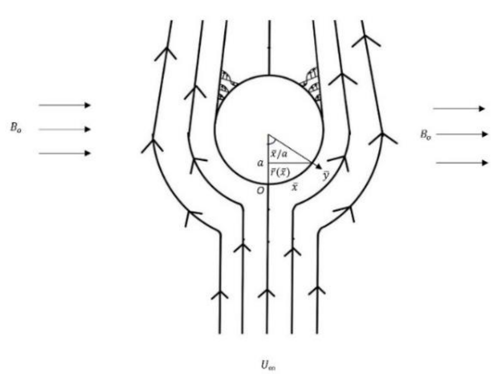

Fig. 1. Physical coordinate of the problem

Fluid phase:
Continuity equation:

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}}+\frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}}=0, \tag{1}$$

x-momentum equation:

$$\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{x}}+\frac{\mu}{\rho}\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right)-\frac{\sigma B_0^2}{\rho}\bar{u}+\frac{\rho_p(\bar{u}_p-\bar{u})}{\rho\tau_v}, \tag{2}$$

y-momentum equation:

$$\frac{\partial\bar{v}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}=-\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}}+\frac{\mu}{\rho}\left(\frac{\partial^2\bar{v}}{\partial\bar{x}^2}+\frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right)-\frac{\sigma B_0^2}{\rho}\bar{v}+\frac{\rho_p(\bar{v}_p-\bar{v})}{\rho\tau_v}, \tag{3}$$

Dust phase:
Continuity equation:

$$\frac{\partial(\bar{r}\bar{u}_p)}{\partial\bar{x}}+\frac{\partial(\bar{r}\bar{v}_p)}{\partial\bar{y}}=0, \tag{4}$$

x-momentum equation:

$$\frac{\partial\bar{u}_p}{\partial\bar{t}}+\bar{u}_p\frac{\partial\bar{u}_p}{\partial\bar{x}}+\bar{v}_p\frac{\partial\bar{u}_p}{\partial\bar{y}}=-\frac{(\bar{u}_p-\bar{u})}{\tau_v}, \tag{5}$$

y-momentum equation:

$$\frac{\partial\bar{v}_p}{\partial\bar{t}}+\bar{u}_p\frac{\partial\bar{v}_p}{\partial\bar{x}}+\bar{v}_p\frac{\partial\bar{v}_p}{\partial\bar{y}}=-\frac{(\bar{v}_p-\bar{v})}{\tau_v}, \tag{6}$$

Subject to the initial and boundary conditions

$$\begin{aligned}&\bar{t}<0:\ \bar{u}=\bar{v}=\bar{u}_p=\bar{v}_p=0 \text{ for any } \bar{x} \text{ and } \bar{y},\\ &\bar{t}\geq 0:\ \bar{u}=\bar{v}=0 \text{ at } \bar{y}=0,\\ &\qquad \bar{u}=\bar{u}_e(\bar{x}),\ \bar{u}_p\to 0,\ \bar{v}_p\to\bar{v} \text{ as } \bar{y}\to\infty.\end{aligned} \tag{7}$$

where $\bar{u},\bar{v}$ are the dimensional velocity components of fluid along $\bar{x}$ and $\bar{y}$ directions while $\bar{u}_p,\bar{v}_p$ are the dimensional velocity components of dust particle along $\bar{x}$ and $\bar{y}$ directions. Then, $\bar{r}$ is the dimensional radial distance from symmetrical axis to surface of the sphere, μ is the dynamic viscosity, $\bar{t}$ is dimensional time, $\bar{p}$ is the dimensional pressure, ρ is fluid density, ρ is dust particle density, σ is the electrical conductivity, B_0 is the applied magnetic field and τ_v the relaxation time of dust particle.

As indicated below, some dimensionless variables are provided to simplify and ease the solving procedure of the governing equations.

$$t=U_\infty\frac{\bar{t}}{a},x=\frac{\bar{x}}{a},y=Re^{\frac{1}{2}}\frac{\bar{y}}{a},u=\frac{\bar{u}}{U_\infty},v=Re^{\frac{1}{2}}\frac{\bar{v}}{U_\infty},u_p=\frac{\bar{u}_p}{U_\infty}, \tag{8}$$

$$v_p = Re^{\frac{1}{2}} \frac{\bar{v}_p}{U_\infty}, p = \frac{\bar{p}}{\rho U_\infty^2}, r(x) = \frac{\bar{r}(\bar{x})}{a}.$$

where a is the radius of the sphere, $Re = \frac{\rho U_\infty a}{\mu}$ is the Reynolds number and $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity.
Outside the boundary layer region,

$$-\frac{dp}{dx} = u_e \frac{du_e}{dx} + Mu_e - N(u_p - u_e). \tag{9}$$

where $u_e = \frac{3}{2}\sin(x)$ is the free stream velocity for sphere. Consequently, the nondimensional governing equations as such are produced by include these dimensionless variables in the governing equations.

