# Development of an Epidemiological Model with Transmission Matrix to Understand the Dynamics of Tuberculosis Spread

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**Abstract.** Tuberculosis remains a major challenge in the field of healthcare. The spread of tuberculosis depends on complex interactions between individuals within a population, involving factors such as mobility, physical contact, and age groups. Each age group has unique characteristics that influence how tuberculosis spreads among the population and how each group responds to the infection. To understand the dynamics of tuberculosis spread, an epidemiological model is required. This study aims to develop an epidemiological model based on a transmission matrix that can represent the pattern of tuberculosis spread within a population. The transmission matrix is used to describe the interactions between individuals and subpopulations, taking into account the transmission rate and incubation period. After building the model and transmission matrix, model calibration and validation are conducted. In this stage, model parameters are adjusted to ensure that the model can accurately replicate the observed epidemiological data. Subsequently, analysis is performed using the model and transmission matrix to understand the dynamics of disease spread, followed by interpretation of the results. The findings of this study indicate that the use of the transmission matrix provides valuable insights into the dynamics of tuberculosis spread and helps identify high-risk subpopulations.

## 1 Introduction

Tuberculosis is an infectious disease caused by the bacterium Mycobacterium tuberculosis [1]. According to the World Health Organization, tuberculosis remains one of the top ten causes of death, with millions of new cases diagnosed each year [2]. The rapid and widespread transmission of tuberculosis is caused by various factors, including human mobility, socioeconomic conditions, and population density. The dynamics of tuberculosis spread are also influenced by the complex interaction between epidemiological and biological factor, which make understanding the transmission patterns of this disease a major challenge [3]. Tuberculosis has a long and variable incubation period, usually several weeks to months after exposure [4].

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Tuberculosis is spread through the air when an infected person coughs, sneezes, or talks, releasing droplets containing the bacteria [5]. Many people infected with tuberculosis do not show symptoms immediately and are in a state of latent tuberculosis. The person cannot transmit tuberculosis but can develop active tuberculosis. The spread of tuberculosis is influenced by several factors, one of which is age groups. Each age group has unique characteristics that affect how tuberculosis spreads within that population and how each age group responds to the infection [6]. In order to, understand and control the spread of the disease, it is necessary to use mathematical models that are able to represent the complex interactions between individuals in a population. One important tool in epidemiology is the transmission matrix, which is used to model interactions between individuals in a population and describe how infectious disease can be transmitted from one individual to another.

Transmission matrices present a mathematical framework that makes it possible to investigate patterns of disease transmission, predict the development of outbreaks, and design effective control strategies. Transmission matrices in epidemiological modelling are used to look at the spread within sub-populations and between sub-populations Manna et al [1]. Mathematical models using transmission matrices help predict how different strategies affect tuberculosis disease in populations that are also affected by HIV [7]. This model identifies a dynamic tuberculosis case-finding policy in the context of an epidemic tuberculosis epidemic. [8] developed a mathematical model that use a transmission matrix to understand the spread of tuberculosis in highly endemic region in Asia-Pacific. The transmission matrix was used to capture the dynamics of social contact and disease spread across different age groups and socio-economic conditions. In addition, a transformation matrix contact data was developed to evaluate the risk of tuberculosis spread in Zambia and Western Cape, South Africa [9]. The results showed that focusing on young age groups can be effective in controlling the spread of tuberculosis.

In a transmission matrix, each entry represents the transmission rate from one individual to another, with various factors affecting this transmission rate including physical contact, population mobility, individual immunity levels and the effectiveness of control interventions. Transmission matrix can be used in epidemic models that are more expressive and able to capture behavioural heterogeneity Dunbar [2]. Using epidemiological data, behavioural science, and mathematical modelling techniques, transmission models can be built that can provide predictions about the number of new cases, the rate of spread, and the impact of various intervention scenarios. However, transmission matrices also have a number of challenges and limitations. Individual variability, data uncertainty and the complexity of human interactions are some of the many factors that need to be considered in the use of transmission matrix. The use of transmission matrices makes it possible to model the complexity of individual interactions within the population. Thus the use of transmission matrices can help in identifying hotspots where diseases may spread rapidly. Therefore, a deep understanding of the strengths and limitations of transmission matrix is essential in the development of infectious disease control strategies.

