Search method for optimal interpolation of thermomechanical coefficients for non-ferrous metals and high-alloy steels

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Abstract. This study explores the optimization of interpolation techniques for thermomechanical coefficients in non-ferrous metals and high-alloy steels. These coefficients, including the temperature coefficient (K_t), deformation degree coefficient (K_ε), and deformation rate coefficient (K_u), play a crucial role in characterizing the deformation resistance of materials under varying conditions. The challenge lies in the non-linear nature of these coefficients and their current representation in graphs and tables, which hinders their integration into automated control and optimization systems. To address this issue, we propose the development of smooth interpolation functions using the least squares method. Our research aims to identify the most precise interpolation approach for a wide range of non-ferrous metals and high-alloy steels, facilitating more accurate mathematical modeling of their thermomechanical properties. This advancement will enable improved prediction and control of material behavior in complex manufacturing processes, ultimately enhancing the efficiency and precision of metal forming operations

1 Introduction

One of the key factors determining the behaviour of a metal under the influence of temperature and pressure includes mechanical properties which provide resistance to a deformation. Thermomechanical coefficients are actively used to mathematically describe these properties which determine the direct effect on the deformation resistance. There are several research articles [1-2] describing the interpolation of a small quantity of non-ferrous metals and high-alloy steels to create various kinds of automated design systems. However, most of these articles do not clarify the reasons behind the selection of specific interpolation methods, and the overall number of steels considered is lacking, which raises concerns about the entire body of research in the area of thermomechanical coefficient interpolation. Additionally, mathematical models are similarly utilized in microstructure-based finite element modelling [3-6]. However, achieving similar accuracy with these mathematical models can be challenging [7-8].

This challenge underscores the critical importance of developing accurate thermo-deformation mathematical models and their integration into automated control systems for thermo-deformation processes. The ability to precisely interpolate thermomechanical coefficients is fundamental to creating robust models that can effectively predict and control material behavior under various thermal and mechanical conditions. Addressing this issue is essential for advancing the field of metal forming and optimizing industrial processes that rely on precise control of material properties during deformation.

In fact, all these coefficients such as temperature coefficient (K_t), deformation degree coefficient (K_ε) and deformation rate coefficient (K_u) are functions of temperature, deformation degree and deformation rate respectively. Thermomechanical coefficients are extensively utilized in mathematical calculations that describe complex thermo-deformation states, particularly in the area of thermo-mechanical methods to creating non-breakable joints for non-ferrous metals. However, they are presented in a format that is not conducive to calculations, specifically as graphs and tables. As a result, they cannot be utilized in automated control and optimization algorithms [9]. Obviously, creating smooth interpolation functions based on these tables and graphs will help solve this problem. However, there is another issue concerning the question of the method which is both the most accurate and the simplest for interpolating thermomechanical coefficients.

This study addresses two key aspects of interpolating thermomechanical coefficients. The first aspect explores suitable methods for interpolation, while the second and primary focus evaluates the accuracy of these methods for specific non-ferrous metals and high-alloy steels. The research aims to identify the most effective interpolation techniques for each material under consideration, providing valuable insights into the optimal approaches for predicting thermomechanical properties across various metallic compositions. The work focuses on the development of a diverse array of interpolation models associated with thermo-deformation processes.

2 Materials and methods

The initial phase of the study involves establishing a framework for representing the source data. This data comprises tabulated thermomechanical coefficient values, structured according to specific increments:

- Temperature: 100-degree intervals
- Deformation: 10% increments
- Deformation time:
    - 0.01 steps from 0.01 to 0.1
    - 0.1 steps from 0.1 to 1
    - 1 step from 1 to 10
    - 10 steps from 10 to 80

This approach typically yields between 6 and 10 discrete values for each material examined. The study also incorporates temperature limits, which serve as interpolation boundaries. These datasets are compiled in Tables 1, 2, and 3.

Subsequently, the research identifies suitable interpolation techniques for the tabular data. The selected methods include:

1. Linear interpolation
2. Quadratic interpolation
3. Logarithmic interpolation
4. Cubic interpolation
5. Fourth-degree polynomial interpolation

The final objective is to evaluate the accuracy of each interpolation method by quantifying deviations from the original tabulated data. This assessment employs the least squares method to determine the most precise smooth function representation for the thermomechanical coefficients.