Fluid phase:
Continuity equation:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{10}$$

x-momentum equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e \frac{du_e}{dx} + \frac{\partial^2 u}{\partial y^2} + M(u_e - u) + N(u_e - u), \tag{11}$$

Dust phase:
Continuity equation:

$$\frac{\partial(ru_p)}{\partial x} + \frac{\partial(rv_p)}{\partial y} = 0, \tag{12}$$

x-momentum equation:

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = N_p(u_p - u), \tag{13}$$

Subject to the initial and boundary conditions

$$\begin{aligned} &t < 0: u = v = u_p = v_p = 0 \text{ for any } x \text{ and } y, \\ &t \geq 0: u = v = 0 \text{ at } y = 0, \\ &\qquad u = u_e(x),\ u_p \to 0,\ v_p \to v \text{ at } y \to \infty. \end{aligned} \tag{14}$$

where $M = \frac{\sigma B_0^2 a}{\rho U_\infty}$ is the magnetic parameter, $N = \frac{a\rho_p}{U_\infty \rho \tau_v}$ is the parameter of fluid particle interaction in fluid and $N_p = -\frac{a}{U_\infty \tau_v}$ is the parameter of fluid particle interaction in dust phase.

Using the following relations, the partial differential derivative of the stream function ψ in two-dimensional flow can be used to describe the velocity in x- and y- direction at a specific location.

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, v = -\frac{1}{r}\frac{\partial \psi}{\partial x}, u_p = \frac{1}{r}\frac{\partial \psi_p}{\partial y}, v_p = -\frac{1}{r}\frac{\partial \psi_p}{\partial x}. \tag{15}$$

In terms of stream function, the resulting governing equation is
Fluid phase:

$$\frac{1}{r}\frac{\partial^2 \psi}{\partial y\,\partial t} + \frac{1}{r^2}\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{1}{r^3}\frac{dr}{dx}\left(\frac{\partial \psi}{\partial y}\right)^2 - \frac{1}{r^2}\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = u_e\frac{du_e}{dx} + \frac{1}{r}\frac{\partial^3 \psi}{\partial y^3} + M\left(u_e - \frac{1}{r}\frac{\partial \psi}{\partial y}\right) + N\left(u_e - \frac{1}{r}\frac{\partial \psi}{\partial y}\right), \tag{16}$$

Dust phase:

$$\frac{1}{r}\frac{\partial^2 \psi}{\partial y\,\partial t} + \frac{1}{r^2}\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial y \partial x} - \frac{1}{r^3}\frac{dr}{dx}\left(\frac{\partial \psi}{\partial y}\right)^2 - \frac{1}{r^2}\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = N_p\left(u_p - \frac{1}{r}\frac{\partial \psi}{\partial y}\right), \tag{17}$$

The initial and boundary conditions

$$\begin{aligned} &t < 0:\ \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = \frac{\partial \psi_p}{\partial y} = \frac{\partial \psi_p}{\partial x} = 0 \text{ for any } x \text{ and } y, \\ &t \geq 0:\ \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial x} = 0\ at\, y = 0, \\ &\quad \frac{\partial \psi}{\partial y} = u_e(x),\ \frac{\partial \psi_p}{\partial y} \to 0,\ \frac{\partial \psi_p}{\partial x} \to \frac{\partial \psi}{\partial x} \text{ as } y \to \infty. \end{aligned} \tag{18}$$

Around the sphere, a boundary layer with a thickness of $O(\nu t)^{\frac{1}{2}}$. At $t = 0$, the boundary layer has zero thickness at first and increases with time squared. For small time $(t \leq t^*)$, the following similarity variables are used to convert the governed stream function.
Fluid phase:

$$\psi = t^{\frac{1}{2}} u_e(x) f(x, \eta, t), \text{and } \eta = \frac{y}{t^{\frac{1}{2}}}, \tag{19}$$

Dust phase:

$$\psi_p = t^{\frac{1}{2}} u_e(x) F(x, \eta, t), \text{and } \eta = \frac{y}{t^{\frac{1}{2}}}. \tag{20}$$

Hence, the governing for small time case become
Fluid phase:

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2}\frac{\partial^2 f}{\partial \eta^2} + t\frac{du_e}{dx}\left(1 - \left(\frac{\partial f}{\partial \eta}\right)^2 + f\frac{\partial^2 f}{\partial \eta^2}\right) + Mt\left(1 - \frac{\partial f}{\partial \eta}\right) + Nt\left(1 - \frac{\partial f}{\partial \eta}\right) = t\frac{\partial^2 f}{\partial \eta \partial t} + tu_e\left(\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta \partial x} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial \eta^2} - \frac{1}{r}\frac{dr}{dx} f\frac{\partial^2 f}{\partial \eta^2}\right), \tag{21}$$