In this study, an epidemiological model with a transmission matrix is analysed to understand the dynamics of tuberculosis spread. Through this approach, it is expected that a deeper understanding of the factors affecting the transmission of tuberculosis can be obtained and the most effective controls can be identified to reduce the spread of this disease.

## 2 Research Methods

The methodology used in this research is a literature study, which involves searching for references in books and journals. The research begins by studying epidemiological modelling and how transmission matrices represent interactions between subpopulations, as well as how the spread of infectious diseases is represented using transmission matrices. The next step is

to establish the research objective, which is to model the spread of tuberculosis among certain subpopulations and to understand the disease dynamics within those populations. Based on this objective, data collection is conducted, along with the formulation of several assumptions and the definition of variables and parameters, such as the number of infectious disease cases, the level of interaction/contact between subpopulations, the transmission rate of the infectious disease, and the incubation period.

Based on these assumptions and parameter identification, an epidemiological model that aligns with the research objectives and disease characteristics is developed. From the data obtained, a transmission matrix is created that includes the number of transmission cases between different subpopulations within the population. After building the model and the transmission matrix, model calibration and validation are conducted. In this stage, model parameters are adjusted to ensure that the model can accurately replicate the observed epidemiological data. Subsequently, analysis is performed using the model and transmission matrix to understand the dynamics of disease spread, and the results are interpreted.

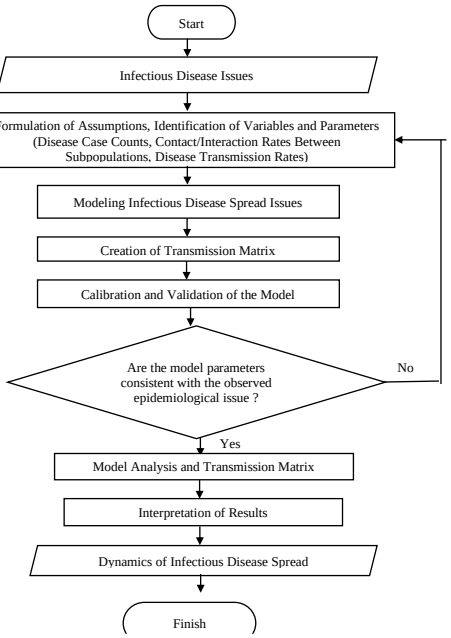


**Fig. 1.** Research method flowchart

## 3 Results and Discussions

Tuberculosis is an infectious disease caused by the bacteria Mycobacterium tuberculosis. Tuberculosis has a long and variable incubation period usually several weeks to months after exposure. Tuberculosis is spread through the air when an infected person coughs, sneezes, or talks, releasing droplets containing the bacteria. Many people infected with Tuberculosis do not show symptoms immediately and are in a state of Latent TB. The person cannot transmit tuberculosis but can develop active TB. The spread of tuberculosis is influenced by several factors, one of which is the age group. Each age group has unique characteristics that affect how tuberculosis spreads among the population and how each age group responds to the infection.

The teenagers age group (15-19 years old) begins to have wider social interactions such as school, playgrounds, which increases the risk of exposure to tuberculosis. Infected

teenagers can transmit the disease and tend to realise symptoms sooner. However, sometimes the social stigma associated with tuberculosis can cause teenagers to not report their symptoms. The Adult age group (20 - 44 years) is at the peak of social and economic activity. This results in increased social interaction with many people and increased risk of spread. Adults infected with tuberculosis tend to be highly contagious. This is exacerbated by the presence of other chronic diseases such as diabetes or HIV. In the Elderly age group (45 years and above) the immune system declines. In addition, the presence of chronic diseases such as diabetes, heart disease, or malnutrition increases the susceptibility of the elderly to tuberculosis. In addition, elderly people who have been infected with latent TB may be at risk of reactivation of the infection when immunity declines. Tuberculosis in the elderly may be less contagious than in adults, but this age group can be a source of infection. In this age group, symptoms are difficult to diagnose as tuberculosis symptoms may mimic other chronic diseases common in the elderly. Mortality from Tuberculosis is higher in the elderly due to weakened immunity and complications from other chronic illnesses.