3 Results and discussion

The method yielded optimal representations of thermomechanical coefficients as smooth functions. There are temperature coefficient (K_t) deformation degree coefficient (K_ε) and deformation rate coefficient (K_u). Functions are bounded on partial intervals of temperature, deformation degree and deformation rate of the metal. The data are presented in the following tables.

Table 1 presents the optimal smooth functions for the temperature coefficient (K_t) across various steel grades. The data reveals that the temperature dependence of K_t is best represented by different function types for different steels, ranging from linear to fourth-degree polynomials. This variation highlights the complex nature of temperature effects on deformation resistance across different steel compositions. The accuracy of these interpolations, quantified using the least squares method, demonstrates the effectiveness of the chosen approach in capturing the non-linear behavior of K_t within specified temperature ranges.

Table 1. Optimal smooth function representations of temperature coefficient (K_t) for conventional and low alloy steels.

Material	**K_t**	**Constraints K_t**
CO	$-1.37046\times10^{-7} x^3 + 0.000113762 x^2 - 0.034728 x + 4.51$	$150<x<340$
M1	$-4.5\times10^{-8} x^3 + 0.0000517604 x^2 - 0.0278411 x + 6.88926$	$450<x<950$
AMF	$-3.98148\times10^{-9} x^3 + 9.02183\times10^{-6} x^2 - 0.00822814 x + 3.53872$	$450<x<950$
L62	$8.18519\times10^{-8} x^3 - 0.0000835868 x^2 + 0.0303394 x - 1.30406$	$450<x<950$
L68	$6.61111\times10^{-8} x^3 - 0.0000658854 x^2 + 0.0238169 x - 0.88056$	$450<x<950$
L70	$5.16071\times10^{-6} x^2 - 0.00986214 x + 5.05256$	$450<x<950$
L90	$-4.44444\times10^{-8} x^3 + 0.0000523438 x^2 - 0.029871 x + 7.79447$	$450<x<950$
NPA1	$-7.13625\times10^{-9} x^3 + 9.03187\times10^{-6} x^2 - 0.00763281 x + 4.03315$	$600<x<1270$
NPAN	$-2.55971\times10^{-9} x^3 + 0.0000109547 x^2 - 0.0156304 x + 7.80036$	$800<x<1250$
NMZHMc28	$-4.44444\times10^{-9} x^3 + 0.0000145816 x^2 - 0.017393 x + 7.75541$	$600<x<1200$
NZHMc30	$-5.5674\times10^{-8} x^3 + 0.0000631527 x^2 - 0.0339433 x + 8.7334$	$600<x<1050$
MN19	$-1.83542\times10^{-7} x^3 + 0.000246321 x^2 - 0.147512 x + 34.4521$	$600<x<1050$
MNC15-20	$9.01639\times10^{-9} x^3 - 0.0000171066 x^2 + 0.00593033 x + 2.5518$	$600<x<950$
AD	$-1.33333\times10^{-8} x^3 + 0.0000248571 x^2 - 0.0183524 x + 5.22886$	$300<x<500$
AMc	$3.14286\times10^{-6} x^2 - 0.00537429 x + 2.65114$	$300<x<500$
Amg6	$3.\times10^{-6} x^2 - 0.00607 x + 2.9525$	$300<x<450$
D16	$9.\times10^{-6} x^2 - 0.01109 x + 3.9875$	$300<x<450$
H18N9T	$3.5\times10^{-6} x^2 - 0.00985 x + 7.35$	$900<x<1200$
HN78T	$4.25\times10^{-6} x^2 - 0.011735 x + 8.4705$	$900<x<1200$
HN75MBTYU	$4.\times10^{-6} x^2 - 0.0112 x + 8.19$	$900<x<1200$
VZH98	$4.5\times10^{-6} x^2 - 0.01201 x + 8.513$	$900<x<1200$
HN70YU	$5.\times10^{-6} x^2 - 0.01354 x + 9.517$	$900<x<1200$
EI661	$5.\times10^{-6} x^2 - 0.01327 x + 9.273$	$1000<x<1200$

Table 2 showcases the optimal representations of the deformation degree coefficient (K_ε) as smooth functions. The results indicate that the relationship between deformation degree and its coefficient is often more complex than that of temperature, with higher-order polynomials frequently providing the best fit. This complexity reflects the intricate microstructural changes that occur during deformation processes.

