Precise Localization of Single Rocket Debris Sonic Boom Sources Using TDOA and Particle Swarm Optimization

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Abstract. This paper addresses the precise localization of airborne sonic boom sources from rocket debris. A mathematical model utilizing Time Difference of Arrival (TDOA) technology is constructed to determine the exact location (longitude, latitude, altitude) and time of sonic boom occurrence. The model first defines the relative time difference between the shockwave reception at monitoring devices and reference devices, converting it into distance difference using the speed of sound. For distance calculation, the model employs the cosecant formula to compute the great circle distance between two points on the Earth's surface, incorporating a proportional correction method to estimate the elevation's impact on distance. The objective function is defined as minimizing the sum of squared deviations between predicted propagation time and actual observation time. Analysis of the four unknowns (three-dimensional coordinates and time) indicates that at least four monitoring devices are required for solution. Ultimately, the study employs a particle swarm optimization (PSO) algorithm to solve the multivariate optimization problem. Simulation results show that the objective function value decreases significantly during the initial phase and rapidly approaches 0, indicating the algorithm is converging toward a global optimum. The calculated sonic boom source coordinates and timing are: longitude 110.631°, latitude 27.145°, elevation 960.1 meters, time offset −8.944 seconds.

1. Introduction

With the continuous advancement of rocket technology, the accurate localization and rapid recovery of rocket debris have become critical issues requiring urgent solutions. This is not only vital for environmental safety but also crucial for subsequent scientific research. During a rocket's descent, debris crossing the speed of sound generates a sonic boom, which can be captured by ground-based vibration wave monitoring equipment [1-2]. By analyzing the time differences in shockwave signal reception across monitoring stations, triangulation principles can be applied to calculate the precise location and timing of the shockwave occurrence in

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the debris's flight path. This section therefore aims to establish a mathematical model for accurately determining the positional coordinates (longitude, latitude, altitude) and timing of a single debris object's shockwave event in flight, while analyzing the minimum number of monitoring stations required to solve this problem. Previous localization methods often lacked efficient global optimization techniques when addressing the multivariate nonlinear problem involving four unknowns: the three-dimensional coordinates of the sonic boom source and the time. The innovations in this section are as follows: First, a systematic mathematical model for debris sonic boom localization based on TDOA technology was constructed, integrating the semi-normal vector formula and altitude correction to precisely calculate the distance between debris and monitoring equipment. Second, through analysis of model parameters and equation quantities, it establishes that at least four monitoring devices are required to solve for the four unknowns; Third, it employs the particle swarm optimization (PSO) algorithm—a global optimization technique—to efficiently solve the nonlinear objective function, verifying its convergence[3]. The research approach in this section comprises three steps: First, constructing the TDOA positioning model by defining relative time differences and distance differences; second, calculating distances using the semi-polar equation and elevation correction; finally, defining the objective function and employing the PSO algorithm for solution and result analysis.

2. Time Difference of Arrival Localization

Assume the longitude, latitude, elevation, and occurrence time of the rocket wreckage sonic boom are lon, lat, Z, and t_0 respectively. Calculate the relative time difference:

For other devices, calculate the relative time difference between their reception of the sonic boom and the reference device:

$$\Delta t_i = t_i - t_0 \tag{1}$$

Where t_i is the time when the i-th device receives the sonic boom.

Convert time difference to distance difference: Use the speed of sound and time difference to calculate the distance difference from the reference device:

$$\Delta d_i = c \cdot \Delta t_i \tag{2}$$

This distance difference represents the distance difference of the sonic boom source relative to the reference device.

3. Haversine Formula for Calculating Latitude and Longitude Distance

The Haversine formula is a method used to calculate the great-circle distance between two points on the Earth's surface. This formula assumes the Earth is a perfect sphere and is suitable for approximating distances between two points [4-5].

Given two points with latitude and longitude coordinates (x_1, y_1) and (x_2, y_2), where x represents longitude and y represents latitude (usually in radians), the Haversine formula for calculating the distance d between these two points is:

$$d = 2r \cdot \arcsin\left(\sqrt{\sin^2\left(\frac{\Delta y}{2}\right) + \cos(y_1) \cdot \cos(y_2) \cdot \sin^2\left(\frac{\Delta x}{2}\right)}\right) \tag{3}$$

Where:

r is the average radius of the Earth, approximately 6371 km.

Δx is the difference in longitude between the two points, i.e., $\Delta x = x_2 - x_1$.