The mathematical model of the spread of the Tuberculosis virus is formed based on a population divided into 4 age groups, namely children (under 14 years), teenagers (15-19), adults (20-44) and the elderly (45 and above). The population is divided into four sub-populations, namely susceptible individuals, individuals who have been exposed to tuberculosis bacteria and are in the latent phase (exposed), individuals who have active tuberculosis and can transmit the disease (infected), and individuals who recover from tuberculosis (recovered). These sub-populations are denoted by S for susceptible, E for exposed, I for infected, and R for recovered, respectively.

The susceptible sub-population contains individuals who are not yet infected, but are susceptible to infection, and will become infected if they come into direct contact with infected individuals. The exposed sub-population contains individuals who have been exposed to tuberculosis bacteria but are not yet infected and cannot spread to other individuals. The infected sub-population contains individuals who have been infected and can transmit to other sub-populations. While the Recovered sub-population contains individuals who have successfully recovered, but still do not have permanent immunity, meaning that individuals who have recovered can still be infected again. Some assumptions that will be used to form the model are:

1. The number of individuals born equals the number of individuals who die (closed population)
2. Total population ($N$) is the total number of individuals in each sub-population. The number of individuals in the susceptible, exposed, infected, and recovered sub-populations are denoted by $S, E, I$, and $R$, respectively. The total population can be expressed as $N = S + E + I + R$ and is considered constant. The population is divided into age groups Children (0-14 years), teenagers (15-19 years), adults (20 - 44 years) and elderly (45 years and above).
3. Tuberculosis spreads through contact between susceptible and infected individuals.
4. Exposed individuals ($E$) are not immediately infectious until they turn into infected individuals.
5. Infected individuals may recover or die from tuberculosis.
6. Recovered individuals may lose immunity and become susceptible again.
7. There is natural mortality in each compartment. Death from tuberculosis occurs only in infected individuals.
8. Close contact with individuals infected with tuberculosis has a higher probability of transmission.
9. Contact between age groups (children, teenagers, adults and elderly) differs in terms of frequency and intensity.

Based on the above assumptions, the disease spread model is formed as follows:

$$\begin{cases} \frac{dS_i}{dt} = \Lambda N_i - \frac{\sum_{j=1}^{4} \beta_{ij} S_i I_j}{N_i} + \rho R_i - \mu_i S_i \\ \frac{dE_i}{dt} = \frac{\sum_{j=1}^{4} \beta_{ij} S_i I_j}{N_i} - \sigma E_i - \mu_i E_i \\ \frac{dI_i}{dt} = \sigma E_i - (\mu_i + d_i) I_i - \tau I_i \\ \frac{dR_i}{dt} = \tau I_i - (\mu_i + \rho) R_i \end{cases} \quad (1)$$

With $N_i = S_i + E_i + I_i + R_i$, is the total population in age group $i$, with $i = 1$ children, $i = 2$ teenagers, $i = 3$ adults dan $i = 4$ elderly. The parameters of the model are presented in Table 1.

**Table 1.** Parameters of the model

| Parameter | Description |
|---|---|
| $\Lambda$ | Recruitment rate |
| $\beta$ | Effective Tuberculosis contact rate |
| $\rho$ | Rate at which recovered individuals become susceptible |
| $\sigma$ | Rate at which exposed individuals become infected |
| $\tau$ | Rate at which infected individuals become recovered |
| $\mu$ | Natural death rate |
| $d$ | Death rate because of Tuberculosis |

An equilibrium point is a point that is constant and does not change with time. The equilibrium point can be determined if the following conditions are met:

$$\frac{dS_i}{dt} = 0; \frac{dE_i}{dt} = 0; \frac{dI_i}{dt} = 0; \frac{dR_i}{dt} = 0 \quad (2)$$

Two kinds of equilibrium points for the Tuberculosis disease spread are obtained as follows:

1) Disease Free Equillibrum Point ($E_0$)

This equilibrium point occurs when there are no individuals infected with TB disease in a population, namely when $E = I = R = 0$. So that the equilibrium point is obtained as follows

$$E_0 = (S, E, I, R) = \left(\frac{\Lambda N_i}{\mu_i}, 0,0,0\right) \quad (3)$$

2) Endemic Disease Equillibrum Point ($E_1$)