Table 2. Optimal smooth function representations of deformation degree coefficient (K_ε) for conventional and low alloy steels.

Material	kε	Constraints k_ε
CO	0.394818 log(x) - 0.464986	10<x<80
M1	-8.5724×10^{-7} x^3 + 0.0000113099 x^2 + 0.0145341 x + 0.473528	10<x<80
AMF	-1.6735×10^{-7} x^3 - 0.0000689076 x^2 + 0.0161361 x + 0.495848	10<x<80
L62	-6.18169×10^{-7} x^3 - 0.0000287122 x^2 + 0.0187957 x + 0.317096	10<x<80
L68	1.53347×10^{-6} x^3 - 0.00031968 x^2 + 0.0289574 x + 0.245753	10<x<80
L70	-8.70902×10^{-7} x^3 - 0.0000168427 x^2 + 0.0194235 x + 0.310489	10<x<80
L90	1.81011×10^{-6} x^3 - 0.000417859 x^2 + 0.0366287 x + 0.0788335	10<x<80
NPA1	2.42145×10^{-6} x^3 - 0.000478224 x^2 + 0.0390278 x + 0.0602175	10<x<80
NPAN	2.04577×10^{-6} x^3 - 0.000404343 x^2 + 0.0360069 x + 0.0804887	10<x<80
NMZHMc28	-1.26708×10^{-6} x^3 + 0.0000398093 x^2 + 0.0184868 x + 0.316201	10<x<80
NZHMc30	1.02801×10^{-6} x^3 - 0.000222864 x^2 + 0.0209824 x + 0.461384	10<x<80
MN19	-5.94262×10^{-7} x^3 - 0.0000150221 x^2 + 0.0160949 x + 0.413569	10<x<80
MNC15-20	-0.000143953 x^2 + 0.0253931 x + 0.237855	10<x<80
AD	7.20584×10^{-8} x^4 - 6.69377×10^{-6} x^3 + 0.0000867965 x^2 + 0.0193334 x + 0.802534	5<x<50
AMc	-1.07104×10^{-8} x^4 - 3.4898×10^{-7} x^3 - 0.000106212 x^2 + 0.0216289 x + 0.794626	5<x<50
Amg6	-1.51658×10^{-7} x^4 + 0.0000205272 x^3 - 0.00108171 x^2 + 0.039158 x + 0.698435	5<x<50
D16	3.45042×10^{-6} x^3 - 0.000346971 x^2 + 0.0206349 x + 0.825305	5<x<50
H18N9T	0.0000101639 x^3 - 0.00128108 x^2 + 0.0543317 x + 0.574559	5<x<40
HN78T	0.489311 log(x) - 0.173769	5<x<25
HN75MBTYU	-0.000133333 x^2 + 0.0348 x + 0.651333	5<x<25
VZH98	0.498286 log(x) - 0.189068	5<x<25
HN70YU	0.0000533333 x^3 - 0.00297143 x^2 + 0.0818095 x + 0.41	5<x<25
EI661	0.00002 x^3 - 0.00104286 x^2 + 0.0462857 x + 0.622	5<x<25

Table 3 presents the optimal smooth functions for the deformation rate coefficient (K_u). The variety of function types observed across different steel grades for K_u underscores the diverse strain rate sensitivities of various alloys. These findings provide crucial data for developing more accurate thermo-deformation models and optimizing metal forming processes across a wide range of operational conditions.

Table 3. Optimal smooth function representations of deformation rate coefficient (K_u) for conventional and low alloy steels.