Δy is the difference in latitude between the two points, i.e., $\Delta y = y_2 - y_1$.

The arcsin function in the formula is the arcsine function, while the sin and cos functions are the sine and cosine functions respectively.

Note that when using the Haversine formula, all angles should be expressed in radians. If the original data is provided in degrees, it needs to be converted to radians before substituting into the formula. The conversion method is to multiply the degree value by $\frac{\pi}{180}$. For example, if a latitude is given in degrees, the conversion should be performed as follows before using the formula:

In this way, the Haversine formula can be used to calculate the great-circle distance between two points. Since elevation is also involved, adjustments to the Haversine formula are required when considering its impact on distance calculation. Changes in elevation can affect the straight-line distance between two points, especially when the elevation difference between the two points is significant. Here, since the elevation change is not particularly significant, a proportional correction method can be used to estimate the impact of elevation on distance. Correct the distance based on the elevation difference. If the elevations of the two points are h_1 and h_2 respectively, a simple proportional correction can be adopted:

$$d_{corrected} = d \cdot \sqrt{1 + \left(\frac{h_1 - h_2}{d}\right)^2} \tag{4}$$

The above solves the calculation of the distance difference of the sonic boom source relative to the reference device [6-7].

4. Objective Function Definition

The objective function aims to measure the deviation between the predicted propagation time and the actual observed time. The time at which each device receives the sound is key data. By minimizing the error between the predicted time and these data, the position and occurrence time of the sound source can be inferred more accurately. The mathematical model of the objective function is as follows:

$$\text{Objective} = \sum_{i=1}^{n} \left(\frac{S_i}{c} + t_0 - t_i\right)^2 \tag{5}$$

Where:

S_i represents the distance from the sonic boom point to the i-th observation device.

t_0 is the time when the sonic boom occurs, which is one of the unknown quantities to be estimated.

t_i is the actual time when the i-th device receives the sonic boom.

c is the speed of the shock wave, usually approximately 340 meters per second in air.

In the objective function, $\frac{s_i}{c}$ represents the sound propagation time from the sonic boom point to the device. The predicted time is $\frac{s_i}{c} + t_0$, which is the time it takes for the sound to propagate from the sonic boom occurrence to the i-th device. The objective function sums the squares of the time errors of all devices (n devices), aiming to find the optimal estimates of t_0 and S_i by minimizing this sum.

5. Particle Swarm Optimization Solution

The Particle Swarm Optimization (PSO) algorithm was initially developed by Kennedy and Eberhart through the analysis, modeling, and simulation of the collective behavior of birds, drawing on Heppner's biological model. Similar to most models in the study of collective behavior, its model assumes that in the initial stage, the bird flock is in a state of unordered flight with no goals or planned paths. When a bird flies back to its habitat, if the set habitat threshold is higher than continuing to follow the flock, all birds will independently fly to the habitat, forming another group. Birds determine their flight position and speed based on simple rules. When one bird leaves the flock and returns to the habitat, others will also fly to the habitat. Once a habitat is found, they will not leave, and the entire flock will stay there. The bird that first finds the habitat and stays will cause the surrounding flock to fly to its location, greatly increasing the probability that the entire flock finds the habitat. Such problems are similar to the method of finding the optimal solution for specific problems. Therefore, Kennedy and Eberhart modified Heppner's model to enable it to search the space and find the optimal solution [8-10].

The standard PSO algorithm is a global optimization algorithm that integrates the principles of "swarm" and "optimization" and searches for the optimal solution based on the fitness value of particles. The PSO algorithm retains the population-based global search strategy, treating each individual as a particle without weight or volume in an n-dimensional search space, flying at a certain speed in the search space. The flight speed is dynamically adjusted based on the individual's flight experience and the swarm's flight experience. In each iteration, each particle adjusts its flight speed and position according to the following equations:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_{1j}[p_{vj}(t) - x_{ij}(t)] + c_2 r_{2j}[p_{gj}(t) - x_{ij}(t)] \tag{6}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \tag{7}$$

Where:

j represents the j-th dimension of the particle.

i represents the i-th particle.

t represents the t-th generation.

c_1 and c_2 are acceleration constants.

$r_1 \sim U(0,1)$ and $r_2 \sim U(0,1)$ are two mutually independent random functions.