Dust phase:

$$\frac{\eta}{2}\frac{\partial^2 F}{\partial \eta^2} - t\frac{du_e}{dx}\left(\left(\frac{\partial F}{\partial \eta}\right)^2 + F\frac{\partial^2 F}{\partial \eta^2}\right) + N_p t\left(1 - \frac{\partial F}{\partial \eta}\right) = t\frac{\partial^2 F}{\partial \eta \partial t} + tu_e\left(\frac{\partial F}{\partial \eta}\frac{\partial^2 F}{\partial \eta \partial x} - \frac{\partial F}{\partial x}\frac{\partial^2 F}{\partial \eta^2} - \frac{1}{r}\frac{dr}{dx} F\frac{\partial^2 F}{\partial \eta^2}\right), \tag{22}$$

Subject to the initial and boundary conditions

$$t < 0:\ f = \frac{\partial f}{\partial \eta} = F = \frac{\partial F}{\partial \eta} = 0 \text{ for any } x \text{ and } \eta,$$
$$t \geq 0:\ f = \frac{\partial f}{\partial \eta} = 0 \text{ at } \eta = 0,$$
$$\frac{\partial f}{\partial \eta} = 1,\ \frac{\partial F}{\partial \eta} \to 0,\ F \to f \text{ as } y \to \infty. \tag{23}$$

By applying forward stagnation point, $x = 0^{\circ}$. Thus, $u_e = 0$ and $\frac{du_e}{dx} = \frac{3}{2}$. The governing equation are written as
Fluid phase:

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2}\frac{\partial^2 f}{\partial \eta^2} + \frac{3}{2}t\left(1 - \left(\frac{\partial f}{\partial \eta}\right)^2 + f\frac{\partial^2 f}{\partial \eta^2}\right) + Mt\left(1 - \frac{\partial f}{\partial \eta}\right) + Nt\left(1 - \frac{\partial f}{\partial \eta}\right) = t\frac{\partial^2 f}{\partial \eta \partial t} \tag{24}$$

Dust phase:

$$\frac{\eta}{2}\frac{\partial^2 F}{\partial \eta^2} - \frac{3}{2}t\left(\left(\frac{\partial F}{\partial \eta}\right)^2 + F\frac{\partial^2 F}{\partial \eta^2}\right) + N_p t\left(1 - \frac{\partial F}{\partial \eta}\right) = t\frac{\partial^2 F}{\partial \eta \partial t}. \tag{25}$$

Subject to the initial and boundary conditions

$$t < 0:\ f = \frac{\partial f}{\partial \eta} = F = \frac{\partial F}{\partial \eta} = 0 \text{ for any } x \text{ and } \eta,$$
$$t \geq 0:\ f = \frac{\partial f}{\partial \eta} = 0\ at\ \eta = 0,$$
$$\frac{\partial f}{\partial \eta} = 1,\ \frac{\partial F}{\partial \eta} \to 0,\ F \to f \text{ as } y \to \infty. \tag{26}$$

3 Results and discussion

In the MATLAB environment, the Keller-Box method is utilised to solve the governing equations numerically. By introducing new variables in similarity transformation, the higher order partial differential equation (PDE) is reduced to first order PDE. Then, the discretization by Finite Difference method (FDM) and linearization by Newton method are applied. The system of governed equations is solved by forming the block tri-diagonal structure. The velocity profile and magnetic field parameter of dusty boundary layer problem is graphically analysed.

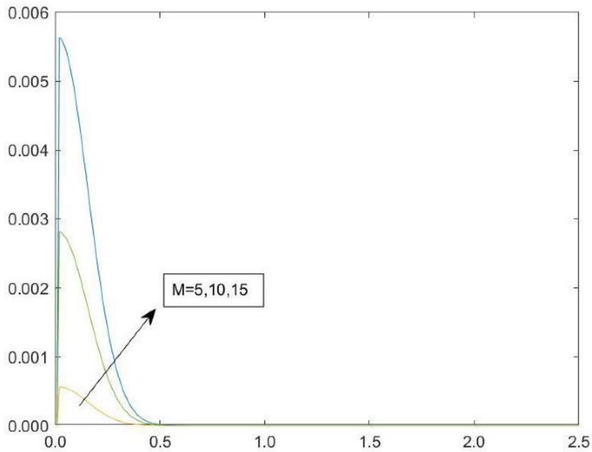

Fig. 2. Velocity profile vs coordinate distance y for different magnetic parameter, M.

Figure 2 shows how the magnetic parameter affects velocity profiles. The fluid and dust particles motion will be slowed down by the electromagnetic body force known as Lorentz force. When the magnetic parameter is large, the velocity boundary layer's thickness drops. As a result, the velocity distribution becomes less with increasing magnetic field. Since this study focuses on the forward stagnation point ($x = 0^{\circ}$), it can be seen that for a fixed value of time, t, the velocity increases with the increases of M. Different behaviour is observed near the forward stagnation point. At early times, the value of velocity increases as the value of M increases. As t increases, the opposite behaviour is seen. Thus, this study concluded that the increasing of magnetic field will increase the velocity of fluid flow. This paper recommended the future researcher to study the rear stagnation point over a fluid flow past a sphere by considering the flow reversal, separation of fluid flow and drag friction for unsteady dusty MHD fluid flow past a sphere. Then, study the fluid flow past another blunt geometry such as cylinder.

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