Material	ku	Constraints ku
CO	$-0.0273224 x^3 - 0.315889 x^2 + 0.734447 x + 0.308733$	0.3<x<1
	$-0.000131648 x^4 + 0.00344765 x^3 - 0.0351374 x^2 + 0.198791 x + 0.53432$	1<x<10
	$0.0115714 x + 1.03$	10<x<40
M1	$-0.0535714 x^2 + 0.189286 x + 0.744$	0.2<x<1
	$-0.00187446 x^2 + 0.0426397 x + 0.840404$	1<x<10
	$0.00735714 x + 1.005$	10<x<40
AMF	$0.104167 x^3 - 0.223214 x^2 + 0.25119 x + 0.728$	0.2<x<1
	$-0.0000401639 x^4 + 0.000988265 x^3 - 0.00968176 x^2 + 0.0613683 x + 0.807759$	1<x<10
	$0.005 x + 0.99$	10<x<40
L62	$0.104167 x^3 - 0.276786 x^2 + 0.390476 x + 0.582$	0.2<x<1
	$0.0000901128 x^3 - 0.00446193 x^2 + 0.073557 x + 0.730588$	1<x<10
	$0.00892857 x + 1.025$	10<x<40
L68	$0.104167 x^3 - 0.223214 x^2 + 0.30119 x + 0.608$	0.2<x<1
	$0.000112399 x^3 - 0.00542615 x^2 + 0.0849627 x + 0.710484$	1<x<10
	$0.00764286 x + 1.065$	10<x<40
L70	$-0.125 x^2 + 0.325 x + 0.6$	0.2<x<1
	$-0.000011817 x^4 + 0.000587489 x^3 - 0.0106596 x^2 + 0.0985258 x + 0.711419$	1<x<10
	$0.180337 \log(x) + 0.683092$	10<x<40
L90	$-0.208333 x^3 + 0.375 x^2 - 0.0666667 x + 0.68$	0.2<x<1
	$-0.00355904 x^2 + 0.0744822 x + 0.711267$	1<x<10
	$0.180337 \log(x) + 0.683092$	10<x<40
NPA1	$-0.416667 x^3 + 0.696429 x^2 - 0.144048 x + 0.604$	0.2<x<1
	$-0.000148178 x^4 + 0.0036776 x^3 - 0.03475 x^2 + 0.183941 x + 0.589242$	1<x<10
	$0.180337 \log(x) + 0.733092$	10<x<40
NPAN	$-0.142857 x^2 + 0.381429 x + 0.55$	0.2<x<1
	$0.00015032 x^3 - 0.00579811 x^2 + 0.0848916 x + 0.711502$	1<x<10
	$0.00992857 x + 1.035$	10<x<40
NMZHMc28	$0.115 x + 0.707$	0.2<x<1
	$0.000193895 x^3 - 0.00574307 x^2 + 0.0715206 x + 0.755479$	1<x<10
	$0.115416 \log(x) + 0.820912$	10<x<40
NZHMc30	$0.104167 x^3 - 0.276786 x^2 + 0.390476 x + 0.582$	0.2<x<1
	$-0.0000787568 x^4 + 0.00181793 x^3 - 0.0172125 x^2 + 0.108306 x + 0.707683$	1<x<10
	$0.00921429 x + 1.015$	10<x<40
MN19	$-0.0535714 x^2 + 0.189286 x + 0.694$	0.2<x<1
	$-0.000144126 x^4 + 0.00330644 x^3 - 0.0278644 x^2 + 0.12949 x + 0.726114$	1<x<10
	$0.00557143 x + 1.05$	10<x<40
MNC15-20	$-0.125 x^2 + 0.325 x + 0.6$	0.2<x<1
	$0.0000764747 x^3 - 0.00402114 x^2 + 0.0723983 x + 0.731099$	1<x<10
	$0.0100714 x + 1.045$	10<x<40
AD	$0.0419668 \log(x) + 0.889973$	0.5<x<1
	$-0.000101755 x^3 + 0.000935775 x^2 + 0.017649 x + 0.871556$	1<x<10
	$4.77387\times10^{-6} x^2 + 0.00608794 x + 0.977839$	10<x<60