The steps for numerical solution based on PSO are as follows:

5.1 Initialize the particle swarm

Generate m particles, each representing an estimated value of the possible occurrence time t_0 of the sonic boom, i.e., t_{0j} where $j = 1,2,\ldots,m$.

5.2 Calculate the fitness of each particle

For each particle j, calculate its corresponding objective function value F_j = Objective (t_{0j}).

5.3 Determine the individual optimal and global optimal

For each particle, if its current fitness is better than the individual optimal fitness, update the individual optimal time $t_{0jBest} = t_{0j}$. Among all particles, find the particle with the minimum objective function value, and its corresponding time t_{0gBest} is taken as the current global optimal solution.

5.4 Update particle speed and position

For each particle j, update its speed v_{0j} and position t_{0j} based on the individual optimal and global optimal:

$$v_{0j}^{(k+1)} = wv_{0j}^{(k)} + c_1 r_{1j} \cdot \left(t_{0jBest} - t_{0j}^{(k)}\right) + c_2 \cdot r_{2j} \cdot \left(t_{0gBest} - t_{0j}^{(k)}\right) \tag{8}$$

$$t_{0j}^{(k+1)} = t_{0j}^{(k)} + v_{0j}^{(k+1)} \tag{9}$$

Where k represents the current number of iterations, w is the inertia weight, c_1 and c_2 are acceleration constants, and r_{1j} and r_{2j} are random numbers in the range [0,1].

5.5 Iteration and convergence check

Repeat steps 2-4 until the stopping condition is met, such as reaching the preset number of iterations or the improvement of the objective function being less than a certain threshold.

5.6 Output the optimal solution

Output the global optimal time t_{0gBest} as the estimated value of the sonic boom occurrence time. Flow chart of particle swarm optimization algorithm

6. Analysis of Required Number of Monitoring Devices

When discussing the problem of rocket wreckage sonic boom localization, a core issue is determining the number of ground monitoring devices required. To accurately locate the three-dimensional coordinates (longitude, latitude, elevation) of the sonic boom source and its occurrence time, sufficient equations are constructed through Time Difference of Arrival (TDOA) technology to solve the involved multi-variable optimization problem.

6.1 Parameter and equation requirements

Firstly, determining the sonic boom source requires solving four basic parameters: three are spatial coordinates (longitude, latitude, and elevation), and the fourth is the occurrence time of the sonic boom. These four unknowns need to be solved through corresponding equations. The time recorded by each monitoring device when receiving the sonic boom can provide important information about the distance from the sonic boom source to the device, which is converted into a mathematical equation about position and time.

6.2 Construction and independence of equations

Construct relative distance difference equations by calculating the time difference between the reception time of other devices and the sonic boom of the wreckage. Each device thus provides an independent equation relative to the wreckage. These equations are usually nonlinear in form, expressed as:

$$\sqrt{(x - x_i)^2 + \left(y - y_i\right)^2 + (z - z_i)^2} = c \times (t_i - t) \tag{10}$$

Where (x, y, z) represents the coordinates of the sonic boom source, t is the occurrence time of the sonic boom, (x_i, y_i, z_i) and t_i are the coordinates and reception time of the i-th device respectively, and c is the speed of sound.

6.3 Minimum requirement for the number of devices

From the perspective of the number of equations and the complexity of solving the problem, if there are only three monitoring devices, three equations can be constructed to solve the three spatial coordinates. However, since time is also an unknown quantity, three equations alone cannot solve four unknowns. Therefore, at least four monitoring devices are required, so that three distance difference equations based on time differences can be obtained, plus one device to determine the time reference, totaling four equations to solve four unknowns.

In summary, to accurately locate the position and time of the sonic boom source, at least four monitoring devices are required. Such a device configuration can not only meet the basic mathematical equation requirements but also increase the system's robustness to measurement errors and non-ideal environmental factors through redundant data, thereby improving the accuracy and reliability of localization.