This equilibrium point occurs if there is contact between individuals in the susceptible and infected TB subpopulations, which means that the exposed subpopulation, infected subpopulation and cured subpopulation remain. The following equilibrium point is obtained:

$$E_1 = (S^*, E^*, I^*, R^*) \quad (4)$$

$$S^* = \frac{N\left((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma)\right)}{\beta\sigma}$$

$$E^* = \frac{(\Lambda\beta\sigma-\mu((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma))N(d+\mu+\tau)(\mu+\rho)}{\beta\sigma\left((d+\mu)(\mu+\rho)(\mu+\sigma)+\mu\tau(\mu+\rho+\sigma)\right)}$$

$$I^* = \frac{(\Lambda\beta\sigma-\mu((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma))N(\mu+\rho)}{\beta\left((d+\mu)(\mu+\rho)(\mu+\sigma)+\mu\tau(\mu+\rho+\sigma)\right)}$$

$$R^* = \frac{N\tau(\Lambda\beta\sigma-\mu((d+\mu)(\mu+\sigma)+\tau(\mu+\sigma))}{\beta\left((d+\mu)(\mu+\rho)(\mu+\sigma)+\mu\tau(\mu+\rho+\sigma)\right)}$$

After determining the equilibrium point, next determine the transmission matrix. To create a transmission matrix for the spread of disease, it is necessary to understand some basic concepts regarding the spread of Tuberculosis disease in the population. Transmission matrices are used in epidemiological models to describe how diseases spread between individuals or groups in the population. The transmission matrix includes elements that describe direct transmission from infected individuals to susceptible individuals ($S$) who become exposed ($E$). The Transmission matrix contains the value of $\beta_{ij}$ which is the rate of transmission from age group j to age group i Since there are two infected classes ($E$ and $I$) for 2 age groups of the population, the Transmission matrix is

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta_{11}}{N_1}S_1 & \frac{\beta_{12}}{N_1}S_1 & \frac{\beta_{13}}{N_1}S_1 & \frac{\beta_{14}}{N_1}S_1 \\ 0 & 0 & 0 & 0 & \frac{\beta_{21}}{N_2}S_2 & \frac{\beta_{22}}{N_2}S_2 & \frac{\beta_{23}}{N_2}S_2 & \frac{\beta_{24}}{N_2}S_2 \\ 0 & 0 & 0 & 0 & \frac{\beta_{31}}{N_3}S_3 & \frac{\beta_{32}}{N_3}S_3 & \frac{\beta_{33}}{N_3}S_3 & \frac{\beta_{34}}{N_3}S_3 \\ 0 & 0 & 0 & 0 & \frac{\beta_{41}}{N_4}S_4 & \frac{\beta_{42}}{N_4}S_4 & \frac{\beta_{43}}{N_4}S_4 & \frac{\beta_{44}}{N_4}S_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

The disease spread transmission matrix provides an overview of how disease spreads between groups in the population based on contact rates and transmission probabilities. This matrix is very important in epidemiological models to understand the dynamics of disease spread and to design control strategies. Since it's at the disease-free equilibrium point (3) and the population is closed, so the transmission matrix is

$$T = \begin{pmatrix} 0 & 0 & 0 & 0 & \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & 0 & 0 & 0 & \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & 0 & 0 & \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & 0 & \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (6)$$

After determining the transmission matrix we next determine the basic reproduction number. The basic reproduction number is the average of the number of new infections in the susceptible subpopulation produced by an infected person. The basic reproduction number ($R_0$) can be calculated using the transmission matrix and the infection transition matrix ($V$). The $V$ matrix describes the rate at which individuals transition from susceptible to infected. The V matrix consists of the transition rate and the recovery rate from infection. Thus there are $\sigma$ infection rate, death rate from tuberculosis ($d_i$), recovery rate ($\tau$) and natural death ($\mu_i$).