AMc	$0.0419668 \log(x) + 0.849973$	0.5<x<1
	$-0.00184508 x^2 + 0.0394447 x + 0.81083$	1<x<10
	$-0.0000184673 x^2 + 0.00626508 x + 0.960201$	10<x<60
Amg6	$0.1 x + 0.77$	0.5<x<1
	$-0.000998137 x^2 + 0.028651 x + 0.843734$	1<x<10
	$0.00652542 x + 0.965424$	10<x<60
D16	$0.1 x + 0.75$	0.5<x<1
	$-0.00175265 x^2 + 0.0359774 x + 0.81534$	1<x<10
	$-0.0000280151 x^2 + 0.0080892 x + 0.924523$	10<x<60
H18N9T	$-0.144096 x^2 + 0.305178 x + 0.698733$	0.1<x<1
	$-0.0000111566 x^4 + 0.0000661848 x^3 + 0.000157845 x^2 + 0.0188523 x + 0.841044$	1<x<10
	$-7.94171\times10^{-9} x^4 + 1.5059\times10^{-6} x^3 - 0.000105299 x^2 + 0.00713132 x + 0.938058$	10<x<100
HN78T	$0.235443 x^3 - 0.534267 x^2 + 0.52537 x + 0.613009$	0.1<x<1
	$-0.00199627 x^2 + 0.0393021 x + 0.805469$	1<x<10
	$-1.39413\times10^{-8} x^4 + 3.24731\times10^{-6} x^3 - 0.000276614 x^2 + 0.0132032 x + 0.892611$	10<x<100
HN75MBTYU	$-0.147034 x^2 + 0.317128 x + 0.640308$	0.1<x<1
	$0.000271368 x^3 - 0.00674768 x^2 + 0.0651013 x + 0.752394$	1<x<10
	$2.52049\times10^{-9} x^4 - 5.00244\times10^{-7} x^3 + 9.70172\times10^{-6} x^2 + 0.00501848 x + 0.949328$	10<x<100
VZH98	$-0.187446 x^2 + 0.406397 x + 0.602404$	0.1<x<1
	$0.000271368 x^3 - 0.00674768 x^2 + 0.0651013 x + 0.762394$	1<x<10
	$8.65483\times10^{-8} x^3 - 0.0000299114 x^2 + 0.00539654 x + 0.963707$	10<x<100
HN70YU	$-0.207423 x^4 + 0.390437 x^3 - 0.378331 x^2 + 0.391077 x + 0.614254$	0.1<x<1
	$0.0819543 \log(x) + 0.810602$	1<x<10
	$-4.01639\times10^{-9} x^4 + 9.88265\times10^{-7} x^3 - 0.0000968176 x^2 + 0.00713683 x + 0.937759$	10<x<100
EI661	$0.0731465 \log(x) + 0.84781$	0.1<x<1
	$0.000228822 x^3 - 0.00516461 x^2 + 0.0479025 x + 0.809049$	1<x<10
	$-0.0000215678 x^2 + 0.00549043 x + 0.947436$	10<x<100

4 Conclusion

The development of smooth functions for thermomechanical coefficients marks a significant advancement in materials science and engineering, providing a robust foundation for creating sophisticated mathematical models that accurately describe the complex interplay of thermal and deformation processes in metals. These functions enable the creation of advanced algorithms for optimal control of thermo-deformation processes, allowing for more precise manipulation of material properties during manufacturing and potentially leading to improved product quality and consistency.

The data and methodologies developed in this study are invaluable for the advancement of computer-aided design (CAD) and computer-aided control systems, enabling engineers to simulate and predict material behavior with unprecedented accuracy. This breakthrough streamlines the design process and reduces the need for costly physical prototypes. Furthermore, these smooth functions and interpolation methods serve as powerful tools for both researchers and educators, providing a solid framework for further investigations into material behavior under various conditions and offering students a more intuitive understanding of complex thermomechanical relationships. The findings of this research have immediate practical applications in various industries, including aerospace, automotive, and manufacturing, where the ability to accurately predict and control material behavior under different thermal and mechanical conditions can lead to optimized production processes, reduced material waste, and the development of more efficient and durable products.

While this study has made significant strides in the interpolation of thermomechanical coefficients for non-ferrous metals and high-alloy steels, it also opens up several avenues for future research, including expansion to other material classes, integration with machine learning algorithms for real-time process optimization, investigation of the relationship between microstructure evolution and thermomechanical coefficients, and development of user-friendly software tools for widespread industrial use.

In essence, this research not only provides a comprehensive set of tools for modeling thermomechanical properties but also lays the groundwork for future innovations in materials processing and manufacturing technologies, representing a crucial step towards more efficient, precise, and sustainable industrial processes.

This work was supported by the Ministry of Science and Higher Education of the Russian Federation (Grant No. 075-15-2022-1121).

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