7. Analysis of Planning and Solution Results

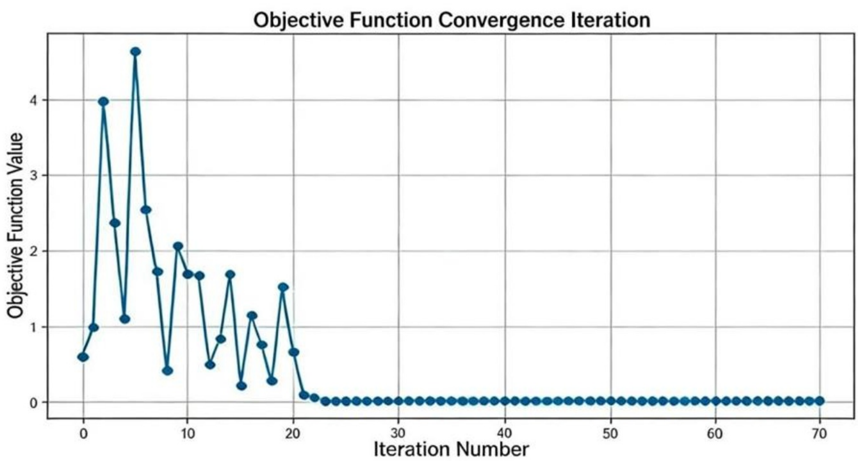

Fig. 1 Convergence iteration analysis of the objective function

The above figure 1 is an iteration chart of the objective function value during the model optimization process. The chart shows the change of the objective function value in each iteration. It can be seen that as the number of iterations increases, the objective function value gradually decreases, indicating that the optimization algorithm effectively approaches the minimum value of the problem. The stable downward trend shown in the figure reflects the stability and efficiency of the algorithm in finding the position and time of the sonic boom source. During the optimization process, the objective function value decreases significantly especially in the initial stage, and then gradually stabilizes. At this time, the function value is basically close to 0, indicating that the algorithm has approached the global optimal solution. Through the optimized model, the predicted position and relevant time information of the sonic boom source are obtained as follows:

Longitude: 110.631°

Latitude: 27.145°

Elevation: 960.1 meters

Time offset: -8.944 seconds

The predicted longitude and latitude indicate that the sonic boom source is located in the possible fall area of the rocket wreckage. The elevation of 960.1 meters may reflect the height of the wreckage when the sonic boom occurred or the average elevation of the terrain in the area. The time offset value of -8.944 seconds indicates that the actual occurrence time of the sonic boom is approximately 8.944 seconds earlier than the 0 hour of the observation system clock. A 3D graph is drawn with longitude, latitude, and elevation as the x, y, and z axes as shown in figure 2.

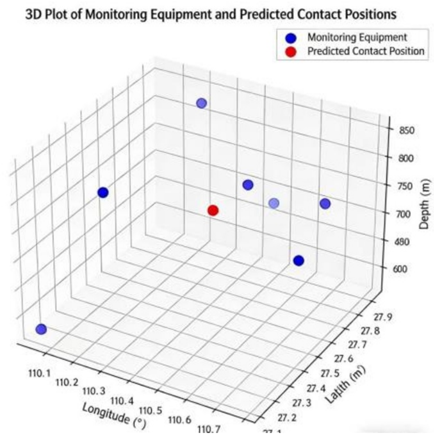

Figure 2 Three-dimensional diagram of monitoring devices and predicted wreckage positions

8. Conclusions

This section successfully established an accurate positioning model for a single rocket debris sonic boom source, effectively resolving the challenge of determining the location and timing of an individual debris sonic boom source. Based on TDOA technology, the model solves for four unknown parameters: the longitudinal, latitudinal, altitudinal, and temporal coordinates of the sonic boom source. For distance calculation, the model employs the cosecant formula to compute latitudinal and longitudinal distances, incorporating a proportional correction method to account for the influence of altitude on distance. Through analysis of the four unknowns of the sonic boom source, the study clearly indicates that at least four monitoring devices are required for localization, considering both the number of equations and the complexity of the problem. Ultimately, the Particle Swarm Optimization (PSO) algorithm was employed to solve the objective function aimed at minimizing the deviation between the predicted propagation time and the actual observation time. The optimization process exhibited stable convergence characteristics, with the objective function value ultimately approaching 0. The solution yielded the predicted location of the sonic boom source (longitude 110.631°, latitude 27.145°, elevation 960.1 m) and a time offset of −8.944 seconds.

8.1 Limitations of the Model and Future Research Prospects

Several aspects of the model's development and solution remain open for improvement. In distance calculations, the model employs the cosecant formula, which assumes the Earth is a perfect sphere. Additionally, a simple proportional correction method is used to account for elevation's impact on distance calculations. This simplification may compromise positioning accuracy in complex geographical environments or areas with significant elevation differences. Future research should focus on optimizing the distance calculation model to better accommodate complex terrain and pronounced elevation variations, thereby enhancing the accuracy and reliability of debris localization.

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