$$TV^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta_{11}}{\mu_1+d_1+\tau} & \frac{\beta_{12}}{\mu_2+d_2+\tau} & \frac{\beta_{13}}{\mu_3+d_3+\tau} & \frac{\beta_{14}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & \frac{\beta_{21}}{\mu_1+d_1+\tau} & \frac{\beta_{22}}{\mu_2+d_2+\tau} & \frac{\beta_{23}}{\mu_3+d_3+\tau} & \frac{\beta_{24}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & \frac{\beta_{31}}{\mu_1+d_1+\tau} & \frac{\beta_{32}}{\mu_2+d_2+\tau} & \frac{\beta_{33}}{\mu_3+d_3+\tau} & \frac{\beta_{34}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & \frac{\beta_{41}}{\mu_1+d_1+\tau} & \frac{\beta_{42}}{\mu_2+d_2+\tau} & \frac{\beta_{43}}{\mu_3+d_3+\tau} & \frac{\beta_{44}}{\mu_4+d_4+\tau} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (7)$$

$R_0$ is the largest eigenvalue (spectral radius) of the matrix $TV^{-1}$. Mathematically

$$R_0 = \rho(TV^{-1}) \quad (8)$$

where $\rho$ denotes the largest eigenvalue. If $R_0 > 1$, then the infection has the potential to spread within the population. If $R_0 < 1$, then the infection will wane and eventually disappear.

## 4 Numerical Solutions

Numerical simulation of the SEIRS model for tuberculosis spread provides insights into the dynamics of transmission across different age groups. The simulation was conducted using Runge-Kutta method by providing each parameter value can be seen in Table 2.

**Table 2.** The parameter values of the Mathematical model for the spread of tuberculosis

| Parameter | value | | Parameter | value |
|---|---|---|---|---|
| $\wedge$ | 0,00904450447 | | $\beta_{31}$ | 0,001 |
| $\mu_1$ | 0,0003575985 | | $\beta_{32}$ | 0,01 |
| $\mu_2$ | 0,00120022 | | $\beta_{33}$ | 0,02 |
| $\mu_3$ | 0,005742038 | | $\beta_{34}$ | 0,005 |
| $\mu_4$ | 0,007114707 | | $\beta_{41}$ | 0,005 |
| $\sigma$ | 0,1916113 | | $\beta_{42}$ | 0,005 |
| $\beta_{11}$ | 0,02 | | $\beta_{43}$ | 0,005 |
| $\beta_{12}$ | 0,01 | | $\beta_{44}$ | 0,01 |
| $\beta_{13}$ | 0,005 | | $\rho$ | 0,01 |
| $\beta_{14}$ | 0,005 | | $\tau$ | 0.563878595 |
| $\beta_{21}$ | 0,01 | | $d_1$ | 0,007150002 |
| $\beta_{22}$ | 0,02 | | $d_2$ | 0,00239978 |
| $\beta_{23}$ | 0,01 | | $d_3$ | 0.011480915 |
| $\beta_{24}$ | 0,005 | | $d_4$ | 0,007114707 |

These parameter values were derived from Tuberculosis data in Kalimantan Barat. Based on Table 1, the transmission matrix is as follows:

$$T = \begin{pmatrix} 0{,}02 & 0{,}01 & 0{,}005 & 0{,}005 \\ 0{,}01 & 0{,}02 & 0{,}01 & 0{,}005 \\ 0{,}001 & 0{,}01 & 0{,}02 & ,0{,}005 \\ 0{,}005 & 0{,}005 & 0{,}005 & 0{,}01 \end{pmatrix} \quad (9)$$

Based on this matrix, children transmit tuberculosis to other children at a rate of 0,02 per year. Children transmit tuberculosis to teenagers at a rate of 0,01 per year. Children transmit tuberculosis to adults at a rate of 0,005 per year. Children transmit tuberculosis to the elderly at a rate of 0,005 per year.
The graph of the tuberculosis disease spread with the parameters from Table 1 can be seen in Fig 2.

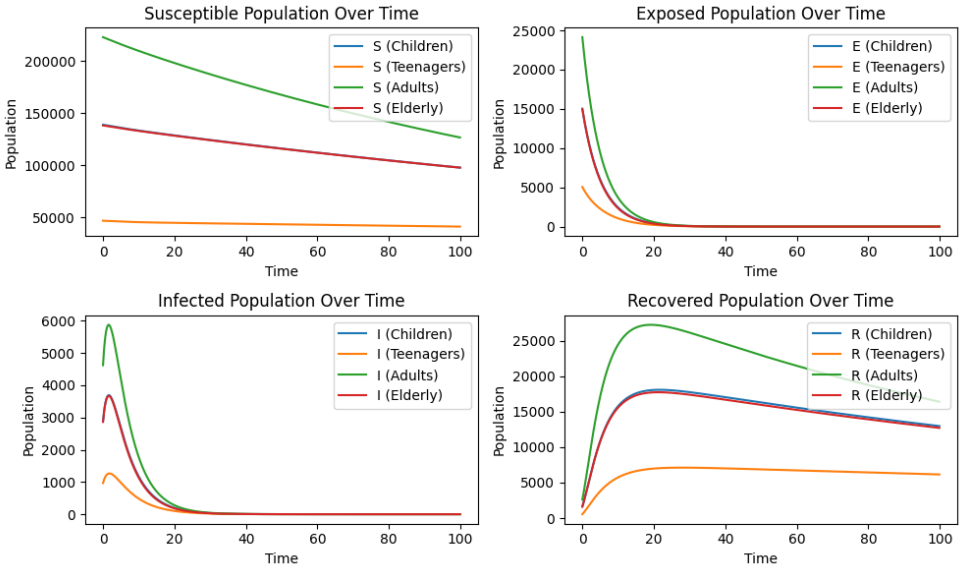


**Fig 2.** Graphs of Tuberculosis Spread by Age Group

Based on Fig 2, it can be observed that in children (0 -14 years), the number of susceptible individuals initially starts high, reflecting a large proportion of children who have not yet been exposed to tuberculosis. Over time, this number decreases as children become exposed or infected. The exposed subpopulation shows an initial increase as susceptible children come into contact with infected individuals and are exposed to tuberculosis. Subsequently, the infected subpopulation rises, following the trend of the exposed group, reaching a peak before eventually declining due to recovery or progression to other states. The recovered subpopulation gradually increases as children recover from tuberculosis. This increase reflects the role of immunity in reducing the susceptible population. In teenagers (15 – 19 years), the observed pattern is similar to that of children but differs in magnitude due to variations in transmission dynamics and contact rates. Teenagers tend to experience a faster decline in the susceptible population and a sharper peak in infections, likely caused by increased social interactions.

Among adults (20 – 44 years), the infection peak is higher, driven by more frequent interactions and a larger initial population base. The susceptible subpopulation declines more rapidly in this age group, while the exposed and infected populations remain significant for

longer periods due to ongoing transmission dynamics. In the elderly (45 years and above), distinct dynamics are observed. The susceptible population declines more slowly, and infections tend to last longer, reflecting weaker immune responses and higher tuberculosis-induced mortality rates. Additionally, the number of recovered individuals increases more slowly, indicating a slower recovery process in this age group.

The model effectively captures the progression of tuberculosis epidemics with initial increases in exposed and infected populations, followed by recovery. The susceptible population steadily decreases, indicating the spread of the disease and a reduction in vulnerable individuals. Based on Figure 2, it is evident that the epidemic peaks at different times across age groups, emphasizing the need for control strategies tailored to specific age groups. Targeting particular groups would be more effective in managing the epidemic.

Factors influencing the dynamics of disease spread include varying transmission rates across age groups, influenced by their respective interaction patterns. Additionally, recovery and loss of immunity play crucial roles in shaping the long-term dynamics of the epidemic and determining the potential for future outbreaks. Variations in transmission rates between age groups reflect the impact of differing social interaction patterns on disease dynamics.

## 5 Conclusion

The use of transmission matrices in tuberculosis disease spread models allows for a deeper understanding of how the disease spreads among different age groups. The basic reproduction number is calculated using the transmission matrix and the infection transition matrix, which indicates the average number of new infections caused by an infected individual in a susceptible population. The transmission matrix depicts the rate at which tuberculosis is transmitted from one group to another. By understanding these interaction patterns, we can identify groups that are at higher risk of transmitting or contracting the disease.

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